1. (15 pt) Suppose you are given a full color RGB image where each color component is represented by 8 bits. We would like to quantize the color values so that each pixel is represented by 8 bits only for all color components. Instead of quantizing R,G,B separately, given that we know the human eye is more sensitive to the luminance component than to the chrominance, we can convert the RGB values to YCbCr values and quantize the Y component to 4 bits, and Cb and Cr each to 2 bits. The RGB to YCbCr conversion is given below. Suppose a pixel has R,G,B values of R=100, G=200, B=50. What is the Y,Cb,Cr values before quantization, what are the quantized Y,Cb,Cr values? What are the corresponding quantized RGB values? Use uniform quantizers in the range of 0-256, for quantizing Y,Cb,Cr values.

\[
\begin{bmatrix}
Y \\
C_b \\
C_r
\end{bmatrix} =
\begin{bmatrix}
0.257 & 0.504 & 0.098 \\
-0.148 & -0.291 & 0.439 \\
0.439 & -0.368 & -0.071
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
+ \begin{bmatrix}
16 \\
128 \\
128
\end{bmatrix}
\]

2. (10pt) Suppose an image has a probability density function given in Fig. (a). We would like to modify it so that it has a probability density function given in Fig. (b). Derive the transformation function \( g(f) \) that will accomplish this. For simplicity, assume both the original image and the modified image can take on gray levels in the continuous range of (0,255).

![Fig. (a)](image1.png) ![Fig. (b)](image2.png)
3. (15pt) The histograms of three images are illustrated in Figs. (a) to (c). For each image, choose one of the three transformations given in Figs. (d) to (f) such that the transformed image has a nearly flat histogram. Explain briefly your reasoning.

4. (15pt) A 2D filter $H$ is given below, where the center position corresponds to $m=n=0$:

$$H = \frac{1}{4} \begin{bmatrix} -1 & -2 & -1 \\ -2 & 16 & -2 \\ -1 & -2 & -1 \end{bmatrix}$$

a) Is the filter separable? If so, give the one dimensional filters in horizontal and vertical directions.

b) If this filter is not separable, can you decompose it into an all pass filter plus another separable filter? If yes, write down the decomposed form.

c) Determine the DTFT $H(u,v)$ of the filter, and sketch the one dimensional profiles $H(u,0)$, $H(u,1/2)$, $H(0,v)$, and $H(1/2,v)$. You can either compute the DTFT directly or making use of the observations you made in part a) and b).

d) What is the function of this filter? Explain why.
5. (10 pt) Consider performing circular convolution in the 2D DFT domain. Suppose the image size and the DFT size are both \( N \times N \) and the filter mask in the DFT domain is as given below (also illustrated in the figure below):

\[
H(u, v) = \begin{cases} 
1, & u = 0, ..., K - 1 \text{ and } v = 0, ..., K - 1 \\
1, & u = 0, ..., K - 1 \text{ and } v = N - K, ..., N - 1 \\
1, & u = N - K, ..., N - 1 \text{ and } v = 0, ..., K - 1 \\
1, & u = N - K, ..., N - 1 \text{ and } v = N - K, ..., N - 1 \\
0, & \text{otherwise}
\end{cases}
\]

What is the corresponding spatial domain filter \( h(m,n) \)? Illustrate the filter in the horizontal direction (indicating locations of zero crossings if any).

Hint: both the frequency domain and spatial domain filter are separable. You just need to work out the filter in one dimension and then write the combined filter in 2D. Explain what is the function of this filter and what is impact of the parameter \( K \). What may be the dominating artifact in the resulting image?
6. (15 pt) Consider the following two ways of convolving an image \( f(m,n) \) of size \( N \times N \) with a filter \( h(m,n) \) of size \( K \times K \). Assume \( K \) is an odd number, \( K < N \), and the filter’s origin is at the center of the filter mask.

a. Do the 2D convolution directly and denote the resulting image by \( g(m,n) \). What is the size of the resulting image \( g(m,n) \)? What is the required number of multiplications? Assume the filter may not possess any symmetry.

b. Do the convolution through 2D DFT, i.e., \( z = \text{IDFT} \left( \text{DFT}(f) \cdot \text{DFT}(h) \right) \). The DFT and IDFT in the above equation are both \( N \times N \) points and are implemented using a 1D \( N \)-point FFT algorithm. What is the size of the resulting image \( z(m,n) \)? For what values of \( (m, n) \) does \( z(m, n) \) equal \( g(m, n) \)? What is the required number of multiplications? (assume an \( N \)-pt 1D FFT takes \( N \log_2 N \) multiplications)

c. For what value of \( K \) will method b) require less computation than method a)?

7. (20pt) (a) List the major steps needed for performing histogram equalization on an image. (b) Write a MATLAB function that will implement these steps. You should not use the MATLAB built-in function for histogram computation and histogram equalization. Write your program as a Matlab function of the following syntax: \( \text{outimg} = \text{equalize}(\text{inimg}) \), where \( \text{inimg} \) is a 2D array storing a gray-scale image, and \( \text{outimg} \) stores the equalized image. Assume the \( \text{inimg} \) takes integer values in the range of 0 to 255, and make the equalized image also take integer values in the same range.