1. Find the 9-point inverse DFT of $X^d(k) = [2, 2, 5, 2, 2, 2, 2, 2, 2]$, $0 \leq k \leq 8$.

2. Suppose $x$ is a 5-point sequence and $h$ is a 4 point sequence. Zero pad both $x$ and $h$ up to length 7. Take the DFT of each 7-point sequence and multiply them together, point by point, and then take the inverse DFT to get the 7-point sequence $y(n)$, $0 \leq n \leq 6$. Which values of $y(n)$ are the same as the linear convolution of $x$ and $h$?

3. The analog signal $x(t)$ is bandlimited to 60 Hz.

\[ X(f) = 0 \quad \text{for} \quad |f| > 60 \text{ Hz}. \]

The signal $x(t)$ is sampled with a sampling rate of 90 Hz and 500 samples are collected. You then zero pad the 500 samples with 100 zeros, and then take the DFT of the 600 values. Which DFT coefficients $X^d(k)$, $0 \leq k \leq 599$, are free of aliasing?

4. Shown below are six spectrum plots all obtained from the same signal $x(n)$, where two different numbers of samples are collected, with and without zero padding, with and without windowing.
These six plots were obtained with the following Matlab commands:

\[
\begin{align*}
N &= 30; \\
N2 &= 60; \\
x &= \cos(...) \\
\text{stem}([0:N-1]/N, \text{abs}(\text{fft}(x(1:N),N)),'.') \\
\text{stem}([0:N2-1]/N2, \text{abs}(\text{fft}(x(1:N),N2)),'.') \\
\text{stem}([0:N2-1]/N2, \text{abs}(\text{fft}(x(1:N2),N2)),'.') \\
\text{stem}([0:N-1]/N, \text{abs}(\text{fft(hanning(N)}.*x(1:N),N)),'.') \\
\text{stem}([0:N2-1]/N2, \text{abs}(\text{fft(hanning(N)}.*x(1:N),N2)),'.') \\
\text{stem}([0:N2-1]/N2, \text{abs}(\text{fft(hanning(N2)}.*x(1:N2),N2)),'.')
\end{align*}
\]

Match the Matlab commands to the graphs, in a one-to-one correspondence.
(Note: \texttt{fft(x,N)} zero pads \( x \) to length \( N \) if the length of \( x \) is less than \( N \), then performs the FFT.)
5. Consider a Type IV impulse response (with real-coefficients) of length 12. If $H(z)$ has zeros

$$z_1 = -3 + 2j, \quad z_2 = 0.5 + 0.5j, \quad z_3 = 0.75$$

find the remaining zeros of $H(z)$ and make a sketch of the zero-diagram.

6. Suppose $h(n)$ is a Type II FIR lowpass filter with the frequency response $H^f(\omega)$ fitting a template:

$$1 - \delta_p \leq |H^f(\omega)| \leq 1 + \delta_p \quad 0 \leq \omega \leq \omega_p$$

$$0 \leq |H^f(\omega)| \leq 1 + \delta_p \quad \omega_p \leq \omega \leq \omega_s$$

$$0 \leq |H^f(\omega)| \leq \delta_s \quad \omega_s \leq \omega \leq \pi.$$ 

(A) Draw this template in which the frequency response lies.

(B) Define the impulse response $g(n) = (-1)^n h(n)$. Which of the four linear-phase filter types is $g(n)$?

(C) Draw the template in which the frequency response $G^f(\omega)$ lies.

7. Backwards/forwards filtering. A technique to perform filtering with no phase distortion (i.e. linear-phase filtering) is the following. ($h(n)$ is real-coefficient filter, not nec. lin-phase.)

(1) Filter the data $x(n)$ with the filter $h(n)$.

(2) Reverse the filtered data; call it $g(n)$.

(3) Filter the data $g(n)$ with the filter $h(n)$.

(4) Reverse the resulting data again.

Show that this has the over all effect of filtering with a linear-phase filter. What is the total impulse response?