Midterm

- 3 single-sided pages of notes.
- Closed book.
- Calculators are allowed but probably unnecessary.
- Simplify your answers where applicable.

1. Let \( x(n) \), for \( 0 \leq n \leq N - 1 \), be a real \( N \)-point signal. The DFT coefficients are \( X(k) \), for \( 0 \leq k \leq N - 1 \).

   (a) Show that \( X(0) \) is a real number.
   (b) Assume \( N \) is an even number. Is \( X(N/2) \) real, imaginary, or a generic complex number? Show your explanation.

2. Consider the following 8-point signals, \( 0 \leq n \leq 7 \).
   (a) \([1, 1, 1, 0, 0, 0, 1, 1]\)
   (b) \([1, 1, 0, 0, 0, -1, -1]\)
   (c) \([0, 1, 1, 0, 0, -1, -1]\)
   (d) \([0, 1, 1, 0, 0, 1, 1]\)

   Which of these signals have a real-valued 8-point DFT? Which of these signals have a imaginary-valued 8-point DFT?

3. Which is likely to be faster?
   (a) A 96-point FFT,
   (b) or an 87-point FFT?

   Explain your answer.

4. An analog signal is bandlimited to 40 Hz.

   \[ X(f) = 0, \quad \text{for} \quad |f| > 40 \text{ Hz} \]

   The signal \( x(t) \) is sampled at 50 Hz for 2 seconds. Then these samples are zero padded with 50 zeros. Then the DFT of this zero padded sequence is computed. Which DFT values are free of aliasing?
5. An analog signal is bandlimited to 50 Hz.

\[ X(f) = 0, \quad \text{for} \quad |f| > 50 \text{ Hz} \]

You need to filter the signal with a lowpass filter with a cut-off frequency at 30 Hz. You will do this by sampling the analog signal and applying a digital lowpass filter. You will sample the signal with a sampling rate of 150 Hz. In terms of normalized frequency, what should be the cut-off frequency \( \omega_o \) of the digital filter? (0 < \( \omega_o \) < \( \pi \)).

6. Let \( h(n) \) be the impulse response of a type II filter.

Suppose the transfer function of another filter is given by \( F(z) = H(-z^2) \). What is \( f(n) \) in terms of \( h(n) \)? Is \( F(z) \) a linear-phase filter? If so, what is its type? (Explain)

7. The Savitsky-Golay FIR filters (or windows) are a family of lowpass digital filters obtained by minimizing an unweighted square error subject to side constraints.

For example, the following problem leads to a Savitsky-Golay filter when the desired response \( D(\omega) \) is set equal to the zero function; \( D(\omega) = 0 \).

Minimize

\[ \int_0^\pi (A(\omega) - D(\omega))^2 \, d\omega \quad (1) \]

subject to the side constraint

\[ A(0) = 1. \quad (2) \]

The frequency response amplitude of a length-21 Savitsky-Golay filter is shown in the following figure.

(a) How would you solve the constrained minimization problem stated in equations (1) and (2)? (Write down the equations you need to solve.)

Extra credit: Solve the equations and determine \( h(n) \).

(b) You can see that the stop-band ripples are rather large. By using only an error weighting function how could you reduce the sizes of the stopband ripples without changing the length of the filter? (Explain.) What happens to the other characteristics of the frequency response?
8. Several linear-phase FIR filters are shown in the following figure. Match each impulse response with its zero diagram and frequency response. Also classify each as Type I, II, III, IV.
9. The following figures show a signal and its spectrogram, computed with different sets of parameters,

\[ R \in \{30, 60\}, \quad N \in \{64, 256\} \]

where

- \( R = \) block length
- \( N = \) FFT length (\texttt{nfft} in the Matlab \texttt{specgram} function). (Each block is zero-padded to length \( N \).)

(a) For each of the spectrograms, indicate what you think \( R \) and \( N \) are, and explain your choices.

(b) Describe how the spectrogram would change if the time-skip (\( L \) in the notes), is increased.