1. What is the circular convolution of the following two sequences?

\[ x = [1, 2, 3, 4, 0, 0, 0] \]

\[ g = [0, 0, 1, -1, 1, 2, 0] \]

2. An analog signal is bandlimited to 208 Hz. You are to compute the value of its spectrum at the following frequencies,

\[ f_k = 5k \text{ Hz, } \quad 0 \leq k \leq 10. \]

You will do this by collecting \( N \) samples at a sampling rate of \( F_s \) Hz, and by taking the \( N \)-point DFT of these \( N \) samples.

(a) Choose minimal values for \( N \) and \( F_s \).

(b) Choose minimal valued for \( N \) and \( F_s \) under the constraint that \( N \) is a power of 2.

Note: You are not required to reconstruct the analog signal from its samples. The goal is only to evaluate the spectrum at the specified frequencies. Some aliasing is acceptable. The minimal value of \( F_s \) is not given by the sampling theorem.
3. Four linear-phase FIR filters are shown in the following figure. Match each impulse response with its zero diagram and frequency response. Also classify each as Type I, II, III, IV.
4. The following figures show four signals and their spectrograms (out of order). Match each signal to its spectrogram.
5. The following figures show a signal and its spectrogram, computed with four different sets of parameters,

\[ L \in \{1, 10\}, \quad N \in \{32, 256\} \]

where

\( L = \) time lapse between blocks.
\( N = \) FFT length (\texttt{nfft} in the Matlab \texttt{specgram} function). (Each block is zero-padded to length \( N \).)

In each case, the block length is 30 samples.

(a) For each of the four spectrograms, indicate what you think \( L \) and \( N \) are, and explain your choices.

(b) Describe how the spectrogram would change if the block length (\( R \) in the notes), is increased, or decreased.
6. The ideal discrete-time differentiator has the following frequency response amplitude,

\[ A(\omega) = \omega, \quad |\omega| < \pi, \]

and linear-phase.

(a) For a Type III linear-phase FIR differentiator filter, draw the amplitude response \( A(\omega) \) over the extended range \(-2\pi < \omega < 2\pi\). (Use properties of linear-phase FIR amplitude functions.)

(b) For a Type IV linear-phase FIR differentiator filter, draw the amplitude response \( A(\omega) \) over the extended range \(-2\pi < \omega < 2\pi\).

(c) In designing an FIR digital differentiator using the IRT method, what differences would you expect to see between Type III and Type IV solutions?

7. In the notes, for the design of Type I linear-phase FIR filters by DFT-based interpolation we had the formula

\[ h(n) = \text{DFT}^{-1}_N \left\{ A \left( \frac{2\pi}{N} k \right) \cdot W_{N}^{-Mk} \right\}. \]

(Recall that the filter length here is \( N = 2M + 1 \).) It turns out that if we leave out the term \( W_{N}^{-Mk} \) the method still works; we just have to modify the result in a simple way. Let

\[ b(n) = \text{DFT}^{-1}_N \left\{ A \left( \frac{2\pi}{N} k \right) \right\}. \]

(a) How is \( b(n) \) related to \( h(n) \)? (Use the DFT properties.)

(b) How can \( h(n) \) be simply obtained from \( b(n) \)?

8. Given the transfer function \( H(z) \), define a new transfer function \( G(z) \) by

\[ G(z) = H(z^2). \]

What linear-phase FIR filter Type is \( G(z) \) if ...

(a) \( H(z) \) is Type-I?

(b) \( H(z) \) is Type-II?

(c) \( H(z) \) is Type-III?

(d) \( H(z) \) is Type-IV?

For (a), suppose \( H(z) \) is a low-pass filter with cut-off frequency at \( \omega_o \). Draw the frequency response of \( G(z) \) and mark the relevant frequencies.