Dealing with User Heterogeneity in P2P Video Conferencing: Layered Coding Versus Receiver Partitioning

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Abstract—Although layered coding solutions elegantly handle bandwidth heterogeneity among users in video conferencing, they also incur significant rate and complexity overheads. An alternative is partitioning the receiver sets and using non-layered coding for each group. In this paper, we investigate how to maximize the received video quality for both systems under uplink and downlink capacity constraints. We first show that any multicast tree is equivalent to a collection of depth-1 and depth-2 trees, under inbound and outbound flow constraints. For the layered system, we propose an algorithm that simultaneously solves for the number of video layers, the rate and distribution tree of each layer. For the receiver partitioning system, we develop an algorithm for determining the receiver partitions and tree construction for each group. Through numerical comparison study, we show that the receiver partitioning system achieves significantly higher video quality than the layered system, because of its higher coding efficiency.

I. INTRODUCTION

Advancements in broadband technologies, along with novel video encoding methods, enabled Multi-Party Video Conferencing (MPVC) applications, such as Skype Group Calls [1], Google Plus Hangout [2] and ooVoo [3], to flourish on the Internet. Along with cloud video conferencing services recently proposed in [4], most existing MPVC solutions are server-centric [5], in which a user’s video is first sent to a server before being relayed to other receivers. Such a “backhaul” design totally ignores the network and geographic locality of users in the same conference. Users located far away from servers are forced to traverse long network paths with large delay and small bandwidth, leading to poor conferencing experience. Meanwhile, MPVC is built around realtime group interaction between users. The natural delivery solution is Peer-to-Peer (P2P), where users send their voice and video to each other directly. Several P2P MPVC solutions have recently been proposed [6], [7], [8], [9] to offload servers and exploit user locality to achieve low delay and high rate.

P2P MPVC, where multiple users multicast voice and video with intense bandwidth requirements in real-time, has to deal with the inherent heterogeneity among users in the same conference. Compared with a user sitting in front a desktop computer with high speed wireline connection, a user dialing in from a smartphone with 3/4G connection not only has lower uplink and downlink bandwidth, which limits how much video she can upload and download, but also has less computation power and energy supply, which imposes constraints on the video format she can encode and decode in realtime. To deal with peer heterogeneity, it is very important to design video generation and distribution in an integrated fashion. One solution is layered coding, where each source encodes multiple video layers using the recent layered video coding techniques [10], and receivers downloading more layers will receive better video quality. An alternative solution is receiver partitioning, where each source generates multiple video versions using the traditional single-layer video coding techniques, receivers are partitioned into different groups, with receivers in each group receive the same video version. In server-centric MPVC, layered coding is adopted by Google Plus Hangout, while Skype employs receiver partitioning [5]. For P2P MPVC, layered coding and receiver partitioning enable different P2P sharing opportunities. With layered coding, receivers receiving different subsets of layers can still share their common video layers. With receiver partitioning, only receivers watching the same version can share video with each others. Most proposed P2P MPVC solutions, e.g., [7], [8], chose layered coding over receiver partitioning to achieve higher P2P sharing efficiency. However, the flexibility of layered coding comes at the price of non-negligible rate overhead, that is, to achieve the same perceptual quality, layered coding has to use higher bit rate than non-layered coding [10]. Layered coding also has much higher encoding and decoding complexity, and consumes more CPU cycles and energy, which limits its adoption by mobile devices.

In this paper, we formally study the achievable performance by layered coding and receiver partitioning in P2P MPVC through analysis and numerical simulations. One salient feature of our study is that we investigate the interplay between video coding and P2P video distribution. We develop different distribution algorithms for layered system and receiver partition system. Instead of assuming an idealized layered coding scheme with zero overhead, as was done in most existing P2P MPVC studies, we consider realistic layered coding schemes with practical encoding overhead ratios obtained using the H.264/SVC codec, the latest layered coding standard [10]. To simplify P2P distribution design, most existing P2P MPVC studies assume that peer downlinks are never bottlenecks, and only focus on how to utilize peer upload bandwidth to maximize the system-wide video quality. Such an assumption is too crude to model user heterogeneity in MPVC, in which each user needs to download multiple streams, so the downlink may potentially become a bottleneck for some users, even if their uplink capacity is higher than their downlink capacity. This scenario becomes more severe with wireless/mobile users. Another salient feature of our study is that we consider both peer uplink and downlink bandwidth constraints in video generation and distribution. Our main contributions are summarized as the following:

- We first develop the general formulation for tree-based P2P MPVC distribution under peer uplink and downlink bandwidth constraints. We show that any distribution tree can be reduced to a collection of depth-1 and depth-2 trees, which greatly reduces the computational complexity...
for searching the optimal trees.

- For layered coding, we design an integrated video encoding and distribution algorithm that simultaneously solves for the number of video layers and the rate of each layer to be generated on each source, as well as the subset of layers each receiver should receive from each source.
- For receiver partitioning, we study the optimal receiver partitioning problem which partitions receivers of the same source to multiple groups and uses single-layer coding in each group. We propose a fast heuristic algorithm for determining the receiver partition.
- We compare the achievable video rate and quality with layered coding and receiver partitioning through numerical simulations of different number of users and user bandwidth profiles. In our simulations, layered coding can always achieve the optimal video rates. Somewhat surprisingly, in most simulated cases, our receiver partitioning heuristic can also achieve video rates very close to the theoretical bounds. Because layered coding incurs substantial rate overhead, the receiver partitioning system significantly outperforms the layered system in terms of video quality.

The rest of the paper is organized as follows. We briefly discuss the related work in Section II. The general formulation for P2P MPVC and the two-hop optimality result are presented in Section III. The integrated video encoding and distribution algorithm for layered coding is developed in Section IV. In Section V, we study the optimal partitioning problem with non-layered video coding. We then present a fast heuristic partition algorithm. Numerical simulation results comparing layered coding with receiver partitioning are reported in Section VI. The paper is concluded in Section VII.

II. RELATED WORK

The problem of optimal flow configuration in a P2P MPVC system has largely been explored under the assumption that only the uplink constraints of the peers present a bottleneck. [6] presents the optimality result for a special set of trees, often called the Mutualcast trees in the literature, in a single-rate and single-source setting. In [7], this work is extended to include multi-source case and distributed algorithms are presented to find optimal tree and video rates. The same authors then make a further extension in [8], where a multi-rate solution is considered with only layered video in uplink-throttled P2P networks without any downlink capacity constraints. [9] investigates a resource sharing solution among different MPVC swarms leading to a higher multiplexing gain. In [11], the solution is once again extended to focus on both bandwidth and delay aspects of MPVC design by reducing the set of distribution trees by removing trees with extensive delay and considering the underlay, as well as the overlay. Almost all of the prior studies assume P2P MPVC systems are constrained by peer uplink, and only focus on video distribution design without considering video coding overhead.

In contrast, we consider the problem of optimal flow configuration in a multi-source, multi-rate video conferencing scenario in a P2P network with both uplink and downlink capacity constraints. As a result, previous application layer flow design solutions are rendered either inapplicable or inefficient. We obtain new formulations regarding the dissemination of source videos with layered and single-layer coding, in order to consider the impact of video coding. Through numerical studies, we demonstrate that optimizing video distribution in P2P MPVC without considering video generation can be misleading.

III. PROBLEM STATEMENT AND MULTICAST TREES

A. General Formulation for P2P MPVC

We examine an MPVC scenario where each participant transmits its own video to all other participants. The end-users are connected through an overlay P2P network such as the one shown in Fig. 1, which is represented by a node-capacitated, complete, directed graph \( G = (N, A) \), where \( |N| = n \). Henceforth, we use the terms users, nodes, peers and participants interchangeably. Our assumption that each node maintains \( n - 1 \) connections with the rest of the nodes can be justified due to the small number \( n \) of participants in a typical MPVC session. Only the user uplink and downlink capacities create bottlenecks in the network, bounding the incoming and outgoing flows of a node. We label the users \( \{1, \ldots, n\} \) in ascending order of their download capacities so that node \( i \in N \) has upload and download capacities \( U_i \) and \( D_i \), respectively and the downlink capacities are ordered as \( D_1 \leq \cdots \leq D_n \). Each user concurrently hosts an application layer multicast session to distribute its own video sequence to others, i.e., the receiver set \( R_i \) for node \( i \) is \( N \setminus \{i\} \).

![Fig. 1. An example MPVC overlay, with given upload & download capacities.](image)

As for the video encoding methods, we consider layered and non-layered techniques. Non-layered encoding of a video sequence produces a single video stream, where the video chunks must be received entirely before decoding. Thus, all the users in a non-layered video multicast group receive the same video stream at the same bit rate. Unlike non-layered coding, layered video encoding [10] produces a bitstream that consists of multiple layers that can be decoded progressively. Video chunks consist of sub-chunks that correspond to different quality layers that can be decoded in a nested fashion, starting from the base layer and then the enhancement layers. Therefore, the users may receive the same video sequence at different rates. The cost of this flexibility is what is called the coding overhead of layered video, which is defined as the additional video rate needed to achieve the same quality as a non-layered coder. For example, to achieve the same video quality, the H.264/SVC coder, which is a layered coding standard, requires up to 30% higher video rate than the H.264/AVC coder, which is a single-layer coding standard (Fig. 5).

In order to multicast its video, peer \( i \) makes use of a number of directed distribution trees, which we denote generally by \( T \in T \). These multicast trees are rooted at peer \( i \) itself and spanning, in general, a subset of the receiver set \( R_i \). Packets originating from the sources are routed along these trees, where
nodes on the trees replicate the packets and send them to their downstream nodes. We denote the source node of a directed tree \( T \) by \( s(T) \) and its vertex set by \( V(T) \). We assume that the packet flow rate, denoted by \( x(T) \), is equal along all arcs of a given tree \( T \), since any tree with unequal flows on its arcs can be decomposed into sub-trees with equal flows. Thus, the total communication rate \( r_{ij} \) from user \( i \) to user \( j \) is simply the sum of the rates of trees that are rooted at node \( i \) and cover node \( j \),

\[
r_{ij} = \sum_{T: s(T) = i, j \in V(T)} x(T).
\]

Therefore, for an arbitrary set \( T \) of multicast trees and an arbitrary concave utility function \( f \) that measures the video quality at rate \( r \), a general application layer flow configuration problem can be formulated as

\[
\max_{\forall T \in T, \sum_{i \in N} \sum_{j \in B_i} f(r_{ij})} \sum_{i \in N} \sum_{j \in B_i} f(r_{ij})
\]

subject to

\[
\sum_{T \in T: i \in V(T)} c(i, T) x(T) \leq U_i, \quad \forall i \in N
\]

\[
\sum_{j : j \in B_i} r_{ij} \leq D_i, \quad \forall i \in N
\]

where \( r_{ij} \) is the rate of node \( i \)'s video that node \( j \) receives. \( c(i, T) \) is the number of children nodes that node \( i \) has on tree \( T \) and \( r_{ij} \) is given as in Eq(1). The notation used is summarized in Table I. For simplicity, we assume that the videos from all participants in a conference have similar characteristics and thus simply use \( f(r) \) to represent the quality-rate relation of such a video. The problem with this formulation is that the number of potential trees that can be considered is very large. Note that the following constraints always hold for any multicast tree set chosen in any P2P MPVC system.

\[
\sum_{(i,j)} r_{ij} \leq \sum_{m} U_m
\]

\[
\sum_{i} r_{ij} \leq D_j, \quad \forall j \in N
\]

\[
\max_{j} r_{ij} \leq U_i, \quad \forall i \in N
\]

Therefore, the region defined by these constraints presents, in general, a loose upper bound on the achievable video rates.

B. Optimal Multicast Trees

A crucial design problem for a P2P-based MPVC system is then to determine which trees should be used in a given node-capacitated complete graph. It was shown in [7] that employing the two types of trees in Figure 2, introduced in [6], is sufficient to maximize the throughput and the utility in a multi-source P2P scenario without helper nodes, under the assumption that the network is uplink-throttled. Hereafter, we call such trees Mutualcast (MC) trees. MC trees for node \( i \) are all rooted at \( i \) and consists of a 1-hop tree that reaches all \( j \in R_i \) and \( R_i \) 2-hop trees, each passing through a particular \( j \in R_i \) and then branching to the rest. In such an uplink-throttled setting, all receivers of a source node \( s \) receive the video at the same rate. However, to the best of our knowledge, there is no optimality result for any trees in an uplink- and downlink-throttled network. At this point, we present the following theorem, which shows that any given tree with flow \( f \) can be replaced by MC trees covering the same node set as before.

**Theorem 1.** Any node-capacitated, directed multicast tree \( T = (N \cup \{s\}, E) \) rooted at \( s \) can be replaced by 1-hop and 2-hop MC trees that are rooted at \( s \) and span \( N \cup \{s\} \), and the aggregate download and upload rate of each node in all the MC trees are exactly the same as in the original tree \( T \).

**Proof:** Let \( |N| = n \) and let the flow along \( T \) be equal to \( x \). We denote the number of outgoing branches at node \( j \) by \( b_j \), therefore node \( j \) is a leaf iff \( b_j = 0 \). Clearly, the total outgoing flow of \( j \) is \( k_j x \). Then, for any node \( j \neq s \) with \( k_j \neq 0 \), build a 2-hop MC tree rooted at \( s \) and going through \( j \) with a flow of \( k_j x / (n - 1) \) and a 1-hop MC tree with a flow of \( (k_s - 1)x / (n - 1) \). We are done if we can show that the total incoming and outgoing flows for each node is the same as before. Each node is now receiving

\[
r = \frac{x}{n - 1} \left( k_s - 1 + \sum_{j \in N} k_j \right)
\]

\[
= \frac{x}{n - 1} \left( k_s - 1 + n - k_s \right)
\]

(6) follows because for a tree, we have \( k_s + \sum_{j \in N} k_j = |E| = n \). In this new configuration, it is easy to see that the total outgoing and incoming flows of node \( j \) are still equal to \( k_j x \) and \( x \), respectively. Therefore, the same amounts of upload and download capacities are consumed and hence the equivalency with the old configuration.

**Fig. 2.** Depth-1 and depth-2 Mutualcast (MC) Trees

An illustration of the tree construction is given in Figure 3. As we can replace any tree with a combination of MC trees covering the same node set, we need merely consider MC trees covering different node sets. We emphasize this with a remark.
Table 1. Notation Regarding Formulation, Layered Video System and Receiver Partitioning System

Remark 1. In a node-capacitated, directed, complete graph \( G = (N, A) \), any feasible flow configuration achieved by a given set of trees, each of which spans a subset of \( N \), can also be achieved by a combination of 1-hop and 2-hop MC trees that cover the same subsets. Thus, to find the optimal set of multicast trees, it is sufficient to consider only MC trees.

IV. DESIGN OF LAYERED SYSTEM

In this section, we look into the problem of layered video distribution in a fully-connected P2P network with upload and download constraints. We first describe the multicast tree sets to be employed. Afterwards, we formulate the optimal flow configuration problem as a tree-packing problem with continuous rates. For this study, we assume that a video can be coded into an arbitrary number of layers and that each layer depends on the video stream structure. Specifically, for a layered video stream, these sets of nodes are, in fact, receiver sets of particular video layers. Denote the set of users that receive the \( l \)th layer of node \( i \)’s video by \( S_i^{(l)} \), usually referred to as subscribers of the \( l \)th layer in the literature. Thus, only the \( l \)th video layer is distributed through the multicast trees that span \( S_i^{(l)} \). Due to layered coding, \( S_i^{(l)} \) have to be nested, since the nodes need to receive all the layers up to \( l - 1 \) in order to decode the \( l \)th layer. As a result, all receivers subscribe to the first (base) layer. Thus, for user \( i \)’s video stream \( V_i \), we have \( S_i^{(l)} \subseteq S_i^{(l-1)} \subseteq \cdots \subseteq S_i^{(1)} = R_i \), where \( L_i \) is the number of video layers that user \( i \) generates. We do not assume that \( L_i \) is given, nor that it is bounded by source capabilities or user preferences. Rather, \( L_i \) is only bounded by the number of receivers \( R_i \) and will be determined with the following subscriber determination heuristic. For \( S_i^{(l)} \), nodes in \( R_i \) are sorted in ascending order of their total download capacities. Clearly, \( S_i^{(1)} = R_i \). Next, we remove the node(s) with the smallest total download capacity. The remaining nodes make up \( S_i^{(2)} \), i.e., they are receivers of layer 2. We proceed in this fashion until every node is removed. With this heuristic, the number of layers for source \( i \) equals the number of receivers for source \( i \), if all receivers have different downlink capacities. As an example, Figure 4 illustrates the selection of subscriber sets for node 3 in a video conference with 5 participants. Once the subscriber sets and the number of video layers are determined for source peer \( i \), each layer \( l \) of \( V_i \) is distributed with the help of \( |S_i^{(l)}| \) 2-hop trees \( \{ T_m^{(2)} : m \in S_i^{(l)} \} \) and the single 1-hop tree \( T_i^{(1)} \). All these trees allow the users to share their upload bandwidth with other users, increasing throughput and network utility. Let us define \( z_{il} \) as the rate of layer \( l \) of user \( i \)’s video \( V_i \). Then,

\[
z_{il} = x_{iil} + \sum_{j \in S_i^{(l)}} x_{ijl}
\]

Finally, if \( b_{il} \) is the upload bandwidth that user \( i \) requires to drive its own layer \( l \) distribution trees into \( S_i^{(l)} \), we have

\[
b_{il} = |S_i^{(l)}| x_{iil} + \sum_{j \in S_i^{(l)}} x_{ijl} = \left( |S_i^{(l)}| - 1 \right) x_{iil} + z_{il}.
\]

B. Problem Formulation

In this section, we are finally ready to formulate the multi-source, multi-rate flow optimization problem for layered videos, given the layer subscriber sets for each video source.

subject to

\[
\max_{x_{ijk} \geq 0} \sum_{i \in N} \sum_{j \in R_i} Q_{LV} (r_{ij})
\]

(11)

(12a)

(12b)

(12c)
it for helping the other sources for which itself is a receiver. The objective function $Q_{LV}$ given in (11) is a non-decreasing, concave function of $r_{ij}$. Furthermore, feasible region defined by inequalities (12a)-(12c), (8) and (10) is a convex polytope

in tree variables. Using Eq. (1), we can introduce the video rate

by inequalities (12a)-(12c), (8) and (10) is a convex polytope

the polytope onto the

$r_{ij}$ coordinates. Projection preserves convexity, therefore the achievable rate region is also convex.

As a result, the optimization problem in (11)-(12) is a non-

strictly concave optimization problem in the tree rate variables and has an infinite number of solutions. However, if there exists an interval $I$ such that $Q_{LV}$ is strictly concave in $I$ and the optimal video rates lie in $I$, then they are unique, hence the layer rates $z_{ik}$ are also unique. Centralized solution techniques for such concave optimization problems have been well-understood and hence, any one of these solution methods can be employed to find a solution.

In this formulation, the number of variables (number of trees employed) is $O(n^3)$ in the worst case. However, employing a large number of multicast trees could lead to increased jitter in a practical implementation. Therefore, after finding the optimal vector of video rates $r^*$ that maximizes the network-wide video quality, it is of interest to find a configuration of tree rates $x_{ijk}$ that achieves $r^*$ and favors 1-hop trees instead of 2-hop trees, as the packets that are distributed through 1-hop trees suffer less end-to-end delay. In Algorithm 1, we present a method to find a set of feasible tree rates that satisfies the constraints and achieves a given vector of video rates $r_{ij}$, while favoring 1-hop trees over 2-hop trees. Algorithm terminates if the given video rates are not feasible. Note that this algorithm is meant to be used only after the optimal video rates are known by solving (11)-(12). The main idea behind this algorithm is to maximize the rates of 1-hop trees greedily, starting from the base layer. For each peer $i$, given the layer rates $z_{ik}$, we calculate the maximum rate that a 1-hop tree can handle to subject to

$$\sum_{k=1}^{L_i} b_{ik} = \sum_{k=1}^{L_i} \left( (|S_k^{(i)}| - 1) x_{ik} + z_{ik} \right) \leq U_i. \quad (13)$$

After this step, there exists at least one peer that still has excess upload capacity, otherwise the given video rates are infeasible, as we would have $\sum_{(i,j)} r_{ij} > \sum_{i \in N} U_i$. Then, in order to fill the remaining rate gaps, 2-hop trees are constructed that pass through the peers with excess upload capacities. Rates of 2-hop trees are proportional to the upload capacities of the peers that they pass through. It is known that layered video encoding methods present a higher computational complexity than their non-layered counterparts, which might be limiting for mobile devices with computation and power constraints. Furthermore, the use of layered coding has the disadvantage of the **coding overhead**, which will be discussed in the next section. Therefore, we now turn our attention to MPVC systems where the users employ non-layered video encoding techniques, with the hopes of finding an alternative that overcomes these problems.

V. DESIGN OF RECEIVER PARTITIONING SYSTEM

Although layered coding enables generating a flexible stream that offers variable qualities depending on the rate, it is known that the cost of such flexibility is an increased bit rate to achieve a certain quality, which is referred to as the **coding overhead** of layered encoding. As an example, Fig. 5 presents the normalized subjective quality vs. bit rate curves of the Crew video sequence obtained by using H.264/AVC (non-layered) and H.264/SVC (layered) standards, respectively. Coding overhead of layered encoding (up to 30% at some rates), along with its relatively higher computational complexity, motivates the use of non-layered video in MPVC systems.

Clearly, multicasting the same non-layered video to all receivers is suboptimal, starving the receivers with higher download capacities. In order to obtain a multi-rate solution, a source can generate multiple video versions and send different video versions to different users at different rates, matching their download capacities. The drawback of this method in terms of bandwidth is that the source may not have sufficient upload capacity to send out different streams in the first place. Accordingly, we propose creating receiver partitions in each of the $n$ multicast sessions, where the nodes in each group within a partition can share their upload bandwidth using MC trees rooted at the source of the session and spanning all the nodes in the group. Intuitively, this method is able to find a compromise between a video session in which 2-hop MC trees are employed but the video rates are bounded by the minimum download capacity and another video session where the video is distributed at different rates but only through multiple unicast connections, equivalent to 1-hop trees.

A. Problem Formulation

Now, let $R_i$ be partitioned such that the groups in the partition are denoted by $G_k^{(i)}$ and $P_i = \{G_k^{(i)}, k = 1, \ldots, K_i\}$ is the partition with $\bigcup_k G_k^{(i)} = R_i$. Each node $j$ in a given group $G_k^{(i)}$ receives the video at the same group rate $g_k^{(i)} = r_{ij}$, but the nodes in different groups have different rates. Hence, the users with higher download capacities can receive more, resulting in a higher average video quality. Now, assuming that we are given a specific collection $P = \{P_i : i \in N\} \subset \mathcal{P}$, where $P_i$ is the partition of receivers of source $i$, we can formulate the multi-source, multi-rate video quality maximization problem with non-layered encoding as,

$$\max_{u_{ij}, b_{ik}^{(i)}, g_k^{(i)}} \sum_{i \in N} \sum_{k=1}^{K_i} |G_k^{(i)}| Q_{NL}(g_k^{(i)}) \quad (14)$$
subject to
\[
\begin{align*}
g_k^{(i)} & \leq b_k^{(i)}, \quad \forall i \in N, k = 1, \ldots, K_i \quad (15a) \\
|G_k^{(i)}|g_k^{(i)} & \leq b_k^{(i)} + \sum_{j \in G_k^{(i)}} u_{ji}, \quad \forall i \in N, \forall k \quad (15b) \\
\sum_{k=1}^{K_i} b_k^{(i)} & \leq u_{ii}, \quad \forall i \in N, k = 1, \ldots, K_i \quad (15c) \\
\sum_{j=1}^{n} u_{ij} & \leq U_i, \quad \forall i \in N \quad (15d) \\
\sum_{j : i \in G_k^{(j)}} g_k^{(j)} & \leq D_i, \quad \forall i \in N. \quad (15e)
\end{align*}
\]

Here, \(b_k^{(i)}\) and \(u_{ij}\) denote the portions of the total upload capacity of node \(i\) that is allocated for use in its own multicast group \(G_k^{(i)}\) and in \(G_k^{(j)}\), where \(i \in G_k^{(i)}\), respectively. Again, the objective function \(Q_{NL}\) in (14) is a non-decreasing, concave function of the video rate and the feasible region defined by (15) is convex. Hence, the formulated problem above is a non-strictly concave optimization problem with linear constraints. Similar to (11), it has a unique solution in the group rates \(g_k^{(i)*}\), assuming the optimal solution lies where \(Q_{NL}\) is strictly concave, whereas \(u_{ij}\) and \(g_k^{(i)*}\) are not unique. The number of variables, which depends on \(P\), is \(O(n^2)\) in the worst case. Determination of the MC tree rates is straightforward once we have \(u_{ij}\) and \(b_k^{(i)*}\); the 2-hop multicast tree rooted at node \(i\) and passing through node \(j \in G_k^{(i)}\) has rate \(x_{ij} = u_{ji}/(|G_k^{(i)}| - 1)\) and for the 1-hop tree we have \(x_{ii} = (b_k^{(i)*} - \sum_{j \in G_k^{(i)}} x_{ij})/|G_k^{(i)}|\).

The difficulty with employing non-layered coding in MPVC systems is that we do not readily know the optimal collection \(P^*\) of receiver partitions for each \(i \in N\). The size \(|P|\) of the set of all possible receiver partition collections is given by \(B_{n-1}^m\), where \(n\) is the number of participants and \(B_m\) is the \(m^{th}\) Bell number, equal to the number of ways a set of cardinality \(m\) can be partitioned. Therefore, exhaustively searching among all possible collections of partitions is hopeless even for a small number of users\(^1\). In order to overcome this difficulty, we now propose a simple heuristic algorithm to find a suitable collection of receiver partitions along with the group rates that can be achieved.

\[\text{B. Heuristic Algorithm for Partitioning}\]

The main idea behind the heuristic is to shrink the search space by decomposing the problem of finding the best collection \(P^*\) of partitions into separate problems of finding the best partition \(P^*_i\) for each source \(i \in N\). We start our analysis by assuming that a set of target video rates \(\{r_{ij} : \forall (i, j), i \neq j\}\) is given. Let us define the total rate needed to multicast source \(i\)'s video as \(M_i = \sum_{j \in R_i} r_{ij}\). Note that the benefit of using 2-hop multicast trees is that a peer \(i\) can still sustain a video session with total multicast rate \(M_i\), greater than its own upload bandwidth \(U_i\), by exploiting the other peers with abundant upload bandwidths. In this case, additional bandwidth required to drive peer \(i\)'s video session would simply be \(S_i = M_i - U_i > 0\), which can also be thought of as the net bandwidth shift into user \(i\)'s video session. Let

\[\text{us define the set } \epsilon = \{i \in N : S_i > 0\}\]

and call such peers that require additional bandwidth \(\epsilon\)-peers. On the other hand, if there is an \(\epsilon\)-peer in the system, then there is another peer \(j\) with \(U_j > M_j\) or \(S_j < 0\), otherwise we would have \(\sum_{m \in N} U_m < \sum_{(i, j)} r_{ij}\). Let us then call such peers, which provide additional bandwidth, \(\alpha\)-peers and define the set \(\alpha = \{i \in N : S_i < 0\}\). Clearly, it is sufficient for each \(\alpha\)-peer to directly send its video to each of its receivers at rate \(r_{ij}\) through direct one-hop transmission (unicast).

As hinted above, determination of the \(\alpha\)- and \(\epsilon\)-peers, as well as the bandwidth shifts between them, is critical in order to find a good solution. Let \(s_{ij}\) denote the amount of bandwidth provided by node \(i\) to node \(j\). In our heuristic algorithm, we only allow bandwidth shifts to occur from \(\alpha\)-peers to \(\epsilon\)-peers, and the optimal values of these are estimated through the solution of the following optimization problem.

\[
\max_{r_{ij} \geq 0} \sum_{(i,j)} Q_{NL}(r_{ij}) \quad (16)
\]

subject to constraints given in (4).

It is necessary for any feasible \(\{r_{ij}\}\) to satisfy the constraints in (4). Therefore, the optimal solution of (16) under constraints defined in (4) gives also a quality upper bound for any achievable video rate under the given upload and download constraints. Note that this problem can be easily solved using a simple water-filling algorithm: each source sends out equal flows to each receiver, while gradually increasing the flows at the same pace, until either all peers are downlink-saturated or there is no more upload bandwidth. We then calculate \(S_i = M_i - U_i\) for each \(i \in N\) and classify the peers accordingly. Each \(\alpha\)-peer \(i\) offers its own \(\sum_{j \in \alpha} S_j\) units of bandwidth, whereas each \(\epsilon\)-peer \(j\) requires \(S_j\) additional units of bandwidth to support its total multicast rate \(M_j\). The algorithm distributes the bandwidth provided by the \(\alpha\)-peer \(i\) to \(\epsilon\)-peer \(j\) proportionally. So, we have

\[
s_{ij} = \begin{cases} 
S_j, & \text{if } i \in \alpha \text{ and } j \in \epsilon \\
0, & \text{otherwise}
\end{cases} \quad (17)
\]

Next, we set limits on the total rate that a peer is allowed to receive in a particular video session. Specifically, we assume that the download capacity of a peer is equally divided between the remaining video sessions. As a result, from the point of view of source \(i\), a receiving peer \(j\) has a download capacity of \(D_j/(n-1)\). Now, all that remains is to find a suitable receiver partition for each source. For any source \(i\), this is performed by searching through only the ordered partitions, starting with the single-group partition \(P_i = \{R_i\}\) that includes all the receivers. Here, a receiver partition \(P_i = \{G_k^{(i)}, k = 1, \ldots, K_i\}\) is ordered if we have \(D_k \leq D_{k'}\) for all \(k, k' \in G_k^{(i)}\) and \(\ell < \ell'\). The remainder of our heuristic can be regarded as a distributed algorithm, as each user \(i\) performs a search to find a suitable receiver partition on \(R_i\). A steepest-ascent hill climbing method is employed by each user \(i\) to search for a local maximum by examining the neighboring ordered partitions. We consider two partitions as neighbors if only if the number of groups they contain differs by at most one. At each step of the greedy search, only the neighbor partitions containing one more group are examined. More specifically, for each candidate partition \(P_i\), peer \(i\) solves the following

---

\(^1\)For \(n = 5\), \(|P| = 759375\) and for \(n = 6\), \(|P| = 1.97 \times 10^{10}\)
optimization problem.

$$\max_{g_k^{(i)} \geq 0} \sum_{k=1}^{K_i} G_k^{(i)} |QNL(g_k^{(i)}) \tag{18}$$

subject to

$$g_k^{(i)} \leq b_k^{(i)}, \quad k = \{1, \ldots, K_i\} \tag{19a}$$

$$G_k^{(i)} |g_k^{(i)} \leq \begin{cases} b_k^{(i)} + \sum_{j \in G_k^{(i)}} s'_{ji} & \text{if } |G_k^{(i)}| > 1 \[12pt] b_k^{(i)} & \text{if } |G_k^{(i)}| = 1 \end{cases} \tag{19b}$$

$$\sum_{k=1}^{K_i} b_k^{(i)} \leq U^{(eff)}_i \tag{19c}$$

$$s'_{ji} \leq s_{ji}, \quad \forall j \in R_i \tag{19d}$$

Here, $U^{(eff)}_i$ is the effective upload capacity that node $i$ is allowed to use. Clearly, if node $i$ is an $\alpha$-peer, $U^{(eff)}_i = M_i$, otherwise we have $U^{(eff)}_i = U_i$. As before, $b_k^{(i)}$ is the portion of the effective upload capacity of user $i$ that is allocated to group $G_k^{(i)}$, and $s'_{ji}$ is the bandwidth provided to node $i$ by node $j$, bounded by $s_{ji}$. If there is no $\alpha$-peer in the group, then $s_{ji}$ is necessarily zero, since we only allow the $\alpha$-peers to provide bandwidth for the video distribution. After the examination of all the ordered neighboring partitions, the one that yields the highest average session quality $Q(P_i)$ is selected as the new local maximum candidate. The algorithm stops when there is no neighbor partition that yields a higher average session quality. Note that the distributed phase of the whole process, which is summarized in the pseudocode of Algorithm 2, can be easily implemented in any device by using any convex optimization algorithm.

**Algorithm 2** Receiver partition selection heuristic

1: Find the solution of (16)-(4) \textit{▷ Initialization}
2: Calculate $S_i = M_i - U_i$, for $\forall i \in N$
3: Create matrix $[s_{ij}]$ according to Eq.(17)
4: for all $i \in N$ do \textit{▷ Distributed phase}
5: $P_i^{(best)} \leftarrow \{R_i\}$, $Q_i^{(best)} \leftarrow Q(\{R_i\})$
6: repeat
7: $P_i^{(current)} \leftarrow P_i^{(best)}$, $Q_i^{(current)} \leftarrow Q_i^{(best)}$
8: for all $P_i \in \text{neighbors}(P_i^{(current)})$ do
9: \quad Find the solution of (18)-(19)
10: if $Q(P_i) > Q(P_i^{(current)})$ then
11: \quad $P_i^{(best)} \leftarrow P_i$, $Q_i^{(best)} \leftarrow Q_i(P_i)$
12: end if
13: end for
14: until $P_i^{(current)} = P_i^{(best)}$
15: end for

**VI. SIMULATION RESULTS**

In this section, we numerically evaluate the capacity regions achievable through layered coding and receiver partitioning. It is worth restating that our focus is completely on maximizing the video quality of P2P MPVC under two different design choices and we do not explicitly minimize delay. However, end-to-end packet delay in our overlay is controlled since a packet goes through at most 2 hops. In our simulations, we consider layered video distribution scheme, given as the solution of (11)-(12) in Section IV, receiver partitioning video distribution scheme, given as the solution of (14)-(15) with optimal partitions in Section V and the fast heuristic algorithm presented in Algorithm 2 in Section V-B, along with the simpler but no less important schemes of multiple unicast and single-rate multicast. For the clarity of comparison, we will assume that the ratio of the download capacity of a peer to its upload capacity is the same for all peers in the video conference, that is, $w = D_i/U_i$, for all $i \in N$. We first examine a small video conference scenario with only 4 participants under different bandwidth heterogeneity conditions, while keeping the average bandwidth constant. Then, we turn to larger scenarios where the participants are randomly chosen from different user classes with different bandwidth capabilities. In all our simulations, we assume the network is static within the time needed to perform the rate optimization. Participants encode their videos according to H.264/AVC (non-layered encoding) and H.264/SVC (layered encoding) standards. In video conferences, users’ video sequences are likely to have similar features, therefore we associate the same video quality-function with each user. Specifically, in our simulations, we use the following normalized subjective quality model presented in [12]

$$Q(r) = \frac{1 - e^{-\kappa r}}{1 - e^{-\kappa r_{\max}}}, \quad (20)$$

where $r$ is the received video rate, and $\kappa$ and $r_{\max}$ are parameters that depend on the video characteristics and layer configuration. $r_{\max}$ is the video rate needed to code the video at the highest quality (achieved at the highest spatial, temporal, and amplitude resolutions considered). As an example, subjective quality of the Crew video sequence with respect to the bit rate for both H.264/AVC and H.264/SVC encodings can be seen in Figure 5. The sequence is encoded at 5 temporal, 4 quantization and 3 spatial resolutions. For a given rate, the optimal spatial, temporal and amplitude resolutions that maximize the perceptual quality are chosen. For this example, the quality-rate model has the following parameters: $\kappa_{SVC} = 3.121$, $\kappa_{AVC} = 3.4$, $r_{\max}^{AVC} = 2069$ kbps and $r_{\max}^{SVC} = 3515$ kbps.

In the small video conference scenario, we focus on the impact of bandwidth heterogeneity on the average and minimum video qualities achieved in the system. Since the size of the conference is small, we are able to include the optimal receiver partitioning scheme in our comparison. We consider two different upload bandwidth profiles; to establish large and small bandwidth heterogeneity, we set the peer upload bandwidth profiles as $\{0.5, 2, 4, 5.5\}$ Mbps, and $\{1.5, 3, 3, 4.5\}$ Mbps, respectively. The results in Figure 6 and 7 are obtained for relatively more and less heterogeneous upload capacities,
respectively. In all figures, the horizontal axis is the down-load/upload ratio, $w$.

We can observe, for both upload capacity profiles, that the video qualities increase and then stay constant after some value of $w$, as the downlinks cease to be the bandwidth bottleneck and the performance of each scheme is only determined by the uplink constraints. In this regime, video rates achieved by the layered video distribution is equal to those of the non-layered video distribution, that is, the enhancement layer trees are not used and the video from each source is transmitted at a single rate. Nevertheless, SVC dissemination achieves a lower average video quality, due to the coding overhead. In order to show the effect of the coding overhead on the video qualities, we also show the performance of layered video distribution without any coding overhead, which, in all cases, results in the best quality, although with a small difference. Furthermore, we observe that the optimal receiver partitioning strategy with non-layered video distribution is almost as good as layered video distribution without coding overhead. This result shows that, in MPVC systems where the downlinks and uplinks may both present bottlenecks, we can obtain a multi-rate solution by using optimal receiver partitioning and non-layered video without any significant performance loss in terms of the average or minimum video quality, compared even with an ideal layered video distribution scheme with no overhead. Note that the optimal partitioning is done by exhaustively searching among all partition collections for given upload and download profiles. Instead of this computationally demanding task, we see that the proposed heuristic partition algorithm yields a solution that is very close to the optimal partitioning and rate configurations. Finally, we can see that unicast and single-rate distribution schemes perform poorly in face of heterogeneity. For multiple unicast, this is because no 2-hop multicast trees are employed to shift bandwidth from α-peers to help video sessions of ϵ-peers. For the latter, all peers, except for the one with the minimum download capacity, are starved. As expected, as the users become more homogeneous, the performance of the single rate and multi-unicast schemes become more competitive.

Next, we investigate a set of larger video conferences with more peers. Due to its complexity, we exclude the optimal partitioning scheme from each of these simulations. We assume that the end-users can be categorized into 4 different user classes with respect to their upload bandwidth. The considered upload capacities for these different classes are 500, 3500, 6500 and 9500 Kbps. The download capacity for each class can then be calculated for different $w$ ratios. Then, we randomly pick users out of these classes with a uniform distribution and generate the average quality curves for $0.1 \leq w \leq 5$. Figures 8 and 9 depict how the average quality and the average video rate change with respect to $w$. For each $w$, the results are obtained by averaging over 50 randomly selected bandwidth profiles with 6 peers. More specifically, on the left hand side of Figure 8, the average quality performances are shown, whereas on the right, the average quality curves normalized with respect to the performance of the maximum bound solution obtained in (16) are shown. Among all solutions, the proposed partition heuristic comes the closest to the maximum bound in terms of the achieved video quality, although its average video rate falls below the theoretical bound for the average rate, as seen in Figure 9. This rate degradation happens around $w = 1$, where the total download capacity is equal to the total upload capacity. Moreover, although the average rates for all schemes are close to the theoretical maximum bound (except for the single-rate method), the delivered video qualities differ significantly. Especially for layered coding, the achieved rate is as high as the bound, but again the achieved quality is discounted by the coding overhead.

Finally, we examine how the performance of the distribution schemes change with the number of peers in the conference. We make use of the 4 user classes again. However, instead of picking the users from these classes randomly, we pick the same number of peers from each class. As for the number of participants, we try $n = 4, 8, 12, 16, 20$ and show the absolute and relative performance curves with respect to $n$ for each $w = 1, 2, 3$. In Figure 10, the quality curves are shown. Since the number of peers selected from any class is the same for all classes, the total upload bandwidth in the system increases linearly with $n$, keeping the average peer upload bandwidth constant. However, since the number of connections is $n(n - 1)$, the average video rate from a source to its receiver decreases with $n$. We can observe the effect of
We have designed an integrated video encoding and distribution system for the layered and receiver partitioning algorithms for the layered system. For the receiver partitioning algorithm, we have formulated the optimal receiver partitioning problem and proposed a simple partitioning algorithm. Our simulations show that the video rates in both systems are very close to the theoretical bounds, but the receiver partitioning system can achieve significantly higher video quality than the layered system, because of the higher coding efficiency of non-layered coding. Leveraging on our theoretic study here, we are developing distributed P2P MPVC protocols for both layered coding and receiver partitioning. The main challenges are adapting to time-varying peer uplink and downlink bandwidth, real-time recovery from packet losses, and control end-to-end video delays experienced by users.

VII. CONCLUSIONS AND FUTURE WORK

In P2P MPVC systems, using layered coding is the “go-to” method to deal with peer bandwidth heterogeneity. However, it is well-known that layered coders incur significant rate and complexity overheads. Alternatively, one can partition receivers of the same source to multiple groups and distribute single-layer video in each one. In this paper, we have investigated the problem of video quality maximization in P2P MPVC systems for the layered and receiver partitioning systems, under both uplink and downlink capacity constraints. We have shown that any distribution tree can be reduced to a collection of depth-1 and depth-2 trees. Leveraging on this, we have designed an integrated video encoding and distribution algorithm for the layered system. For the receiver partitioning approach, we have formulated the optimal receiver partitioning problem and proposed a simple partitioning algorithm. Our simulations show that the video rates in both systems are very close to the theoretical bounds, but the receiver partitioning system can achieve significantly higher video quality than the layered system, because of the higher coding efficiency of non-layered coding. Leveraging on our theoretic study here, we are developing distributed P2P MPVC protocols for both layered coding and receiver partitioning. The main challenges are adapting to time-varying peer uplink and downlink bandwidth, real-time recovery from packet losses, and control end-to-end video delays experienced by users.

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