1. (a) Three fair dice are rolled three times. Find the probability of rolling a “double five” at least once.

(b) Let $p_A$ represent the probability that a randomly selected item from box $A$ is defective; similarly, $p_B$ represents the probability that a randomly selected item from box $B$ is defective. Boxes $A$ and $B$ are identical. Two items are randomly selected from one of the boxes.

(i) What is the probability that both the items are defective?

(ii) Suppose both the items are found to be defective; what is the probability that they came from box $A$?

2. (a) $X$ is a Poisson random variable with parameter $\lambda$. Define $B = \{X \leq 2\} \cup \{X > 5\}$. Find $P(X = k \mid B), \, k = 0, 1, 2, \cdots$

(b) $X$ is a continuous random variable from $(-\infty, +\infty)$ with $F_X(x)$ and $f_X(x)$ representing its probability distribution and density functions respectively. Find $F_Y(y)$ and $f_Y(y)$ for the transformation $y = g(x)$ shown below.

![Diagram](image-url)
3. The probability density function of the random variable $X$ is given by

$$f_X(x) = \begin{cases} \frac{1}{2} \left(1 - \frac{|x|}{2}\right), & |x| < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Define the new random variable $Y = (X - 1)^2$.

Find $f_Y(y)$. Plot graphs of both $f_X(x)$ and $f_Y(y)$ and verify the area under $f_Y(y)$.

4. (a) Let $P(X = k) = c k^{p-1}, k = 1, 2, \cdots, \infty$.

(i) Find $c$ so that $P(X = k)$ above represents a probability mass function.

(ii) Find the characteristic function of $X$.

(b) Let $f_Y(y) = \begin{cases} 12 y^2 (1 - y), & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$

Find the mean and variance of the random variable $Y$. 

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