Throughput Optimization Using Adaptive Techniques

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Abstract

We present a mathematical framework for maximization of single user throughput in a wireless channel using the symbol rate, the packet length, and the constellation size of MQAM modulation as optimization variables. The throughput is defined as the number of bits per second correctly received. Trade-offs between the throughput and the communication range are observed, and equations are derived for the optimal choice of the design variables. These parameters are location dependent and can be adapted dynamically in response to the mobility of a wireless data terminal. We also look at the joint optimization problem involving all the design parameters together. We find, however, that not all of the three parameters need to be adapted simultaneously: in the high SNR region the maximum throughput is obtained by adapting the packet length and the constellation size together, while in the low SNR region it is achieved by adapting the symbol rate so that the received SNR per symbol stays at some preferred value. We also characterize the optimal triplet of parameter values as functions of received SNR in both AWGN and Rayleigh fading channels with some restrictions on the design parameters.

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I. INTRODUCTION

Throughput, defined as the data rate successfully received, is a key measure of quality of service (QoS) for wireless data transmission systems. Throughput is affected by the channel environment such as the distance between the transmitter and the receiver, the fading state of the channel, and the noise and interference power characteristics. It is also influenced by the choice of design parameters, for example, symbol rate, modulation and coding, constellation size, power level, multiple access scheme, and many others.

Adaptive modulation has been proposed as an efficient technique to improve data rate of such wireless systems by adapting some of those design parameters to the time-varying channel environment to maintain an acceptable bit error rate (BER) [1]-[4]. Most works in adaptive modulation, however, have only aimed to improve the link layer performance. As application requirements become more complex, there is a need to take into account higher layer metrics. For instance, several works in the literature have developed adaptive techniques that support different types of traffic with different QoS requirements [5].

In our work, we look at the throughput at the MAC layer where end-to-end delivery of the packet should be guaranteed. In other words, we include the loss due to the retransmission of the packet in the performance metric of adaptive modulation and study how it affects the design parameters. For example, at low symbol rates, most packets arrive without error and an increase in the symbol rate increases throughput. If the symbol rate is too high, however, packet error probability increases and throughput is limited by frequent retransmission. Thus, we expect that an optimum symbol rate will give a moderate error rate and high throughput. The same holds for the selection of a packet length. If the packet length is small, most packets arrive without error but at the cost of large packet overheads. On the other hand, a large packet is susceptible to error which causes retransmission of the entire packet. The choice of modulation level also affects the throughput. Low bits per symbol means inefficient use of the channel far below its capacity, while high bits per symbol may result in low packet success rate (PSR). In this paper,
we investigate the problem of throughput maximization by looking at the effect of such design parameters. Specifically, we focus our attention on single user wireless systems using MQAM, and derive the optimal selection of symbol rate, packet length, and constellation size for each packet as the channel gain varies.

Although we only treat uncoded MQAM in our work, the results are also applicable to trellis or turbo coded systems [3], because they simply provide additional gain in terms of transmitter power level and do not change the mathematical framework that will follow hereafter. Interested readers are also referred to [4], [6] for studies of adaptive modulation using outer block codes.

The rest of this paper is organized as follows. In Section II, our system model is introduced. In Section III, we derive an optimal adaptation of individual design parameters. In Section IV, we jointly optimize all the design parameters. This result is then extended to Rayleigh fading channels, and also to the case when the design parameters are restricted to take only discrete values. Section V is a discussion of the results.

II. System Model

Consider a communication link which consists of a transmitter, a receiver, and a communication channel with bandwidth $W$. The transmitter constructs packets of $K$ bits and transmits the packets in a continuous stream. To ensure that bits received in error are detected, the transmitter attaches a $C$ bit CRC to each data packet, making the total packet length $K + C = L$ bits. This packet is then transmitted through the air and processed by the receiver. The CRC decoder at the receiver is assumed to be able to detect all the errors in the received packet. (In practice some errors are not detectable, but this probability is small for reasonable value of $C$ and reasonable SNRs.) Upon decoding the packet, the receiver sends an acknowledgment, either positive (ACK) or negative (NACK), back to the transmitter. For ease of analysis we assume this feedback packet goes through a separate control channel, and arrives at the transmitter instantaneously and without error. If the CRC decoder detects any error and issues a NACK, the transmitter uses a selective repeat protocol to resend the packet. It repeats the process until the packet is successfully delivered.

A packet is transmitted symbol by symbol through the channel, where each MQAM symbol has $b$ bits in it and is modulated using fixed power MQAM. Thus, each packet corresponds to $L/b = L_s$ MQAM symbols. For simplicity of analysis, we use a simple path loss model
for the channel with a power fall-off factor $\alpha$. No fading is present unless otherwise stated. (We will consider Rayleigh fading in Subsection IV-A.) We assume additive white Gaussian noise (AWGN) at the receiver front-end, and no interference from other signals. The channel is narrowband (flat fading), so the power spectra of both the received signal and the noise have no frequency dependence, i.e., the channel is characterized by a single path gain variable.

With the above simplifying assumptions, we define the throughput of a system as the number of payload bits per second received correctly [5], [7]:

$$T = \frac{L - C}{L} b R_s f(b, \gamma_s, L)$$

(1)

where $b$ is the number of bits per MQAM symbol, $R_s$ is the symbol rate, $f()$ is the packet success rate (PSR) defined as the probability of receiving a packet correctly, and $\gamma_s$ is the SNR per symbol given by

$$\gamma_s = \frac{E_s}{N_o} = \frac{P_r}{N_o R_s}$$

(2)

where $E_s$, $N_o$, and $P_r$ represent the energy per symbol, the one-sided noise power spectral density, and the received power respectively. In a simple channel model without fading, the received power is given by a path loss relationship as $P_r \propto P d^\alpha$, where $P$ is the transmitter power and $d$ is the distance between the transmitter and the receiver.

In Table I, we list numerical parameters that will be used in the throughput calculation throughout this paper.

### III. Optimization Framework

#### A. Optimal symbol rate

To find the symbol rate $R_s$ that maximizes throughput, we differentiate (1) with respect to $R_s$ and set it to zero to obtain the following condition

$$f(b, \gamma_s, L) = \gamma_s \frac{\partial f(b, \gamma_s, L)}{\partial \gamma_s}$$

(3)

For any $b$ and $L$, there is a unique $\gamma_s^*$ that satisfies this equation. As shown in Figure 1, $\gamma_s^*$ corresponds to the point where a ray from the origin is tangent to $f(b, \gamma_s, L)$ in the $(\gamma_s, f)$ plane [8]. Therefore, for a given PSR characteristic, determined by the channel, modulation scheme, packet length, etc., there exists a preferred SNR per symbol, $\gamma_s^*$, which is independent of the
received power or the location of the receiver. Therefore, (2) implies that the optimal symbol rate is

\[ R_s^* = \frac{P_r}{\gamma_s^* N_0} \]  

(4)

Since any symbol error in the packet results in a loss of the packet, the PSR is given in terms of the symbol error rate \( P_e \) by

\[ f(b, \gamma_s, L) = [1 - P_e(b, \gamma_s)]^{L/b} \]  

(5)

Using (3) and (5), we arrive at an equation for obtaining the preferred SNR value \( \gamma_s^* \):

\[ \gamma_s^* \frac{\partial P_e(b, \gamma_s)}{\partial \gamma_s} \bigg|_{\gamma_s=\gamma_s^*} = -\frac{1 - P_e(b, \gamma_s^*)}{L/b} \]  

(6)

The \( P_e \) of MQAM in AWGN channels is (approximately) given by [9]

\[ P_e(b, \gamma_s) = 4 \left(1 - 2^{-b/2}\right) Q \left( \sqrt{\frac{3}{2^b - 1}} \gamma_s \right) \]  

(7)

The fact that there exists a preferred SNR value makes it necessary for the symbol rate to be adapted to the distance \( d \) between the transmitter and the receiver. This is well observed in Figure 2 where the throughput \( T \) is plotted for different symbol rates. We see that the system can support high symbol rates at small distances, but its throughput rapidly decreases after a certain distance at which point the system should switch to a lower symbol rate to maintain the optimal throughput.

In practice, a variable symbol rate may not be a good choice for building a system because it requires hardware components with variable bandwidths. To avoid such a problem, we may use a fixed symbol rate at \( R_s = W \) and concentrate power in some fraction of the frame \(^1\). Under an average power constraint, this gives the same performance as changing the symbol rate. For example, by using a frame of \( L_f \geq L_s \) symbols, and only using the transmitter during \( L_s \) symbol (i.e. packet length) periods in the frame with a transmit power \( (L_f/L_s)P \), we have \( \gamma_s = (L_f P_r)/(L_s N_0 W) \), which is equal to \( \gamma_s \) of a system that uses a reduced symbol rate \( R_s = (L_s/L_f)W \). Thus, these two schemes are (mathematically) equivalent. Note that both schemes have unused resources, in either frequency or time, leaving room for other users to share the channel in a multiuser system. We will only use the variable symbol rate scheme in this work.

\(^1\)A frame is a fixed block of time that encapsulates the packet and may also have unused symbol durations.
B. Optimal packet length

To find an analytic solution for the optimal packet length \( L^* \), we assume \( L \) to take continuous values. Differentiating (1) with respect to \( L \) and using (5) produce

\[
\frac{\partial T}{\partial L} = C \frac{b R_s f(b, \gamma_s, L)}{L^2} + (1 - \frac{C}{L}) R_s f(b, \gamma_s, L) \ln(1 - P_e(b, \gamma_s))
\]

Setting this to zero and solving the resulting quadratic equation, we have

\[
L^*(b, \gamma_s) = C^2 + \frac{1}{2} \sqrt{C^2 - \frac{4 b C}{\ln(1 - P_e(b, \gamma_s))}}
\]

Thus, the optimal packet length \( L^* \) depends on the constellation size \( 2^b \), the SNR per symbol \( \gamma_s \), and the probability of symbol error \( P_e \). Figure 3 shows \( L^* \) as a function of \( \gamma_s \) for QPSK modulation. Note that in AWGN channels \( L^* \) grows rapidly as \( \gamma_s \) increases\(^2\), which can cause problems in practice because larger packet size means higher delay and more memory requirement. Fortunately, we observe in (1) that at high \( \gamma_s \), \( f(b, \gamma_s, L) \approx 1 \) and the throughput is proportional to \( 1 - C/L \). Therefore, the throughput gain becomes negligible if we increase \( L \) beyond a certain point. In Rayleigh fading channels, \( L^* \) is much smaller and in fact asymptotically proportional to \( \sqrt{\gamma_s} \). Figure 4 shows the throughput of systems with various packet lengths under a fixed symbol rate and constellation size. We see that large packet size gives high throughput at small distances, whereas small packet size gives a longer communication range. Thus, by adaptively changing the packet length, we can achieve both higher throughput and a longer communication range than using a fixed packet length.

However, this doesn’t necessarily mean that the packet length should always be variable. For example, if we adapt the symbol rate and the packet length simultaneously, there is a single \( L^*(b, \gamma_s^*) \) that is optimal regardless of the location of the receiver, because the symbol rate is adapted first to maintain \( \gamma_s = \gamma_s^* \), eliminating the effect of any location change. In this case, one degree of adaptation (i.e., symbol rate) would be enough for the 2-D optimization problem [10]. We will visit this again in Section IV.

C. Optimal constellation size

Constellation size \( 2^b \), or the number of bits per symbol \( b \), is another degree of freedom that can be adapted to variations in received power, to allow packing more bits per symbol when the

\(^2\)This is because in AWGN channels \( P_e \) decays exponentially as \( \gamma_s \) increases.
channel gain is high. For ease of analysis we do not restrict \( b \) to integer values. By differentiating (1) with respect to \( b \) and setting it to zero, and using the relationship in (5), we obtain an equation for the optimal number of bits per MQAM symbol \( b^* \) as:

\[
\frac{\partial P_e(b, \gamma_s)}{\partial b} \bigg|_{b=b^*} = -\frac{1 - P_e(b^*, \gamma_s)}{L} \tag{10}
\]

Figure 5 shows the throughput of systems with various constellation sizes under fixed symbol rate and packet length in AWGN channels. Higher level modulations using MQAM mainly increase the throughput at small distances, but have shorter communication ranges. By adapting the constellation size, therefore, we can boost the throughput at small distances while maintaining the same communication range.

IV. JOINT OPTIMIZATION

The discussion in the previous section implies that the optimal joint adaptation of the parameter triplet \((b, \gamma_s, L)\) is given by the following set of equations.

\[
\gamma_s^* \frac{\partial P_e(b^*, \gamma_s)}{\partial \gamma_s} \bigg|_{\gamma_s=\gamma_s^*} = -\frac{1 - P_e(b^*, \gamma_s^*)}{L^*/b^*} \tag{11}
\]

\[
L^* = \frac{C}{2} + \frac{1}{2} \sqrt{C^2 - \frac{4b^*C}{\ln(1 - P_e(b^*, \gamma_s^*))}} \tag{12}
\]

\[
\frac{\partial P_e(b, \gamma_s^*)}{\partial b} \bigg|_{b=b^*} = -\frac{1 - P_e(b^*, \gamma_s^*)}{L^*} \tag{13}
\]

where the probability of symbol error \( P_e \) in an AWGN channel is given by (7). Then, the optimal throughput is given by

\[
T^* = \frac{L^* - C}{L^*} \frac{P_r}{N_0\gamma_s^*} b^* (1 - P_e(b^*, \gamma_s^*))^{L^*/b^*} \tag{14}
\]

Figure 6 shows the optimal throughput curve as well as suboptimal ones obtained by holding one or more of the three design parameters constant. Also shown is the Shannon capacity of the channel given by \( C = W \log_2(1 + P_r/(N_0W)) \) [11]. Note that the two suboptimal curves coincide with the optimal one at small and large distances, respectively. This leads to an important observation: at small distances the optimal throughput is obtained by adapting the constellation size and the packet length simultaneously, whereas at large distances it is obtained by only adapting the symbol rate while maintaining the optimal packet length. Essentially, there are two regions of operation in terms of the adaptation of design parameters:

August 27, 2003
1) High SNR region: In the region where the channel gain is high, the symbol rate cannot be increased further due to the bandwidth limitation $R_s \leq W$. In this case, the SNR per symbol $\gamma_s$ cannot be maintained constant at the optimal value $\gamma_s^*$. This gives room for packing more bits per symbol into the MQAM modulation, and the corresponding optimal packet length $L^*$ also grows as the channel gain increases. Thus, we use the maximum symbol rate for a given bandwidth and adapt the constellation size and the packet length simultaneously.

2) Low SNR region: In the region where the channel gain is low, the main concern is how to maintain the symbol error rate $P_e$ low enough for the communication link to stay effective. While we cannot further reduce the constellation size below BPSK, we can reduce the symbol rate to bring the SNR per symbol $\gamma_s$ back to the optimal value $\gamma_s^*$. Since $\gamma_s$ is maintained constant in this region, so is the optimal packet length $L^*(b, \gamma_s^*)$. Thus, in this region we adapt only the symbol rate using BPSK modulation.

It is worthwhile to mention that reducing the symbol rate (or the equivalent scheme explained in Section III-A) is not the only way of overcoming the low SNR; schemes such as spread spectrum modulation or error correction coding could be used instead. Specifically, [6] uses forward error correction (FEC) to show that using adaptive FEC gives higher throughput than changing the symbol rate. The common idea in all of these variations, however, is to transmit the packet over an extended period of time to increase the energy per information bit.

Secondly, it is observed from Figure 6 that by optimally adapting the parameters we obtain a near constant gap of about 8 dB from the Shannon capacity regardless of the distance or the received SNR. (The same gap has previously been observed in the high SNR region by only adapting the constellation size [5], but the gap would increase if the choice of the packet length were far from optimum.) This gap could be reduced by use of trellis coded modulation.

A. Fast Rayleigh fading channels

In non-fading or slowly fading channels where the fade duration is longer than a packet period, the system throughput and its optimization are given by (11)-(14) and (7) at each fading state. However, in fast fading channels where the channel gain changes within a symbol period, or in fading channels with interleaving, we need to use average error probabilities instead of (7), and the PSR in (5) also needs to be modified accordingly. In a flat Rayleigh fading channel, the
average probability of symbol error $P_e$ is (approximately) given by [12]

$$P_e(b, \bar{\gamma}_s) = 2 \left(1 - 2^{-b/2}\right) \left(1 - \sqrt{\frac{3\bar{\gamma}_s}{2(2^b - 1) + 3\bar{\gamma}_s}}\right)$$

and the PSR becomes

$$g(b, \bar{\gamma}_s, L) = \left[1 - \bar{P}_e(b, \bar{\gamma}_s)\right]^{L/b}$$

The rest of the analysis is the same as that in AWGN channels with $\gamma_s$, $P_e$, and $f()$ replaced by their averaged quantities $\bar{\gamma}_s$, $\bar{P}_e$, and $g()$, respectively. Figure 7 shows the optimal and suboptimal throughput curves in Rayleigh fading channels. The throughput in Rayleigh fading is lower than that in AWGN channels as expected, but the general optimization framework does not change; one of the two suboptimal policies becomes optimal in high or low SNR regions. In Rayleigh fading channels, $\gamma_s^*$ is higher, $R_s^*$ is slower, $b^*$ is smaller, and $L^*$ is shorter than in AWGN channels.

**B. Constellation size and packet length restrictions**

We now restrict ourselves to square MQAM constellations of an integer number of bits per symbol, $b = 1, 2, 4, 6, \ldots$, and packet lengths of integer multiples of the number of bits per symbol, $L = b, 2b, 3b, \ldots$. For BPSK ($b = 1$), we use $P_e(1, \gamma_s) = Q(\sqrt{2\gamma_s})$ and $\bar{P}_e(1, \bar{\gamma}_s) = 0.5(1 - \sqrt{\gamma_s/(1 + \gamma_s)}$ [9], and for $b \geq 2$, we use (7) and (15). Through exhaustive search over $b$ and $L$, we obtain Figures 8 and 9. These figures fully characterize the optimal selection of parameter triplets $(b, \gamma_s, L)$ as functions of received SNR $P_r/(N_0W)$. It is seen that the low and high SNR regions are not clearly divided, i.e., there are transitional regions ($6-16dB$ in Figure 8 and $8-12dB$ in Figure 9) where the optimal adaptation is to use $b \geq 2$ and $R_s < W$. This is mainly because we cannot continuously adapt $b$; since $b$ can take only discrete values, $b > b^*$ for some SNRs, in which case $R_s$ may need to be reduced to achieve the SNR per symbol required to support the excessive constellation size. The restrictions result in some throughput loss as well; it is found that the loss is less than 28% and 19% in AWGN and Rayleigh fading channels, respectively. Although the results in this subsection are only specific to our system model, similar analysis could be performed for practical systems to tabulate their optimal parameters, which then could be used for the design of adaptive transceivers.
V. Conclusion

We have investigated the effect of the symbol rate, the packet length, and the constellation size on the throughput in single user wireless systems. We have derived the optimal values that maximize the throughput for each of those design parameters. The optimal values depend on the received signal strength and thus should be adapted properly. We have then shown how they can be jointly optimized to achieve a maximum throughput. At high SNR, the maximum throughput is attained by adapting the packet length and the constellation size simultaneously while using the maximum symbol rate. At low SNR, the throughput is maximized by adapting the symbol rate while using the smallest constellation size and some fixed packet length. Finally, we have characterized the optimal adaptation of the parameters in AWGN and Rayleigh fading channels under restrictions on the values that the parameters can take.

The theory and results in this paper apply to single user systems. Typical multiuser systems are interference-limited and should be dealt with differently. For example, schemes that enable orthogonal channel sharing among users through frequency (variable symbol rate), time (variable frame length with fixed symbol rate), or code division (spread spectrum modulation) may have an advantage over the other schemes (variable packet length and/or adaptive FEC).

REFERENCES


**TABLE I**

**NUMERICAL PARAMETERS**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
<th>Value</th>
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<tbody>
<tr>
<td>$W$</td>
<td>Bandwidth</td>
<td>1 MHz</td>
</tr>
<tr>
<td>$P$</td>
<td>Transmitter power</td>
<td>1 W</td>
</tr>
<tr>
<td>$N_0$</td>
<td>One-sided noise PSD</td>
<td>$10^{-15}$ W/Hz</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Path loss exponent</td>
<td>-4</td>
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<tr>
<td>$C$</td>
<td>CRC length</td>
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</tr>
<tr>
<td>$b$</td>
<td>Number of bits per symbol</td>
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</tr>
<tr>
<td>$R_s$</td>
<td>Symbol rate</td>
<td>$\leq 1$ Msp$s$</td>
</tr>
<tr>
<td>$L$</td>
<td>Packet length</td>
<td>$\leq 512$ bits</td>
</tr>
</tbody>
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Fig. 1. Determining $\gamma_s^*$
Fig. 2. Throughput vs distance with variable $R_s$, $L = 100$, and $b = 2$

Fig. 3. $L^*$ vs $\gamma_s$ for QPSK in AWGN and Rayleigh fading
Fig. 4. Throughput vs distance with variable $L$, $R_s = 1\text{M sps}$, and $b = 2$.

Fig. 5. Throughput vs distance with variable $b$, $L = 100$, and $R_s = 1\text{M sps}$.
Fig. 6. Optimal throughput vs distance in AWGN channels

Fig. 7. Optimal throughput vs distance in Rayleigh fading channels
Fig. 8. Optimal adaptation vs received SNR in AWGN channels

Fig. 9. Optimal adaptation vs received SNR in Rayleigh fading channels