Delay Bounds of Peer-to-Peer Video Streaming[*]

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Abstract

Peer-to-Peer (P2P) systems exploit the uploading bandwidth of individual peers to distribute content at low server cost. While the P2P bandwidth sharing design is very efficient for bandwidth sensitive applications, it imposes a fundamental performance constraint for delay sensitive applications: the uploading bandwidth of a peer cannot be utilized to upload a piece of content until it completes the download of that content. This constraint sets up a limit on how fast a piece of content can be disseminated to all peers in a P2P system. In this paper, we theoretically study the impact of this inherent delay constraint and derive the minimum delay bounds for P2P live streaming systems. We show that the bandwidth heterogeneity among peers can be exploited to significantly improve the delay performance of all peers. We further propose a simple snow-ball streaming algorithm to approach the minimum delay bound in P2P live video streaming. Our analysis and simulation suggest that the proposed algorithm has better delay performance and more robust than existing tree-based streaming solutions. Insights brought forth by our study can be used to guide the design of new P2P systems with shorter streaming delays.

1 Introduction

Video-over-IP applications have recently attracted a large number of users on the Internet. YouTube [4], the popular user-generated-content (UGC) video streaming site, serves 100 million distinct videos and $65,000$ uploads daily. While Youtube employs content distribution networks to stream video to end users, Peer-to-Peer (P2P) video streaming solutions utilize the uploading bandwidth of end users to distribute video content at low server infrastructure cost. Several P2P streaming systems have been deployed to provide on-demand or realtime video streaming services over the Internet [6, 25, 18, 19, 21]. Our measurement studies have verified that hundreds of thousands of users can simultaneously participate in these systems [7]. While the initial successes of P2P streaming are impressive, compared with the traditional TV services provided by cable companies, all current P2P streaming systems suffer from long video startup delays and highly variable playback lags among peers [8]. The delay between a video object is chosen by a user and the actual playback starts on his/her screen ranges from several seconds to a couple of minutes. The playback progresses of peers watching the same channel are asynchronous with lags up to tens of seconds.

In traditional client-server based video streaming systems, the video startup delay perceived by a client is determined by the delay and the available bandwidth on the its connection with the server. To deal with delay and bandwidth variations, client-side video buffering is necessary to ensure smooth playback. In P2P video systems, a video stream is divided into video *chunks* and peers collaboratively download/upload

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video chunks from/to each other in P2P fashion to reduce the server workload. While the P2P bandwidth sharing design has been very efficient for bandwidth sensitive applications, such as file sharing, it imposes a fundamental performance constraint for delay sensitive applications: *the uploading bandwidth of a peer cannot be utilized to upload a piece of content until it completes the download of that content.* This constraint sets up a limit on how fast video chunks can be disseminated to all peers.

In this paper, we theoretically study the impact of this inherent delay constraint and derive the minimum delay bounds for generic P2P live video streaming systems. Our analytical results unveil the impact of the bandwidth distribution among peers on their streaming delay performance. We show that the bandwidth heterogeneity among peers can be exploited to improve their delay performance. Even a very small percentage of super peers can significantly reduce the video streaming delays for all peers. We further propose a simple *snow-ball streaming* algorithm to approach the minimum delay bound in P2P live video streaming. The delay performance of the proposed algorithm is compared with existing tree-based streaming solutions. Through analysis and simulation, we demonstrate that the proposed snow-ball streaming algorithm not only can achieve a close-to-optimum delay performance, but also has the potential to do so in face of realistic network impairments, such as long propagation delays and random bandwidth fluctuations. The delay bounds derived in our analysis can serve as delay performance benchmarks for various proposed/deployed P2P streaming systems. Insights brought forth by the study of the snow-ball streaming algorithm can be used to guide the design of new P2P streaming systems with shorter start-up delays and playback lags.

The paper is organized as follows. In Section 2, we provide a short overview on the existing P2P streaming solutions. The bounds on the delay for a single chunk dissemination is established in Section 3 for both homogeneous and heterogeneous P2P network environments. A snow-ball chunk dissemination algorithm is introduced to achieve the delay bound for a single chunk dissemination. In Section 4, we show that the snow-ball chunk dissemination algorithm can be extended to a snow-ball streaming algorithm to achieve the delay bounds in continuous video streaming. The performance of the snow-ball chunk dissemination algorithm under realistic network environment is studied in Section 5. A dynamic snow-ball streaming algorithm is presented in Section 6. Through simulations, we demonstrate that the dynamic snow-ball streaming algorithm can approach the minimum delay bounds in highly variable network environments with a small peer upload bandwidth overhead. The paper is concluded with future work in Section 7.

2 Background and Related Work

Existing P2P streaming solutions can be classified into the following categories.

2.1 Single-Tree Streaming

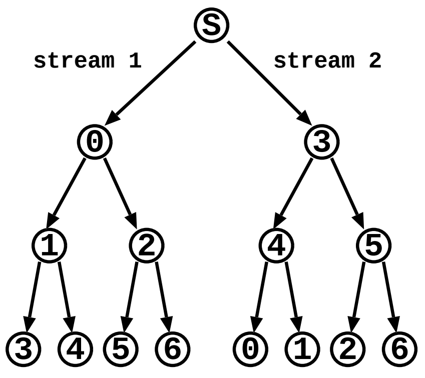
In a single-tree based approach, peers are organized into a tree rooted at the server. Each peer receives the stream from its parent peer and forward to its children peers. The fan-out degree of a peer is limited by its uploading bandwidth. An early example is Overcast [9]. One major drawback of the single-tree approach is that all the leaf nodes don't contribute their uploading bandwidth. Since leaf nodes account for a large portion of peers in the system, this largely degrades the peer bandwidth utilization efficiency.

2.2 Multi-Tree Streaming

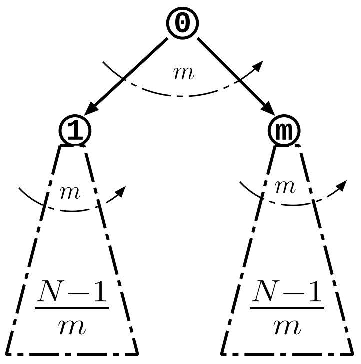
To solve the leaf nodes problem, Multi-Tree based approaches have been proposed [3, 10]. In multi-tree streaming, the server divides the stream into m sub-streams. Instead of one streaming tree, m sub-trees are formed, one for each sub-stream. In a fully balanced multi-tree streaming, the node degree of each sub-tree is m. Each peer joins all sub-trees to retrieve sub-streams. A single peer is positioned on an internal node in only one tree and only uploads one sub-stream to its m children peers in that tree. In each of the remaining

$m-1$ sub-trees, the peer is positioned on a leaf node and downloads a sub-stream from its parent peer. Figure 1(a) shows an example of two-tree streaming for 7 peers.

For any tree-based streaming approach, a chunk is disseminate in a hierarchical way. As illustrated in Figure 1(b), for a $m-$degree tree of N peers, peer 0 sends a chunk to its m children peers at level 1, each of which then is responsible for disseminating the chunk in its own subtree with $(N-1)/m$ peers (including themselves). In terms of time, after m transmissions by peer 0, the task of disseminating a chunk to N peers becomes m sub-tasks of disseminating the chunk to $\frac{N-1}{m}$ peers.



(a) 7-nodes example



(b) hierarchical view

Figure 1: Multi-Tree Based Streaming

2.3 Mesh-based Streaming

The management of streaming trees is challenging in face of frequent peer churns. Mesh-based streaming systems are more robust against peer dynamics. Many recent P2P streaming systems adopt mesh-based streaming approach [25, 17, 23, 15, 24]. In a mesh-based system, there is no static streaming topology. Peers establish and terminate peering relationship dynamically. A peer may download/upload video from/to multiple peers simultaneously. A recent simulation study [16] suggests that mesh-based systems have superior performance than tree-based systems. However, in practice, the delay performance of mesh-based streaming is still not satisfactory. Our analytical results indicate that mesh-based systems have a lower delay bound than that can be achieved by the optimal tree-based systems. One important motivation of the study presented in this paper is to provide some guidelines for the design of peering strategies and chunk scheduling schemes in mesh-based streaming systems.

2.4 Related Work on Delay Performance

Despite of P2P streaming systems' popularity, few studies have addressed their delay performance analytically. One related work was presented in [20]. Authors of [20] studied the trade-off between the server bandwidth cost, the maximum number of peers that can be supported, and the minimum number of streaming hops experienced by a peer. We study the optimal streaming strategy when the server only plays a minimum role in video uploading. The delay bounds obtained through our analysis is much tighter than that predicted in [20], and can be achieved by the proposed snow-ball streaming algorithm. A recent paper [13] studied how one can improve P2P streaming delay performance by constructing streaming trees with minimum depth. Our work focuses on the minimum delay bound imposed by the content bottleneck in P2P streaming. As will be demonstrated in the following sections, mesh-based topologies have shorter delay bounds than tree-based topologies and the proposed snow-ball algorithm can achieve the minimum delay bound for generic streaming topologies.

3 Bound on Single Chunk Dissemination

In a P2P live video streaming session, a sequence of video chunks are continuously generated by the server and disseminated to all peers in the session. The streaming delay is determined by how fast all chunks can be delivered to peers. In this section, instead of developing the streaming delay bound, we assume that there is only one chunk to be disseminated in a P2P video system and develop the delay bound for the single chunk dissemination. Obviously, the single chunk delay bound is a lower bound for streaming delay. We will generalize the analysis for the single chunk dissemination to continuous streaming in Section 4.

Given a P2P system with a server and N peers, one can answer the question: *If the server generates a chunk of content at time $t = 0$, how does one disseminate that chunk to all N peers in the shortest time possible?* The answer depends on the size of the chunk, available bandwidth and the propagation delays among all nodes in the system, including the server and all peers. Without loss of generality, we can normalize the chunk size to be one, and choose the video streaming rate as the bandwidth unit. Consequently, the chosen time unit after the normalization equals to the chunk transmission time on a unit bandwidth link, which in turn equals to the average playback time of video contained in a chunk.[1] For now, let's assume the propagation delay between any two nodes is dominated by the chunk transmission delay and thus can be ignored. We will take propagation delays into account in Section 5.1 when the chunk transmission delay becomes small. Without using P2P dissemination, if the server has a bandwidth of N, the server can upload that chunk to all peers by $t = 1$. However, this is not a scalable solution when N is large. We are interested in the delay performance when peers upload chunks among themselves. Throughout this paper, we assume all peers have enough download bandwidth to receive the whole video stream. Therefore, the download part is never a bottleneck in our analysis.

3.1 Homogeneous case

We start with a homogeneous case where the server and all peers have bandwidth of 1. This corresponds to the *Tit-for-Tat* case in Bittorrent where peers upload roughly the same amount of data as they download. We further assume that the server will upload only one copy of the chunk to one peer and won't participate the chunk dissemination afterward.

3.1.1 Single-Tree Chunk Dissemination

Given the unit bandwidth on all peers, a peer can only have one child. The only possible single-tree based streaming solution is a chain: the server uploads the chunk to peer 0, then peer 0 uploads it to peer 1, and so on until peer $N-2$ uploads it to peer $N-1$. The chunk propagates along the chain from the server to all peers in time N. The average delay is $(N+1)/2$.

3.1.2 Multi-Tree Chunk Dissemination

If the multi-tree approach with degree m is employed, a chunk propagates from the server to all peers along a sub-tree with node degree of m. If there are N peers, the number of levels of each subtree is

[1]The chunk size in P2P live video systems is typically smaller than that of P2P file sharing system. However, to reduce the signaling overhead and chunk scheduling complexity, current P2P live video systems still employ video chunks with considerable size. Our measurement study [7] of one popular commercial system shows that the chunk size is around 10 Kbytes. Given the video streaming rate of 400 Kbps, the playback time of a chunk is around 0.2 second.

$K = \lceil \log_m(N(m-1)+1) \rceil$. The only peer at level 0 downloads the chunk directly from the server, a peer at level i then uploads a video chunk to m children peers at level $i+1$. Let N_i be the number of peers at level i. Then $N_i = m^i$, $0 \le i < K-1$, and $N_{K-1} = N - m^{K-2}$. Since each peer/server only has uploading bandwidth of 1, if the uploading is done in parallel, all children peers of one peer will receive the chunk m time slots after their common parent receives the chunk. The peer at the top level can always receive the chunk from the server after one time slot. For parallel uploading, the peers at the very bottom level will receive the chunk in $(K-1)m+1$ time slots. The average delay among all peers is

$$\bar{D}_p(m) = \frac{1}{N} \sum_{i=0}^{K-1} (im+1) N_i \tag{1}$$

When N is large, the average delay and the worst-case delay are both of the form

$$D_p(m) = m \log_m(N) + o(1) = \frac{m}{\log_2 m} \log_2 N + o(1). \tag{2}$$

To achieve the shortest delay, one can choose tree degree

$$m^* = \underset{m}{\textbf{argmin}}\, D_p(m) = 3,$$

i.e., the server divides the stream into 3 sub-streams, and feeds each stream into one sub-tree with node degree of 3. The minimum delay, in both average and worst-case sense, is $1.89 \log_2 N + o(1)$.

If the uploading is done sequentially, the first child peer will receive the chunk from its parent within 1 time slot, and the last child of a peer will receive the chunk after m time slots. The longest delay at level i is still $im+1$. Therefore the worst-case delay is still $(K-1)m+1$. A degree of 3 can achieve the minimum worst-case delay of $1.89 \log_2 N + o(1)$. The average delay at level i is $(m+1)/2$ time slots more than the average delay at level $i-1$. The peer at the top level can always receive the chunk from the server after one time slot. We can calculate the average delay among all peers as

$$\bar{D}_s(m) = \frac{1}{N} \sum_{i=0}^{K-1} (i(m+1)/2+1) N_i.$$

Again, when N is large, the average delay is

$$\bar{D}_s(m) = \frac{m+1}{2} \log_m(N) + o(1) = \frac{m+1}{2\log_2 m} \log_2 N + o(1).$$

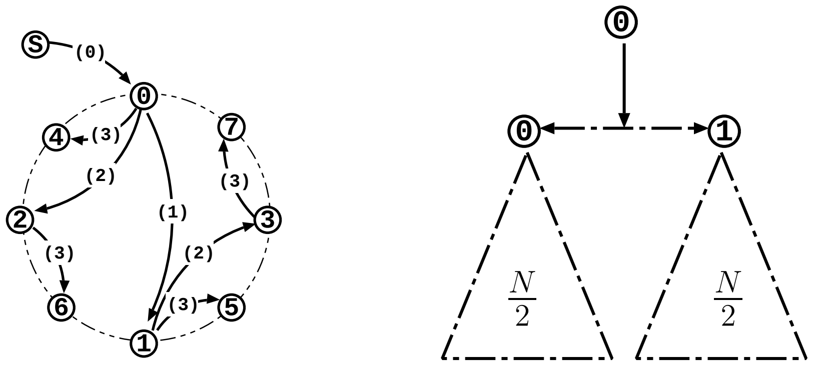
To minimize the average delay, the optimal degree is 4, and the minimum average delay is $1.25 \log_2 N + o(1)$, which is less than $2/3$ of the average delay of parallel uploading.

3.1.3 Snow-Ball Chunk Dissemination

For single chunk dissemination, peers only need to disseminate one chunk, instead of a continuous stream of chunks. After downloading the chunk, a peer can keep uploading that chunk to other peers until all peers receive it. This will largely reduce the chunk dissemination time. The accumulation of the aggregate uploading bandwidth for the chunk mimics the formation of a snow-ball. We refer it as the *snow-ball chunk dissemination* approach. Figure 2(a) illustrates the progress of snow-ball chunk dissemination for eight peers. An arc from node i to node j with a label k represents peer i (or the server) uploads the chunk to peer j in time slot k. The server uploads the chunk to peer 0 in time slot 0. In time slot 1, peer 0 uploads the received chunk to peer 1. In time slot 2, both peer 0 and peer 1 will upload the chunk to peer 2 and 3

respectively. Peer $0, 1, 2, 3$ will upload the chunk to peer $4, 5, 6, 7$ in time slot 3. It takes 4 time slots for all peers receive the chunk.

For general case, the snow-ball approach disseminates a chunk in a recursive way. As illustrated in Figure 2(b), after peer 0 sends a chunk to peer 1, the task of disseminating a chunk to N peers becomes two sub-tasks of disseminating the chunk to $\frac{N}{2}$ peers. Peer 0 continues to lead one sub-task, and peer 1 becomes the leader for the other sub-task. Even though the task splitting degree is only 2, compared with degree m in Figure 1(b), it happens after only 1 chunk transmission, instead of m transmissions in Figure 1(b). We will show that the snow-ball branching is actually the fastest branching process.



(a) 8-nodes (b) recursive view

Figure 2: Snow-ball Chunk Dissemination

Let $x(i)$ denote the number of peers that have the chunk at the beginning of time slot i. In time slot 0, the server uploads the chunk to one peer, therefore, $x(1) = 1$. Afterward, every peer with the chunk will upload it to another peer in one time slot, we have $x(i) = 2 * x(i-1) = 2^{i-1}$. Therefore it takes $K^* = 1 + \lceil \log_2 N \rceil$ time slots for all N peers receive the chunk. One peer receives the chunk after 1 time slot, 2^{i-2} peers receive the chunk after i time slots $\forall 1 < i < K^*$, and $N - 2^{K^*-2}$ peers receive the chunk after K^* time slots. The average delay performance is

$$\bar{D} = \frac{1}{N}\left(1 + \sum_{i=2}^{K^*-1} i2^{i-2} + K^*(N - 2^{K^*-2})\right).$$

If $N = 2^{K^*-1}$, the average delay is: $\bar{D} = \log_2 N + \frac{1}{N}$.

Theorem 1 *In a homogeneous P2P streaming system, the snow-ball chunk dissemination approach simultaneously achieves the minimum average peer delay and the minimum worst-case peer delay.*

Proof: For an arbitrary chunk dissemination approach, let $x(i)$ denote the number of peers that have the chunk at the beginning of time slot i. Since the server will upload the chunk to the first peer at time slot 0, we always have $x(1) = 1$. $x(i)$ is necessarily a non-decreasing function of i. We define a peer delay function $T(k)$ as the delay for the k-th peer to receive the chunk. Then the worst-case peer delay is $T(N)$ and the average delay is $\sum_{k=1}^{N} T(k)/N$. Given $\{x(i), i \geq 1\}$, $T(k)$ can be calculated as

$$T(k) = \min\{i : x(i) \geq k\}, \qquad 1 \leq k \leq N. \tag{3}$$

Due to the homogeneous unit uploading bandwidth among peers, we always have $x(i+1) \leq 2x(i)$, i.e., a peer can at most upload the chunk to another peer within one time slot. By induction, $x(i) \leq 2^{i-1}$.

For the snow-ball approach, $x^*(i) = 2^{i-1}$. Therefore, for any other chunk dissemination approach, $x(i) \leq x^*(i), i \geq 1$. Let $T^*(k)$ be the peer delay function for the snow-ball approach. Since $x(i) \leq x^*(i), i \geq 1$, due to (3), we have $T^*(k) \leq T(k), k \geq 1$. Therefore, the snow-ball chunk approach simultaneously achieves the shortest average and worst-case chunk dissemination delay. ■

Table 1 compares the delay performance of snow-ball chunk dissemination with tree-based and the optimal multiple-tree approach: For a system of 1024 peers, if the transmission delay of a chunk is 0.2 second,

Table 1: Minimum Delay Achieved by Different Streaming Strategies for Homogeneous Case

Peer Delay	Single-Tree	Multi-Tree, Parallel	Multi-Tree, Sequential	Snow-ball Chunk
average	$\frac{N+1}{2}$	$1.89\log_2 N + o(1)$, m=3	$1.25\log_2 N + o(1)$, m=4	$\log_2 N + \frac{1}{N}$
worst-case	N	$1.89\log_2 N + o(1)$, m=3	$1.89\log_2 N + o(1)$, m=3	$\log_2 N + 1$

it takes only 2 second for the snow-ball approach to complete chunk dissemination to all peers, while the minimum delay achieved by multi-tree approach is 3.78 second. Since the single-tree approach degrades to a chain, peers' average delay is around 100 second.

In the snow-ball approach, peers who receive the chunk in the k-th time slot upload the chunk for $K^* - k$ times, the peers who receive the chunk in the last time slot (about half of the peers) don't get a chance to upload the chunk to other peers. Their uploading bandwidth can be utilized to upload other chunks in continuous video streaming when multiple chunks are in transition simultaneously. We will further show in Section 4 that the snow-ball chunk dissemination can be extended to *snow-ball continuous streaming* to continuously disseminate a stream of chunks and the worst-case delay for each chunk is still $1 + \lceil \log_2 N \rceil$. The snow-ball streaming in Section 4 is designed in an optimal way such that the uploading bandwidth of all peers are fully utilized to achieve the minimum delay bound for each chunk.

3.1.4 Effect of Increasing Server Bandwidth

If the server bandwidth is increased from 1 to C, we can divide N peers into C clusters, and let the server upload the chunk to one peer in each cluster within one time slot. Then, within each cluster, we can employ tree, multi-tree or snow-ball approach to disseminate the chunk. For the chain approach, the delay can be reduced by a factor of C. However, for both multi-tree and snow-ball approach, the improvement is only a constant proportional to $\log_2 C$. If the server participates in the snow-ball dissemination, at each round, the server can upload the chunk to C peers. Let $z(k)$ be the number of nodes (including the server) with the chunk, we have

$$z(k) = 2 * z(k-1) + C - 1 = (2^k - 1)C + 1.$$

Therefore the finish time is $\lceil \log_2(\frac{N}{C} + 1) \rceil$. The delay improvement is still bounded by $\lceil \log_2 C \rceil$.

3.1.5 Effect of Increasing Peer Bandwidth

Secondly, if we also increase the bandwidth of each peer from 1 to C, in the tree based approach, the server can simultaneously upload to C peers within one timeslot, and each peer can also upload to C peers within one timeslot. Therefore, we can construct a streaming tree rooted at the server with node degree of C. The delay performance can be calculated in a similar way of the Multi-Tree case. If parallel uploading is employed, both the average and worst-case delay is $\log_C(N) + o(1)$. If sequential uploading is employed, the worst case delay is still $\log_C(N) + o(1)$, the average delay can be reduced to $\frac{C+1}{2C}\log_C(N) + o(1)$.

Multi-Tree approach can still be utilized. Now all the uploading from a peer to its children can be accelerated by a factor of C. The delays can be reduced to $1/C$ of the unit bandwidth case. Then the optimal degrees for parallel uploading and for sequential uploading remain to be 3 and 4 respectively.

For the snow-ball algorithm, at each time slot, each peer can upload to C peers simultaneously, therefore,

$$x(k) = (C+1) * x(k-1) = (C+1)^{(k-1)}.$$

The finish time is $K_{min} = \lceil \log_{C+1} N \rceil + 1$.

In the previous calculation, we assume a peer uploads the chunk simultaneously to C children peers. All C children peers will receive the chunk at the end of the time slot. From the study of multi-tree approaches, we learned that sequential uploading can achieve better average delay performance than parallel uploading. We can adopt sequential uploading in snow-ball approach. A peer uploads the chunk to C other peers *sequentially*, so that the peer receive the chunk first can immediately upload to other peers without wait for the next time-slot. The delay performance in this case is actually $\frac{\lceil \log_2 N \rceil + 1}{C}$. This is because, with bandwidth C and sequential upload, each peer can finish the upload of one chunk within $1/C$ time slot. If we change the time unit to be $1/C$ of the original time unit, the server and peer bandwidth becomes 1, we go back to the homogeneous case in Section 3.1 , all peers can receive the chunk within $\lceil \log_2 N \rceil + 1$ small time slots, which is $\frac{\lceil \log_2 N \rceil + 1}{C}$ original time slots.

3.2 Heterogeneous Cases

In real network environment, different peers have different types of network access, therefore, different uploading bandwidth. From the study so far, the chunk dissemination delay is determined by how quickly peers' bandwidth can be utilized to upload the chunk. We define the system-wide *usable uploading bandwidth* $\mathcal{U}(t)$ for the chunk as the aggregate uploading bandwidth that can be utilized to upload the chunk at any time t. In the homogeneous case, every peer has the same uploading bandwidth. $\mathcal{U}(t)$ is proportional to the number of peers with the chunk $x(t)$. The order at which peers receive the chunk has no impact on how $\mathcal{U}(t)$ grows over time. However, in a heterogeneous environment, the order at which peers receive the chunk determines the growth speed of $\mathcal{U}(t)$, and consequently the chunk dissemination delay. For the quick growth of $\mathcal{U}(t)$, the intuition is to upload the chunk to peers with large uploading capacities first.

In this section, we study the impact of uploading bandwidth heterogeneity among peers on the chunk dissemination delay by studying several typical cases. It will become clear that the peer uploading bandwidth heterogeneity enables the snow-ball approach to achieve a shorter chunk dissemination delay than the homogeneous case.

3.2.1 Case 1: Super-peers and Free-riders

Suppose there are N/C super peers that can upload at rate $C > 1$. All the remaining peers are free-riders and don't participate in the uploading. The chunk can be disseminated by the snow-ball approach to all N/C super peers within $1 + \frac{1}{C}\lceil \log_2(N/C) \rceil$ time slots. Then all super peers can upload the chunk to the remaining $(1 - 1/C)N$ free-riders in $1 - 1/C$ additional time slot. The total delay is $\frac{\lceil \log_2(N/C) \rceil - 1}{C} + 2$. In this case, the average uploading bandwidth of peers are $\bar{u} = 1$. If all peers have the average uploading bandwidth 1, the shortest delay is $\lceil \log_2 N \rceil + 1$, which is around C times of the heterogeneous case. This shows that the heterogeneity of peer uploading bandwidth helps reduce the chunk dissemination delay.

3.2.2 Case 2: Multi-level Bandwidth Hierarchy

In the previous case, peers form a two-level hierarchy according to their uploading contribution. A fraction of $1/C$ super peers with uploading bandwidth C stay at the top level and feed video chunk to the free-riders at the bottom level. In real network environment, peers can be clustered based on the types of their network access. In this case, we extend the two-level hierarchy to accommodate multiple levels and show that even a very small percentage of super peers can bootstrap the chunk dissemination.

Suppose there are N_1 super peers with bandwidth C_1, N_1N_2 medium peers with bandwidth C_2 and $N_1N_2N_3$ slow peers with bandwidth C_3. To quickly disseminate the chunk to all peers, the following chunk scheduling algorithm can be employed:

1. use the snow-ball algorithm to upload to N_1 super peers within time $1 + \frac{1}{C_1}\lceil \log_2 N_1 \rceil$;
2. each of those N_1 super peers acts as a server with bandwidth C_1 and uploads to N_2 other medium peers. As studied in Section 3.1.4, the uploading can finish within time $1 + \frac{\lceil \log_2(N_2/C_1) \rceil}{C_2}$, now N_1N_2 medium peers have the chunk;
3. each of those N_1N_2 medium peers acts as a server with bandwidth C_2 and uploads to N_3 other slow peers within time $1 + \frac{\lceil \log_2(N_3/C_2) \rceil}{C_3}$, now $N_1N_2N_3$ slow peers have the chunk.

The total delay is

$$3 + \frac{\log_2 N_1}{C_1} + \frac{\log_2 (N_2/C_1)}{C_2} + \frac{\log_2 (N_3/C_2)}{C_3}.$$

Without those super and medium peers, the fastest chunk dissemination to $N_1N_2N_3$ slow peers takes time $1 + \frac{1}{C_3}(\log_2 N_1 + \log_2 N_2 + \log_2 N_3)$.

This suggests that the existence of super peers (even if a very small percentage) can dramatically reduce the chunk dissemination delay. For example, to disseminate a chunk to $32k = 2^{15}$ peers with bandwidth 1 need at least 15 time slots. Meanwhile, if $N_1 = N_2 = N_3 = 32$, and $C_1 = 10$, $C_2 = 5$, $C_3 = 1$, in other words, 32 (only 0.1%) of them have bandwidth of 10 and 1024 (only 3%) of them have bandwidth of 5, the time to disseminate a chunk to all $33k$ peers is less than 5.2 time slots. The example can be easily extended to incorporate more than 3 levels. Another insight obtained from this example is that: peers should be organized into tiers according to their uploading bandwidth, peers within each tier should help each other to obtain the chunk in the shortest possible time, then pass it down to the neighboring lower tier. This way, the delay of dissemination to the whole network can be reduced.

3.2.3 General Heterogeneous Case

For general heterogeneous case, one can index peers according to the decreasing order of their uploading capacities. Suppose the sorted uploading capacities of peers are: $u_1, u_2, ...u_N$. To derive a lower bound on the shortest chunk dissemination time, let's allow chunk stripping, namely, multiple peers can upload different portions of a chunk to the same peer simultaneously. If the first k peers have the chunk at time t, the uploading to peer $k+1$ can finish by $\frac{1}{\sum_{j=1}^{k} u_j}$, therefore the lower delay bound can be calculated as

$$\hat{D} = 1 + \sum_{i=1}^{N-1} \frac{1}{\sum_{j=1}^{i} u_j}.$$

However, this is a loose bound. For example, for the homogeneous case, the bound is $\hat{D} = 1 + \sum_{i=1}^{N-1} \frac{1}{i} \leq 2 + \ln(N-2)$. We know the shortest delay without chunk stripping is instead $1 + \log_2(N)$. In this section, we study several variations of the snow-ball algorithm to accelerate the chunk dissemination in general heterogeneous case.

Heterogeneous Parallel Snow-ball Approach:

Assume $\{u_i\}$ are all integers. let $x(k)$ be the number of peers with the chunk at the beginning of time slot k.

1. In time slot 0, the server uploads the chunk to peer 1. x(1)=1;

2. In time slot $k \geq 1$, any peer with ID j, $1 \leq j \leq x(k)$, uploads the chunk in parallel to peers with IDs from $x(k) + \sum_{i=1}^{j-1} u_i$ to $x(k) + \sum_{i=1}^{j} u_i$;

3. $x(k+1) = x(k) + \sum_{i=1}^{x(k)} u_i$. If $x(k+1) < N$, $k = k+1$, go back to step 2; otherwise finishes.

This way, peers with larger uploading bandwidth will receive the chunk first and continuously upload the chunk to other peers until all peers receive the chunk. Let $\bar{u} = (\sum_{i=1}^{N} u_i/N)$ be the average uploading bandwidth among peers. Since peers are sorted according to the decreasing order of their uploading capacities, we have

$$x(k+1) = x(k) + \sum_{i=1}^{x(k)} u_i \geq x(k) + x(k)\bar{u} = (\bar{u}+1)x(k).$$

By induction, we will have $x(k) \geq (\bar{u}+1)^{k-1}$. Therefore the finish time is less than $\lceil \log_{\bar{u}+1} N \rceil + 1$, which is the delay of the parallel snow-ball approach in a system with homogeneous peer uploading bandwidth of $\bar{u}$ as studied in Section 3.1.5. This again demonstrates that snow-ball chunk dissemination approach has even better delay performance when peers have heterogeneous uploading bandwidth.

In this approach, due to parallel uploading, peers receive the chunk at the end of some time slot. Since we know sequential uploading has superior delay performance than parallel uploading, we can also develop a sequential snow-ball approach for heterogeneous systems. After receive the chunk, a peer will continuously upload it to other peers one after another. Since peers have different uploading bandwidth, the finish time of chunk uploading by different peers are no longer aligned. This makes it difficult to coordinate the uploading scheduling among peers. Here we develop a greedy snow-ball scheduling algorithm to achieve short delays in heterogeneous uploading.

Heterogeneous Sequential Snow-ball Approach: Again, index peers in the decreasing order of their uploading capacities. At any time instant t, let $E(t)$ be the ordered set of peers without the chunk, and $U(t)$ the ordered set of uploading peers. At any time, the status of a peer in $U(t)$ can be in either *busy*, meaning it is uploading the chunk to some peer, or *ready*, meaning it is available for next uploading.

1. Initialization: $U(1) = \{1\}$, set peer 1's status to *ready*; $E(1) = \{2, \cdots, N\}$;

2. Choose the first peer i in the ordered set U with status *ready*, pick the first peer j from the ordered set E, let peer i upload the chunk to peer j using its uploading bandwidth u_i, set peer i's status to *busy*, and remove peer j from set E. Repeat this step until either no peers are *ready* in U or E is empty;

3. After peer i completes the uploading to peers j, add j to U, set j's status to *ready*, also set peer i's own status to *ready*. If E is not empty yet, go back to step 2.

However, due to the misalignment of the finish time of uploading events, this algorithm cannot guarantee to achieve the minimum delay. For example, for a system with 5 peers, if peer 1's uploading bandwidth is 10, other peer uploading capacities are 1, 1, 1, 1. When peer 1 finishes the upload to peer 2, peer 1 will upload the chunk to peer 3, and peer 2 will upload the chunk to peer 4. Then peer 4 will receive the chunk after 1.1 time slots. However, if we just let peer 1 upload the chunk to all other peers, every peer can get the chunk by 0.4 time slots. It is possible to develop an optimal uploading schedule for peers by carefully calculating the finish time instants for all possible upload combinations for all peers. We skip the discussion here.

4 Snow-ball Streaming

In single chunk dissemination, any peer can be utilized to upload the chunk after it has downloaded the chunk. In continuous streaming, one new chunk is generated every time slot. When the server capacity is less than N, one chunk cannot be disseminated to all pees within one time slot. Therefore, there will be more than one chunk in transition at any given time. If K^* is the minimum transmission delay for a single chunk, there will be at least K^* chunks in transition at any given time. If the chunk scheduling is not set up appropriately, some chunks cannot be disseminated to all peers within K^* time slots.

4.1 Homogeneous Environment

In this section, we show that, for the homogeneous case, it is possible to set up a chunk schedule such that all chunks can be disseminated to all peers within the minimum delay time. In the snow-ball chunk dissemination approach, the server uploads the chunk to the first peer at time slot 0. Before the beginning of time slot $K^* = \lceil \log_2(N) \rceil + 1$, all N peers will receive the chunk. Let $\phi(j)$ be the number of peers with the chunk at the beginning of time slot j and will upload that chunk in time slot j. We have

$$\phi(j) = \begin{cases} 2^{j-1} & 1 \le j \le K^* - 2 \\ N - 2^{\lceil \log_2(N) \rceil - 1} & j = K^* - 1 \\ 0 & j \ge K^* \end{cases}$$

We call $\Phi^* \triangleq \{\phi(j) : j = 1 \cdots K^* - 1\}$ the *snow-ball chunk dissemination profile*.

Theorem 2 *For a homogeneous P2P streaming system, there exists a continuous streaming schedule such that all chunks in the stream will be disseminated to all peers with the shortest delay K^* achieved by the snow-ball algorithm for single chunk dissemination.*

Proof: Without loss of generality, the server uploads chunk $i \ge 0$ to some peer at time slot i. Let $y_i(k)$ be the number of peers that have chunk i and will upload chunk i to other peers at time slot k. For any feasible schedule, we should have $\sum_{i=0}^{\infty} y_i(k) \le N, \forall k$, i.e., at any time slot the aggregate uploading bandwidth for all chunks is at most N, and $y_i(k+1) \le 2 * y_i(k)$, i. e., each peer can upload to at most one peer within any time slot. A streaming schedule can achieve the optimal delay K^* for each chunk if and only if each chunk can be uploaded according to the snow-ball chunk dissemination profile Φ^* after it is uploaded to some peer by the server, i. e.,

$$y_i(k) = \begin{cases} \phi(k-i) & (i+1) \le k < i + K^* \\ 0 & \text{otherwise} \end{cases}$$

It can be verified that such a schedule satisfies the feasibility constraints:

$$\sum_{i=0}^{\infty} y_i(k) = \sum_{i=k-K^*+1}^{k-1} y_i(k) = \sum_{j=1}^{K^*-1} \phi(j) = N - 1$$

and $y_i(k+1) \le 2 * y_i(k)$.

To complete the proof, for each time slot, we need to construct a uploading schedule for all active chunks. Let $\mathcal{S}$ be the set of all peers. Denote by $\mathcal{S}_i(k)$ the set of peers with chunk i at the beginning of time slot k and will upload the chunk to $|\mathcal{S}_i(k)|$ other peers without chunk i in the time slot. To follow the optimal dissemination profile Φ^*, it is sufficient to have $|\mathcal{S}_i(k)| = y_i(k)$ and $\{\mathcal{S}_i(k), k \ge 1\}$ are pairwise disjoint (since each peer can only upload one chunk in one time slot). We call the previous condition the sufficient condition Λ to achieve the minimum delay streaming. We complete the proof of the theorem by constructing a chunk uploading schedule for each time slot through inductions:

Initial condition: *The server uploads chunk 0 to peer 0 in time slot 0. Therefore, at the beginning of time slot 1, $\mathcal{S}_0(1) = \{0\}$, and $\mathcal{S}_i(1) = \emptyset, i > 0$. It can be easily verified that the sufficient condition Λ is satisfied at the beginning of time slot 1.*

Induction: *If at the beginning of time slot $k \geq 1$, the condition Λ is satisfied, we can construct a schedule in time slot k, such that Λ is still satisfied at the beginning of time slot $k+1$.*

At the beginning of time slot k, according to Λ, $k_o = \max(k - K^* + 1, 0)$ is the ID of the oldest chunk that needs to be uploaded in time slot k. Then $\mathcal{S}_i(k) = \emptyset$, $\forall i < k_o, \forall i \geq k$; and $\{\mathcal{S}_i(k), k_o \leq i < k\}$ are pairwise disjoint, $|S_i(k)| = y_i(k)$. Define a set $\mathcal{F}(k) = \mathcal{S} - \cup_{i=k_o}^{k-1}\mathcal{S}_i(k)$, i.e., the set of peers that don't need to upload any chunk at the beginning of time slot k. The following scheduling will guarantee the Λ condition is still satisfied at the beginning of time slot $k+1$.

I. If $k_1 = k - K^* + 1 \geq 0$, chunk k_1 will be uploaded for the last time in slot k. Since the chunk has been uploaded $1 + \sum_{i=1}^{K^*-2}\phi(i)$ times by the server and peers in the previous $K^* - 1$ time slots, only $\phi(K^* - 1)$ peers don't have it. Let all peers in set $\mathcal{S}_{k_1}(k)$ upload chunk k_1 to those peers and finish the upload of chunk k_1. Peers in $\mathcal{S}_{k_1}(k)$ can be used to upload other chunks in time slot $k+1$. We set $\mathcal{F}(k) = \mathcal{F}(k) \cup \mathcal{S}_{k_1}(k)$. Then $|\mathcal{F}(k)| \geq \phi(K^* - 1)$.

II. If $k_2 = k - K^* + 2 \geq 0$, chunk k_2 will be uploaded for the second-to-last time in slot k. According to Φ^*, $\phi(K^* - 2)$ peers in set $\mathcal{S}_{k_2}(k)$ will upload chunk k_2 to other peers that don't have chunk k_2. In addition, the schedule should guarantee that there will be $\phi(K^* - 1)$ peers available in time slot $k+1$ to upload chunk k_2.

If $\phi(K^* - 1) \leq \phi(K^* - 2)$, let each peer in $\mathcal{S}_{k_2}(k)$ upload chunk k_2 to any peer without chunk k_2, then pick $\phi(K^* - 1)$ peers out of $\mathcal{S}_{k_2}(k)$ to form the set of peers to upload chunk k_2 in next time slot, i.e., $\mathcal{S}_{k_2}(k+1)$. Other peers in $\mathcal{S}_{k_2}(k)$ can be used to upload other chunks in time slot $k+1$. We set $\mathcal{F}(k) = \mathcal{F}(k) \cup \mathcal{S}_{k_2}(k) - \mathcal{S}_{k_2}(k+1)$. We have $|\mathcal{F}(k)| \geq \phi(K^* - 2)$

If $\phi(K^* - 1) > \phi(K^* - 2)$, from step 1, $|\mathcal{F}(k)| \geq \phi(K^* - 1)$, we can take a subset $\mathcal{M}(k)$ of $\phi(K^*-1) - \phi(K^*-2)$ peers out of $\mathcal{F}(k)$, and let $\phi(K^*-1) - \phi(K^*-2)$ peers in $\mathcal{S}_{k_2}(k)$ upload chunk k_2 to peers in $\mathcal{M}(k)$. Remaining peers in $\mathcal{S}_{k_2}(k)$ then upload chunk k_2 to arbitrary peers without chunk k_2. Now peers in $\mathcal{M}(k)$ are ready to upload chunk k_2 in time slot $k+1$, therefore, we set $\mathcal{S}_{k_2}(k+1) = \mathcal{S}_{k_2}(k) \cup \mathcal{M}(k)$; $\mathcal{F}(k) = \mathcal{F}(k) - \mathcal{M}(k)$. We also have $|\mathcal{F}(k)| \geq \phi(K^* - 2)$.

III. Let $k_3 = \max(k - K^* + 3, 0)$. Any chunk i, $i \in [k_3, k-1]$, needs to be uploaded to $\phi(k-i)$ peers by peers in set $\mathcal{S}_i(k)$. We have

$$\sum_{i=k_3}^{k-1} |\mathcal{S}_i(k)| \leq \sum_{j=1}^{K^*-3} \phi(j) = \phi(K^* - 2) - 1 \leq |\mathcal{F}(k)| - 1. \tag{4}$$

Then $\forall i \in [k_3, k-1]$, take a subset $\mathcal{U}_i(k)$ of $|\mathcal{S}_i(k)|$ peers out of $\mathcal{F}(k)$, let all peers in $\mathcal{S}_i(k)$ upload chunk i to peers in $\mathcal{U}_i(k)$, and set $\mathcal{S}_i(k+1) = \mathcal{S}_i(k) \cup \mathcal{U}_i(k)$, $\mathcal{F}(k) = \mathcal{F}(k) - \mathcal{U}_i(k)$. At the end, due to (4), we will have $|\mathcal{F}(k)| \geq 1$.

IV. The server uploads chunk k to some peer m_k in $\mathcal{F}(k)$, and set $\mathcal{S}_k(k+1) = \{m_k\}$.

Following the previous scheduling steps, the sufficient condition Λ will be satisfied at the beginning of time slot $k+1$.

Conclusion: *There exists a schedule such that all chunks can be disseminated with snow-ball chunk dissemination profile Φ^* and achieve the optimal delay K^*.* ■

For the special case of $N = 2^m$, we assign each peer an ID n, $0 \leq n \leq N-1$, and each chunk an ID i, $i \geq 0$. Chunk i will be injected to the system by the server in time slot i. At the beginning of time slot k, half of the peers have chunk $k - m$ and will upload it to the other half in time slot k. The snow-ball streaming schedule can be setup as follows.

1. Let $l(j) = (k+j) \mod m, 0 \leq j < m$;

2. For chunk $i = k - m + j$, $0 \leq j < m$, $\mathcal{S}_i(k) =$\{peers with $bit(l(j)) = 1$, and $bit(l(w)) = 0, 0 \leq w < j$\};

3. A peer in $\mathcal{S}_i(k)$ with ID x uploads chunk i to a peer with ID $x + 2^{l(0)}$;
4. Sever uploads chunk k to the peer with ID $2^{l(0)}$.

Figure 3 illustrates the previous snow-ball streaming schedule in a system with 8 peers. We use a sequence of 9 subfigures to show the snow-ball chunk schedules among all peers within 9 consecutive time slots. Blocks represent chunks and circles represent peers. For time slot k, a white chunk beside a peer is the chunk that the peer has and will be uploaded to another peer within that time slot. An arc from peer i to j indicates peer i uploads its chunk to peer j. A black chunk beside a peer indicates the server will inject that chunk to the peer in time slot k. Chunk 0 is uploaded to all peers by the end of time slot 3 and chunk 1 is uploaded to all peers by the end of time slot 4. The example shows that all chunks can be disseminated to all peers 3 time slots after it is injected by the server. Figure 4 shows the sets of IDs of peers that have different chunks at the beginning of three consecutive time slots.

4.2 Heterogeneous Environment

For heterogeneous case, the delay bound for single chunk dissemination cannot always be achieved in streaming. For example, if the server's upload capacity is 1, and 7 peers' upload capacities are $2, 1, 1, 1, 1, 1, 0$, a single chunk dissemination can be done in 3 time slots, however, no streaming algorithm can achieve this. If peer 0 is still uploading chunk 0 at timeslot 2, chunk 1 cannot be uploaded according to the greedy chunk profile Φ^*. In this case, the first peer with bandwidth 2 becomes the scheduling bottle-neck for adjacent chunks. For the two special heterogeneous cases considered in Section 3.2, we are able to prove the existence of snow-ball streaming to achieve the minimum chunk dissemination delay for all chunks.

Theorem 3 *For a P2P streaming system with N/C super peers and $(1 - 1/C)N$ free-riders, there exists a continuous streaming schedule such that all chunks in the stream will be disseminated to all peers within a delay of $\frac{\lceil \log_2(N/C) \rceil}{C} + 2$ time slots.*

Proof: The idea is to first make sure all chunks can be streamed to all super peers within $1 + \frac{1}{C}\lceil \log_2(N/C) \rceil$ time slots. Then super peers will upload to free-riders whenever they have spare bandwidth. To achieve this, we change the time unit to $1/C$ of the original time slot. Measured in the new time slot, the server generates one new chunk every C time slots. Suppose server only has uploading capacity of 1, and uploads chunk i to some super peer by the end of time slot $C(i + 1)$. For time slot k, let $y_i(k)$ be number of super peers uploading chunk i to other super peers. To achieve the minimum streaming delay among all super peers, let $K^* = \lceil \log_2(N/C) \rceil$, we need

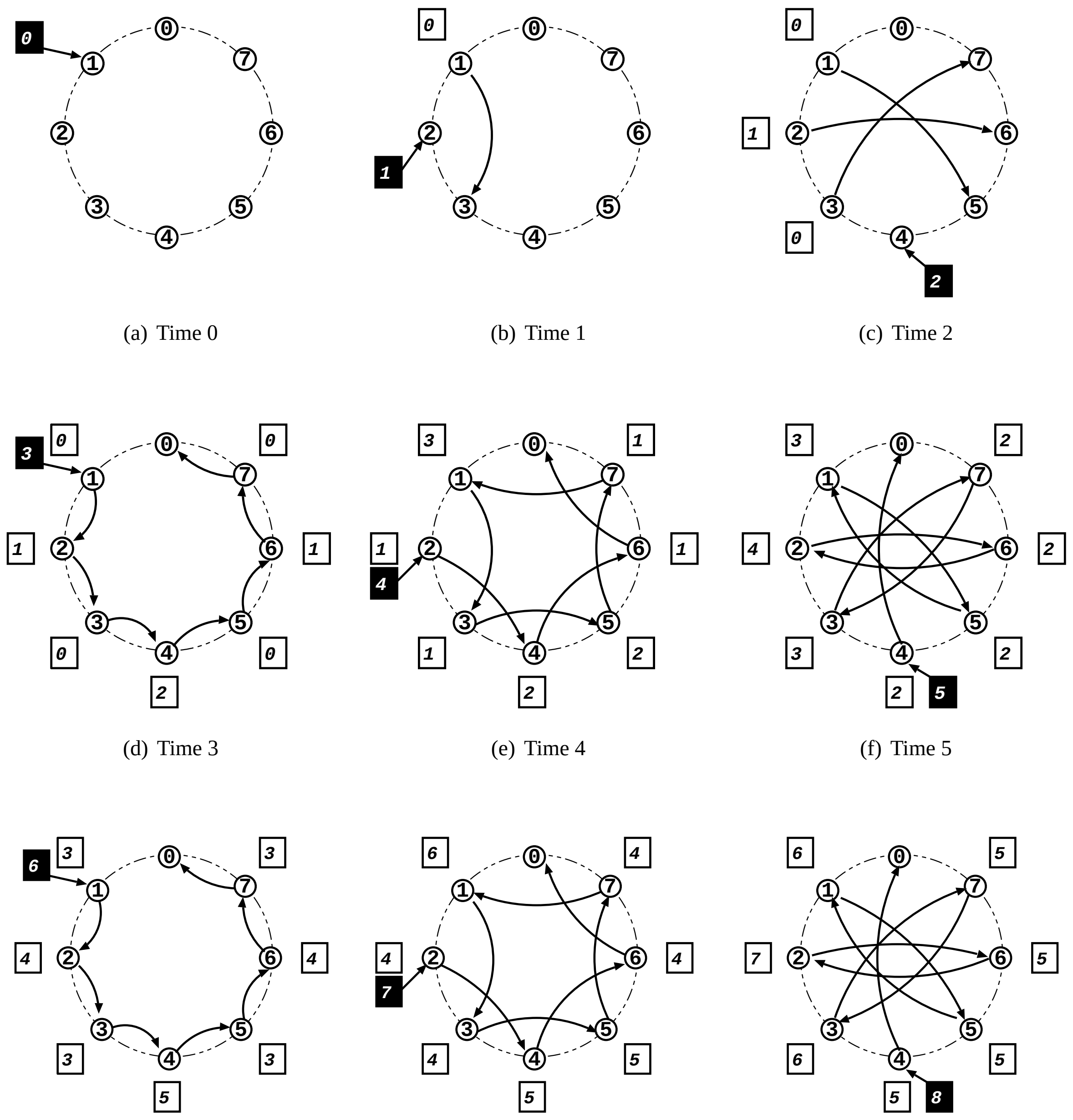
$$y_i(k) = \begin{cases} \phi(k - C(i+1)) & C(i+1) + 1 \le k \le C(i+1) + K^* \\ 0 & \text{otherwise} \end{cases}$$

Let $i_1(k) = \lceil \frac{k-K^*}{C} \rceil - 1$ and $i_2(k) = \lfloor \frac{k-1}{C} \rfloor - 1$, $y_i(k) > 0$ iff $i_1(k) \le i \le i_2(k)$. Then

$$\sum_{i=0}^{\infty} y_i(k) = \sum_{i=i_1(k)}^{i_2(k)} \phi(k - C(i+1)) < N/C - 1$$

and $y_i(k+1) \le 2 * y_i(k)$. According to Theorem 2, there exists a streaming schedule such that all super peers can receive the chunk within $1 + \frac{1}{C}\lceil \log_2(N/C) \rceil$ time slots. In addition, it can be shown that

$$\sum_{j=0}^{C-1} \sum_{i=0}^{\infty} y_i(k+j) = \sum_{j=0}^{C-1} \sum_{i=i_1(k+j)}^{i_2(k+j)} \phi(k + j - C(i+1)) = N/C - 1.$$

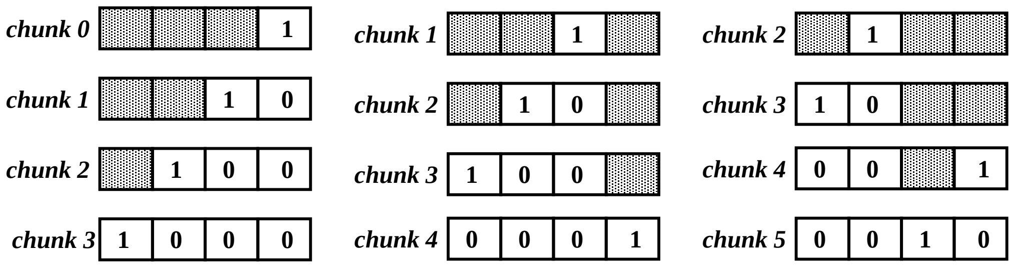


(a) Time 0 (b) Time 1 (c) Time 2

(d) Time 3 (e) Time 4 (f) Time 5

(g) Time 6 (h) Time 7 (i) Time 8

Figure 3: Evolution of Chunk Scheduling of Snow-ball Streaming among 8 Peers



(a) Time 3 (b) Time 4 (c) Time 5

Figure 4: Evolution of the Set of Peer IDs for Different Chunks

In other words, in any C consecutive time slots, the aggregate number of uploading to super peers equals the number of super peers minus one. Since all super peers can upload N times in C time slots (one original time slot), therefore, we have $(1 - 1/C)N + 1$ spare super peer uploading available every C time slots. After all super peers get chunk i at time slot $C(i+1) + K^*$, in the following C time slots, any super peer that is not responsible for uploading new chunks to other super peers can be utilized to upload chunk i to a free-rider, and all free-riders can get the chunk by time slot $C(i+1) + K^* + C$. The achieved streaming delay is $2C + K^*$ sub-time slots, which is $\frac{\lceil \log_2(N/C) \rceil}{C} + 2$ original time slots. ■

We list the chunk schedule for a system with 8 super-peers and 8 free-riders in Table 2. Super-peers are indexed from 0 to 7, each super peer has uploading capacity of 2, free-riders are labeled from a to h. An tuple (x, y) at row i column j means super peer i will upload chunk x to peer y in time slot j. A chunk is uploaded to all super peers first, then it will be uploaded to all free-riders within one additional round. The overall chunk dissemination delay is 3.5 time slots.

Table 2: Schedule between Super-peers and Free-riders

ID	1	1.5	2	2.5	3	3.5	4	4.5	5
0	0, 1	0, 2	0, 4	0, a	2, 1	2, 2	2, 4	2, a	4, 1
1		0, 3	0, 5	0, b		2, 3	2, 5	2, b	
2			0, 6	0, c	0, g	1, a	2, 6	2, c	2, g
3			0, 7	0, d	0, h	1, b	2, 7	2, d	2, h
4				0, e	1, 0	1, c	1, g	2, e	3, 0
5				0, f	1, 1	1, d	1, h	2, f	3, 1
6				1, 4	1, 2	1, e		3, 4	3, 2
7			1, 6	1, 5	1, 3	1, f	3, 6	3, 5	3, 3

Corollary 4 *If peers in a streaming system form a M-level hierarchy with $\prod_{k=1}^{i} N_k$ peers on level i with uploading capacity of C_i, ($C_i > C_{i+1} \geq 1$), there exists a continuous streaming schedule such that chunks can be streamed to all peers with a delay of $M + \sum_{j=1}^{M} \frac{\lceil \log_2(N_i/(C_{i-1}-1)) \rceil}{C_i}$, where $C_0 = 2$.*

Proof: We can construct the chunk scheduling iteratively. Peers at level 1 pick $(C_1 - 1)N_1$ peers from level 2 as their free-riders. Construct a streaming schedule at level 1 according to theorem 3 such that $(C_1 - 1)N_1$ peers from level 2 will receive all chunks with delay $2 + \frac{\log_2(N_1)}{C_1}$. Then each of those peers can lead the snow-ball streaming to $N_2/(C_1 - 1)$ peers at level 2 and $(C_2 - 1)N_2/(C_1 - 1)$ free-riders from level 3, by time $3 + \frac{\log_2(N_1)}{C_1} + \frac{\log_2(N_2/(C_1-1))}{C_2}$, $(C_2 - 1)N_1 N_2$ peers at level 3 will receive the chunk. They continue to do snow-ball streaming from level 3 to 4. The process can continue until all peers at the bottom level receive the chunk. ■

5 Impact of Network Impairments

In real networks, the performance of P2P video streaming is subject to various network impairments. In this section, we evaluate the performance of the snow-ball chunk dissemination in network settings with long propagation delays and random bandwidth variations.

5.1 Impact of Propagation Delays

From the analysis in the previous sections, using smaller chunks in P2P video streaming leads to smaller chunk transmission delay, consequently smaller overall dissemination latency. On the other hand, using smaller chunks increases the signaling overhead and the scheduling complexity among peers. Meanwhile, as the chunk transmission delay getting smaller, the propagation delay between peers will play a more important role. We still use the transmission time of a chunk as the time unit. Now suppose the propagation delay is $P = d - 1$ time slots ($d \geq 2$). The time between a sender begins to upload the chunk and the receiving peer get the whole chunk is d time slots. For the multi-tree approach, if parallel uploading is employed, the chunk transmission delay from a peer to all its children increases from m to $d + m - 1$, the delay performance is $\frac{m+d-1}{\log_2 m}\log_2 N$; if sequential uploading is employed, the worst case delay is still $\frac{m+d-1}{\log_2 m}\log_2 N$, and the average delay is $\frac{2d+m-1}{2\log_2 m}\log_2 N$.

Again, denote by $x(k)$ the number of peers with the chunk at the beginning of time slot k. All the chunks received right before the beginning of time slot k were sent out at the beginning of time slot $k-d$. Therefore we have

$$x(k) = x(k-1) + x(k-d).$$

$x(k)$ is a Fibonacci series with time lag d ($d = 2$ is the standard Fibonacci series). We can solve $x(k)$ by taking Z-Transform:

$$X(z) = \frac{Z^{-1}}{1 - Z^{-1} - Z^{-d}}, \rightarrow x(k) \sim \alpha_*^k,$$

where α_* is the largest root of $1 - Z^{-1} - Z^{-d}$. The finish time is approximately $\log_{\alpha_*} N$, which is $\ln 2/\ln\alpha_*$ times of the snow ball delay without propagation delay. We plot the evolution of chunk dissemination for at different propagation delays in Figure 5. Among them, $P = 0$ corresponds to the case when the propagation delay is ignored as studied in Section 3. As predicted by the Z-transform analysis, the number of peers with the chunk grows exponentially after the first few time steps. For any propagation delay, the exponential growth rate, i.e., the slope of the curve in semi-log plot, is determined by the dominating root of $X(z)$.

We compare the delay performance of multi-tree based strategies and the snow ball strategy in Table 3. The delay performance is measured in the unit of the average delay of snow-ball approach when there is no propagation delay. For parallel multi-tree strategy, we fix the node degree $m = 3$ that minimizes the average and worst-case delay when there is no propagation delay. For sequential multi-tree strategy, at different propagation delays, the node degree is optimized for the average delay. The associated worst-case delay is also calculated. As the propagation delay increases, the delay performance of all three strategies degrades. For parallel multiple tree with fixed degree, its delay increases fastest among the three. As the propagation delay increases, the optimal node degree for sequential multi-tree also increases. This is because propagation delays provide additional chance for pipelining chunk transmissions from a peer to its children. Sequential multi-tree strategy explore this pipelining gain. It increases node degree and a peer will spend more time

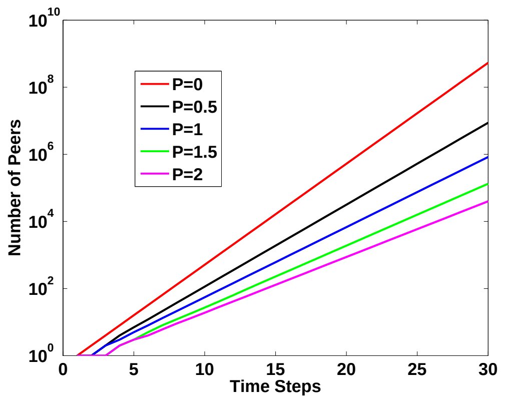


Figure 5: Chunk dissemination speed at different propagation delays.

to upload the same chunk to all its children. This makes it closer to the uploading philosophy of snow-ball streaming: *a peer should keep uploading the same chunk until all peers have it.* As a result, sequential multi-tree has better delay performance than the fixed-degree parallel multi-tree. However, its worst-case delay performance is much worse than that of the snow-ball approach. This is because leaf peers at the bottom level have large delay variations. And leaf peers don't contribute to the uploading of the chunk even if they receive the chunk early. To the contrary, in the snow-ball approach, a peer always contributes to the uploading as long as the chunk is still missing on some peers.

Table 3: Minimum Delay Achieved by Different Streaming Strategies with Propagation Delays.

Prop. Delay	M-Tree, Para., Fix		M-Tree, Seq., Opt.			Snow Ball
	degree	delay	degree	average	worst	
0	3	1.89	4	1.25	2.0	1.0
1	3	2.52	5	1.72	2.58	1.44
2	3	3.15	6	2.13	3.10	1.81
3	3	3.79	7	2.49	3.56	2.15
4	3	4.43	8	2.83	4.0	2.47

More generally, if the propagation delays are random with and the p.m.f of chunk delay (transmission delay of 1 plus a random propagation delay) is $(p_i, \Delta_i), 1 \leq i \leq M$, i.e., a chunk uploaded by a peer at the beginning of time slot k will be received by a peer before the beginning of time slot $k + \Delta_i$ with probability p_i, then in average sense we have

$$x(k) = x(k-1) + \sum_{i=1}^{M} p_i x(k - \Delta_i),$$

Then the average number of peers that receive the chunk in time slot k can be calculated as:

$$X(z) = \frac{X(1)Z^{-1}}{1 - \sum_{i=1}^{M} p_i Z^{-\Delta_i}}$$

If the propagation delay is not a multiple of chunk transmission time, we can pick a fine time unit such that the chunk transmission time is d_1 time units and the propagation delay is d_2 time units. Note that, in this case, a peer can finish the transmission of a chunk in d_1 time units. At the beginning of time slot k, let $x(k)$ be the number of peers with the chunk and $y(k)$ the number of peers that are ready to transmit the chunk. The system evolution can be characterized by:

$$x(k) = \sum_{i=0}^{d_1-1} y(k-i), \quad y(k) = y(k-d_1) + y(k-d_1-d_2),$$

where the first equation is due to the fact that each peer with the chunk will transmit it every d_1 time units; the second equation is because all peers that receive the chunk at the end of time slot $k-1$ are ready to transmit the chunk at the beginning of time slot k, and the received chunk was sent by a peer at the beginning of time slot $k-d_1-d_2$. We have

$$Y(z) = \frac{Y(1)Z^{-1}}{1-Z^{-d_1}-Z^{-(d_1+d_2)}} \quad (5)$$

$$X(z) = \frac{X(1)(Z^{-1}+\cdots+Z^{-(d_1-1)})}{1-Z^{-d_1}-Z^{-(d_1+d_2)}} \quad (6)$$

Again, when k is large, $x(k)$ grows exponentially with some fixed rate.

To study the chunk dissemination speed with random propagation delays, we developed a time-stepped simulator that simulates the progress of snow-ball chunk dissemination with random propagation delays among peers. We simulate a network of $4,000$ peers. At time $t=0$, a peer receives a chunk from a server. Then peers employ snow-ball chunk dissemination to distribute the chunk until all peers receive it. At each time step, we record the number of peers that have received the chunk. The chunk transmission time among peers is constant and set to be 8 simulation time-steps. The propagation delay between two peers follow a truncated normal distribution with mean of $8P$ and standard deviation of $4P$, the upper and lower bound for the random delay is $16P$ and 1 respectively. We conducted simulations for $P=0$ (no delay), $P=1$ and $P=2$. For each case we ran 100 iterations and record the max, min and average number of peers with the chunk at each time step among all iterations. We compare the chunk dissemination speed under random propagation delays with the corresponding constant delay case for $P=1$ and 2. Figure 6 plots the numbers of peers with the chunk at every round of eight time-steps (one chunk transmission time) for all the simulated cases. As predicted by the Z-transform analysis, with constant propagation delays, the number

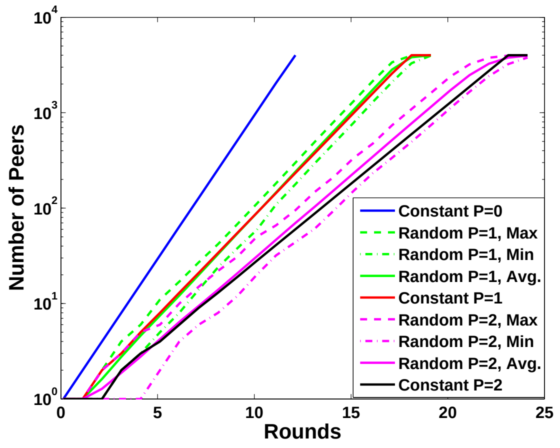


Figure 6: Chunk dissemination speed with constant and random propagation delays.

of peers with the chunk grows exponentially after the first few time steps. For any propagation delay, the exponential growth rate, i.e., the slope of the curve in semi-log plot, is determined by the dominating root of $X(z)$. Random delays did introduce variance in the dissemination speed. However, the snow-ball chunk dissemination still follows the exponential growth trend and the completion time is very close to the constant delay case.

5.2 Impact of Bandwidth Variations

In the previous sections, we assume that peers have constant uploading bandwidth and a chunk transmission completes in constant time: in sequential transmission, a chunk can be transmitted from a peer to another peer in one time slot, in parallel transmission with degree m, a peer can transmit a chunk simultaneously to m children in m time slots. Due to network traffic variations, the available bandwidth on a connection between two peers varies over time. Consequently, the transmission time of a chunk is not constant. In this section, we investigate the robustness of different streaming strategies against the randomness in chunk transmissions.

For the clarity of presentation, we assume all transmission delays are independent and follow the same distribution. We introduce random variable τ^s for sequential transmission time, with $E[\tau^s] = 1$ and $Var[\tau^s] = \sigma^2$; the m-parallel transmission time is τ^p, with $E[\tau^p] = m$ and $Var[\tau^p] = m\sigma^2$.

For the chain-based approach, if peer 0 receives the chunk from the server at time 0, the time for the i-th peer to receive the chunk is $\sum_{j=1}^{i} \tau_i$, where τ_i is the transmission delay from peer $i-1$ to i. And $\{\tau_i\}$ are i.i.d following the distribution of τ^s. Then for the worst-case delay D_N^c is the time for peer N to receive the chunk:

$$E[D_N^c] = N, \quad Var[D_N^c] = N\sigma^2.$$

The average delay among all peers is

$$\bar{D}_c = \frac{1}{N}\sum_{j=1}^{N}\sum_{i=1}^{j}\tau_i = \frac{1}{N}\sum_{j=1}^{N}(N+1-j)\tau_i.$$

Then we have

$$E[\bar{D}_c] = \frac{N+1}{2}; \qquad Var[\bar{D}_c] = \frac{1}{N^2}\sum_{k=1}^{N}k^2\sigma^2 \sim \frac{N\sigma^2}{3}.$$

This suggests that, in a chain topology, the impact of the randomness of individual chunk transmission on the average and worst-case chunk delay performance of all peers is proportional to the number of peers N in the chain.

For the parallel multi-tree approach, all peers at the bottom level will receive the chunk after $\log_m N$ independent parallel chunk transmissions. Then we have for worst-case delay:

$$E[D_N^p] = m\log_m N, \quad Var[D_N^p] = m\log_m N\sigma^2.$$

For the sequential multi-tree approach, there is one peer at the bottom level that will receive the chunk after $m\log_m N$ independent sequential chunk transmissions. We have for worst-case delay:

$$E[D_N^s] = m\log_m N, \quad Var[D_N^s] = m\log_m N\sigma^2.$$

Therefore the mean and variance of the worst-case delay for multi-tree based approaches are proportional to $m\log_m N$.

We can calculate the mean and variance of the average delay performance for multi-tree based approaches using recursions. As illustrated in Figure 1(b), a m-degree tree of N peers consists of the single peer at level 0 and m sub-trees, each of which is rooted at a level 1 peer and has $(N-1)/m$ peers. Denote by $\mathcal{W}(N)$ the aggregate chunk delay of all peers in a m-degree tree with N peers after the root peer receives the chunk. Assume peer 0 received the chunk at time $t = 0$, let t_j be the time when peer j at level 1 receives the chunk. We have

$$\mathcal{W}(N) = \sum_{j=1}^{m} \left(\frac{N-1}{m} t_j + \mathcal{W}_j \left(\frac{N-1}{m} \right) \right), \tag{7}$$

where the first term indicates the delay of peer j contributes to the delays of all peers in its sub-tree, the second term is the aggregate delay to disseminate the chunk in the j-th subtree. For the parallel multi-tree approach, $\{t_j\}$ is just the parallel transmission time from peer 0 to j. They follow the distribution of τ^p. For the sequential multi-tree approach, $t_j = \sum_{i=1}^{j} \tau_i$, where τ_i is the transmission delay from peer 0 to peer i, following the distribution of τ^s.

The average delay of all peers is simply $\bar{D}(N) = \mathcal{W}(N)/N$. It can be verified that for both parallel and sequential multi-tree, $E[\bar{D}_p(N)]$ and $E[\bar{D}_s(N)]$ are the same as the deterministic case. Based on (7), we also calculate the variance $Var[\bar{D}_{\{p,s\}}(N)]$ recursively:

$$Var[\bar{D}(N)] = (1 - \frac{1}{N})^2 \left(\frac{1}{m^2} Var[\sum_{j=1}^{m} t_j] + \frac{1}{m} Var[\bar{D}(\frac{N-1}{m})] \right) \tag{8}$$

For parallel uploading, $Var[\sum_{j=1}^{m} t_j] = m^2\sigma^2$, then

$$Var[\bar{D}_p(N)] = (1 - \frac{1}{N})^2 \left(\sigma^2 + \frac{1}{m} Var[\bar{D}_p(\frac{N-1}{m})] \right), \tag{9}$$

Consequently,

$$Var[\bar{D}_p(N)] \sim \sigma^2 \sum_{i=0}^{\log_m(N)} \frac{1}{m^i} = \frac{m}{m-1}\sigma^2 \tag{10}$$

For sequential uploading, $Var[\sum_{j=1}^{m} t_j] = \sigma^2 \sum_{j=1}^{m} j^2$, then

$$Var[\bar{D}_s(N)] = (1 - \frac{1}{N})^2 \left(\sigma^2 \sum_{j=1}^{m} (\frac{j}{m})^2 + \frac{1}{m} Var[\bar{D}_s(\frac{N-1}{m})] \right), \tag{11}$$

Consequently,

$$Var[\bar{D}_s(N)] \sim \sigma^2 \sum_{j=1}^{m} (\frac{j}{m})^2 \sum_{i=0}^{\log_m(N)} \frac{1}{m^i} = \sigma^2 \sum_{j=1}^{m} (\frac{j}{m})^2 \frac{m}{m-1} \tag{12}$$

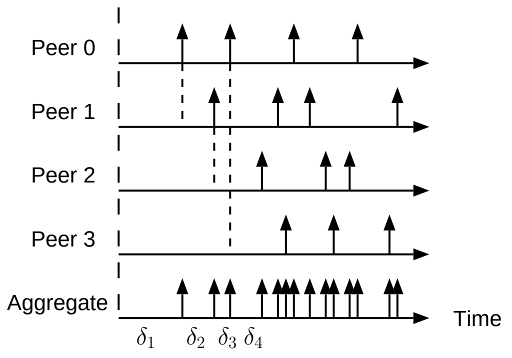
In summary,

$$Var[\bar{D}_p(N)] \approx \frac{m}{m-1}\sigma^2, Var[\bar{D}_s(N)] \approx \frac{\sum_{j=1}^{m} j^2}{m(m-1)}\sigma^2 \tag{13}$$

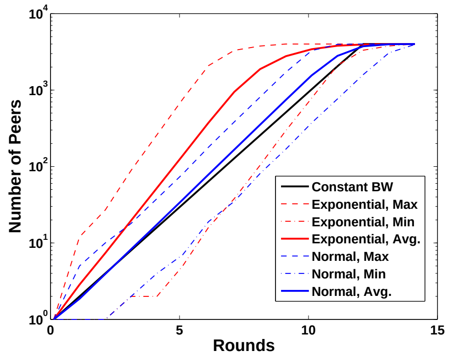
In both cases, the impact of the variability of individual transmissions on the average delay performance is *independent* of the number of peers. And the average delay variance *won't diminishes* as N grows. This is due to the variability at the first few transmission steps will affect almost all peers.

For the snow-ball approach, following the procedure for the tree-based approach, one would calculate the mean and variance of the worst-case delay as proportional to $\log_2(N)$. And referring to Figure 2(b), one would derive a recursive delay formula similar to (7) for multi-tree cases:

$$\mathcal{W}(N) = (N-1)\tau_s + \mathcal{W}_0(\frac{N}{2}) + \mathcal{W}_1(\frac{N}{2}),$$



(a) Chunk dissemination point process with random transmission time.



(b) Chunk dissemination speed with random transmission time.

Figure 7: Impact of random transmission time on delay performance of the snow-ball chunk dissemination algorithm.

where τ_s is the random transmission delay from peer 0 to 1, and $\mathcal{W}_0(\frac{N}{2})$ and $\mathcal{W}_1(\frac{N}{2})$ are the aggregate delays in the sub-tree rooted at peer 0 and 1 respectively. By solving it, one would conclude

$$E[\bar{D}] = \log_2 N + \frac{1}{N}, \quad Var[\bar{D}] \sim 2\sigma^2. \tag{14}$$

However, the previous calculation missed the inherent *adaptiveness* of the snow-ball approach to bandwidth variations. In the snow-ball approach, a peer will keep uploading a chunk until all peers have the chunk. Within one time period, a peer has more bandwidth will upload to more peers than a peer with less bandwidth. Over time, the workload of the same peer is naturally adaptive to its bandwidth: upload more if it has more bandwidth; upload less if its bandwidth reduces. As for the recursive view in Figure 2(b), due to the workload self-adaptiveness, the number of peers in each subtree is no longer $\frac{N}{2}$. What remains to be true is that the uploading in both subtrees will finish around the same time.

To further illustrate, let's assume the chunk transmission time between two peers follows exponential distribution with mean 1. As illustrated in Figure 7(a), after obtaining the chunk, each peer uploads it to other peers. Each arrow on the line for a peer represents the time instant when a chunk delivery completes. Consequently, an arrow on the aggregate line represents the time instant when a new peer receives the chunk. Denote by δ_k the time interval between the time instants when the k-th and the $k+1$-th peer receive the chunk. δ_1 is the transmission time from peer 0 to peer 1, it is an exponential random variable with rate 1. For $k \geq 2$, due to the memoryless property of exponential distribution, δ_k follows an exponential distribution with rate k. Therefore the worst case delay is $D_N = \sum_{k=1}^{N} \delta_k$, which follows a hyper-exponential distribution. We have

$$E[D_N] = \sum_{k=1}^{N} \frac{1}{k} < 1 + \ln N, \quad Var[D_N] = \sum_{k=1}^{N} \frac{1}{k^2} < 2$$

The expected chunk dissemination finish time is only $\ln 2 = 69.3\%$ of the deterministic case. Due to the constant bounded delay variance, for large N, snow-ball approach has better delay performance in random case than in the deterministic case. Similarly, we can calculate the average delay performance

$$\bar{D} = \frac{1}{N} \sum_{k=1}^{N} \sum_{i=1}^{k} \delta_k = \frac{1}{N} \sum_{k=1}^{N} (N-k+1)\delta_k.$$

Then

$$E[\bar{D}] = \sum_{k=1}^{N} \frac{N-k+1}{Nk} < \frac{1+\ln N}{N} + \ln N \tag{15}$$

$$Var[\bar{D}] = \sum_{k=1}^{N} \left(\frac{N-k+1}{Nk}\right)^2 < \sum_{k=1}^{N} \frac{1}{k^2} < 2 \tag{16}$$

Again, the average delay performance is better than the deterministic case. For chunk transmission time follows general distribution with mean 1, the interval between two chunk upload finish times is no longer exponential. However, the superposition of a large number of point processes converges to a Poisson process [2]. For large k, δ_k approximately follows an exponential distribution with rate k. We can apply the previous exponential distribution analysis to study the behavior of large system with generally distributed chunk transmission time. It is our conjecture that for a reasonable large N, e.g., $N \geq 1000$, one can expect snow-ball approach achieves better delay performance than the deterministic case.

This result is somehow counter-intuitive at the first sight. The study in Section 3.2 shows that *the bandwidth heterogeneity among peers will reduce the chunk dissemination delay.* The result obtained here can be considered as a *temporal* heterogeneity result, i.e., *the peer bandwidth variations over time will also reduce the chunk dissemination delay.* To bridge these two results, we can consider an artificial example: for a continuous streaming among N peers over time period of T, divide T into two halves, if in the first half peer 0 to peer $\frac{N}{2}-1$ have bandwidth of 2, peer $\frac{N}{2}$ to peer $N-1$ have no bandwidth; for the second half, peer 0 to peer $\frac{N}{2}-1$ have bandwidth of 0, peer $\frac{N}{2}$ to peer $N-1$ have bandwidth of 2. The average bandwidth of all peers are just 1. According to Theorem 3, the streaming delay of $0.5\log_2 N + 1.5$ can be achieved in both halves, while the minimum delay for the deterministic case when every peer always has uploading bandwidth of 1 is $\log_2 N + 1$.

Using the time-stepped simulator, we simulate the snow-ball chunk dissemination when the upload bandwidth of peers are random and follow truncated exponential and normal distributions respectively. Again, we simulate a network of $4,000$ peers and assume no propagation delay among peers. In the constant bandwidth case, the chunk transmission time is 8 time-steps. For the case of truncated exponential distribution, the mean chunk transmission time is 8 time-steps, the upper bound and lower bound are 24 and 1 respectively. For the case of truncated normal distribution, the mean chunk transmission time is 8 time-steps, the standard deviation is 4 time-steps, and the upper bound and lower bound are 24 and 1 respectively. We run 100 iterations for each distribution and record the max, min and average statistics for all iterations.

Figure 7(b) plots the numbers of peers with the chunk at every round of eight time-steps (one average chunk transmission time) for all the simulated cases. It is shown that snow-ball chunk dissemination can indeed exploit random bandwidth variations and achieve shorter delays than the case with constant bandwidth.

5.2.1 Bandwidth fluctuations on streaming delay performance

So far, we analyze the impact of bandwidth variations on a single chunk dissemination. For continuous streaming, bandwidth variation may cause contention between different chunks. The impact of bandwidth variations on the performance of continuous chunk streaming can be studied through analysis and experiments. Let $x_i(k)$ be the number of peers with chunk i at the beginning of time slot k, $y_i(k)$ be the number of peers that will upload chunk i in time slot k, and let $u_j(i,k)$ be the uploading bandwidth of peer j used to upload chunk i in time slot k. Then we have

$$y_i(k) = \min(x_i(k), N - x_i(k)); \qquad x_i(k+1) = x_i(k) + \sum_{j=1}^{y_i(k)} u_j(i,k).$$

If we assume $u_j(i,k) \sim u_0$, with $E[u_0] = \bar{u}$ and $Var[u_0] = \sigma_u^2$, then

$$\begin{aligned} E[x_i(k+1)] &= E\left[E[x_i(k+1)|x_i(k)]\right] = E[x_i(k) + \bar{u}x_i(k)] = (1+\bar{u})E[x_i(k)] = (1+\bar{u})^k; \\ Var[x_i(k+1)] &= Var\left[E[x_i(k+1)|x_i(k)]\right] + E\left[Var[x_i(k+1)|x_i(k)]\right] \\ &= Var[(1+\bar{u})x_i(k)] + E[x_i(k)\sigma_u^2] = (1+\bar{u})^2 Var[x_i(k)] + \sigma_u^2(1+\bar{u})^{k-1} \end{aligned}$$

Then

$$\begin{aligned} E\left[\frac{x_i(k+1)}{(1+\bar{u})^k}\right] &= 1; \\ Var\left[\frac{x_i(k+1)}{(1+\bar{u})^k}\right] &= Var\left[\frac{x_i(k)}{(1+\bar{u})^{k-1}}\right] + \frac{\sigma_u^2}{(1+\bar{u})^{k+1}} = \sigma_u^2 \frac{\frac{1}{(1+\bar{u})^2} - \frac{1}{(1+\bar{u})^{k+2}}}{1 - \frac{1}{(1+\bar{u})}} \end{aligned}$$

The dissemination delay for chunk i can be calculated as

$$D(i) = \min\{k : x_i(k) \geq N\} = \min\left\{k : \frac{x_i(k)}{(1+\bar{u})^{k-1}} \geq \frac{N}{(1+\bar{u})^{k-1}}\right\}$$

The finish time will have constant variance, which is mostly due to the uploading variances at the first few steps of the chunk dissemination.

As special cases, let's investigate how ON-OFF bandwidth fluctuations affect the delay performance.
Case 1: Synchronized ON-OFF: All peers have uploading capacity of 2 in odd time slot and 0 in even time slot. To deal with this, let's focus on odd time slots, at the beginning of each odd time slots, two new chunks are generated. If each peer uploads those two chunks in parallel, effectively, one unit of bandwidth is allocated to each chunk on each peers, we need $\log_2 N$ odd time slots to finish each chunk, therefore the total delay is $2\log_2 N$ time slots, which is twice the streaming delay when all peers have uploading capacity of 1 at each time slot. To fully utilize the bursty uploading capacity, with each odd time slot, a peer can upload two chunks sequentially. Effectively, we divide each odd time slot into two sub-slots, one new chunk is generated at the beginning of each sub-slot, and a peer can upload a chunk within each sub-slot, according to the study in Section 4, all chunks can be streamed with $\log_2 N$ sub-slots, that is $\frac{\log_2 N}{2}$ original odd time slots, consequently $\log_2 N$ original time slots.
Case 2: Alternative ON-OFF: Half of the peers with odd ID have uploading capacity of 2 in odd time slot and 0 in even time slot, the other half of the peers with even ID have uploading capacity of 2 in even time slot and 0 in odd time slot. The uploading strategy is as follows:

1. Within an odd time slot, server uploads an odd chunk to a peer with an even ID, each peer with odd ID uploads its chunk to two peers with even IDs;

2. Within an even time slot, server uploads an even chunk to a peer with an odd ID, each peer with even ID uploads its chunk to two peers with odd IDs;

It can be shown that

$$\begin{aligned} x_i(k+1) &= x_i(k) + 2y_i(k) \\ y_i(k+1) &= x_i(k) - y_i(k) + 2 * y_i(k) = y_i(k-1) + 2y_i(k) \end{aligned}$$

The uploading of chunk i will finish whenever $\sum_{j=i}^{i+K^*} y_i(j) \geq N/2$. Since $y_i(k+1) > 2 * y_i(k)$, the finish time will be much shorter than $\log_2(N/2)$. It is schedulable among chunks since at most $N/2$ peers are actively uploading chunks within each time slot.

The schedule can be further improved using sequential uploading within each round, e.g., in the first half of an odd round, an odd peer uploads the chunk to another odd peer, then the second half of the odd round, both peers upload to some even peer, then we have

$$\begin{aligned} x_i(k+1) &= x_i(k) + 3y_i(k) \\ y_i(k+1) &= x_i(k) - y_i(k) + 2y_i(k) = x_i(k) + y_i(k) \end{aligned}$$

We list in Table 4 the chunk scheduling between 8 peers with ON-OFF uploading capacity. The key

Table 4: Schedule under ON-OFF bandwidth

ID	1	1.5	2	2.5	3	3.5	4	4.5
0	-	-	0, 4	0, 5	-	-	2, 4	2, 5
1	0, 3	0, 0	-	-	2, 3	2, 0	-	-
2	-	-	0, 6	0, 7	-	-	2, 6	2, 7
3		0, 2	-	-		2, 2	-	-
4	-	-		1, 5	-	-		3, 5
5			-	-	1, 1	1, 0	-	-
6	-	-	1, 4	1, 7	-	-	3, 4	3, 7
7			-	-	1, 3	1, 2	-	-

for scheduling under ON-OFF bandwidth is to send chunk to peers who can upload the chunk in the next round. It is unrealistic to predict peers' upload bandwidth in random network environment. We will propose a dynamic snow-ball algorithm in Section 6 to adaptively find peers with upload bandwidth and push out old chunks for small streaming delay.

Conservation Law for Streaming Delay

Let $Y(k)$ be the aggregate bandwidth utilized by all nodes (including the server) to upload chunks, for a stream of L chunks to N peers, the average chunk delay $\bar{D}(L,N)$ seen by all peers can be calculated as:

$$\bar{D}(L,N) = \frac{1}{NL}\sum_{i=0}^{N-1}\sum_{j=0}^{L-1}(R(i,j) - B(j)),$$

where $B(j) = j$ is the time when chunk j is generated by the server, and $R(i,j)$ is time when peer i receives chunk j. At the end of time slot k, $Y(k)$ chunks will be received by some peers, therefore, $Y(k)$ chunk finish times are $k+1$. Consequently, we have

$$\bar{D}(L,N) = \frac{1}{NL}\sum_{k=0}^{W(L,N)} Y(k)(k+1) - \frac{L-1}{2},$$

where $W(L,N)$ is the last time slot when peers upload some chunk out of L chunks, and

$$\sum_{k=0}^{W(L,N)} Y(k) = NL,$$

i.e., NL copies of L chunks must have been uploaded to all peers till time slot $W(L,N)$. Therefore the average peer chunk delay is

$$\bar{D}(L,N) = \frac{1}{NL}\sum_{k=0}^{W(L,N)} kY(k) - \frac{L-3}{2}.$$

If each time slot all nodes' uploading capacity can be fully utilized, then $Y(k) = N$, $W(L,N) = L-1$, and $\bar{D}(L,N) = 1$. This is only possible if the server has a capacity of N. If all nodes have capacity of 1, then $Y(k) \leq 2^k, 0 \leq k \leq \log_2 N$ and $Y(k) \leq N$. To make $\sum_{k=0}^{W(L,N)} Y(k) = NL$, we must have $\sum_{k=L}^{W(L,N)} Y(k) \geq (\log_2 N - 1) * N$, therefore $W(L,N) \geq L + \log_2 N - 2$, i.e., the uploading of L chunks need at least $L + \log_2 N - 1$ time slots. For the case of half super peers with bandwidth of 2 and half free-riders, we can set the time unit to the half of the original time unit, then the server generates chunk j at time $2 * j$, then

$$\bar{D}(L,N) = \frac{1}{NL} \sum_{k=0}^{W(L,N)} Y(k)(k+1) - (L-1),$$

Measured in the new time slot, we still have $Y(k) \leq 2^k, 0 \leq k \leq \log_2(N/2)$ and $Y(k) \leq N/2$ (assume the server also has capacity of 2), to make $\sum_{k=0}^{W(L,N)} Y(k) = N$, $\sum_{k=2L}^{W(L,N)} Y(k) \geq (\log_2(N/2) - 1) * (N/2)$, therefore, $W(L,N) \geq 2L + \log_2(N/2) - 2$, the uploading of L chunks need at least $L + \frac{\log_2(N/2)-1}{2}$ time slots.

6 Streaming in Dynamic Environment

In the previous section, we studied the delay performance of the snow-ball single chunk dissemination scheme under long propagation delays and network bandwidth variations. However, for continuous streaming, due to the randomness in network bandwidth and propagation delays, we can no longer pre-determine fixed chunk streaming schedules among peers as in the static network case studied in Section 4. Instead, chunk uploading schedules have to be calculated dynamically to adapt to network bandwidth and delay variations. Now we extend the static snow-ball streaming algorithm to *Dynamic Snow-Ball (DSB) streaming* algorithm. We will show through simulations that, with a small peer upload bandwidth overhead, the proposed DSB streaming algorithm can approach the minimum delay bounds in dynamic network environment. The main purpose of this section is to demonstrate the potential of snow-ball type of streaming algorithms to achieve the minimum delay bound. The DSB algorithm is developed as a centralized streaming algorithm. We defer the distributed implementation of DSB algorithms to future work.

6.1 Dynamic Snow-Ball (DSB) Streaming Algorithm

The philosophy of DSB streaming algorithm follows the static snow-ball streaming algorithm. DSB aims at pushing out older chunks as quickly as possible to reduce the chunk dissemination delays, as well as the number of active chunks in transition in the system. At the same time, DSB should also make sure that newer chunks get enough peer upload bandwidth access to quickly grow the usable upload bandwidth for them. In a static network environment, as studied in Section 4, these two seemingly conflicting objectives can be simultaneously achieved by employing a carefully calculated chunk upload schedule among peers. The challenge for DSB streaming in a dynamic network environment is that the chunk transmission complete time is not predictable. Therefore, there is no optimal static streaming schedule that can achieve the minimum delay bound for all chunks in a video stream. Instead, our DSB algorithm is a simple heuristic algorithm that mimics the static snow-ball streaming algorithm and dynamically resolves the conflicts between active chunks in continuous streaming.

The DSB streaming algorithm works in rounds. At each round, let $\mathcal{A}$ be the set of active chunks that have been generated by the video source server, but have not been uploaded to all peers. For any chunk $k \in \mathcal{A}$, let R_k be the number of peers with chunk k, N_k be the number of peers without chunk k. Define the demand factor for chunk k as $d_k = N_k/R_k$, which is the expected workload for each peer with chunk k to upload it to some peers without it. Then for any peer i, let $\mathcal{B}_i$ be the set of chunks in its buffer. The

Algorithm 1: DSB Scheduling in One Round

```
Check whether chunk transmissions scheduled in the previous rounds have finished at the beginning
of this round;
for each newly completed chunk transmission do
    mark the source peer of the transmission idle, add the transmitted chunk to the buffered chunk set
    of the destination peer of the transmission;
end
for each active chunk $k \in \mathcal{A}$ do
    update $R_k$, $N_k$ and $d_k$;
end
for each peer $i$ do
    update buffer set $\mathcal{B}_i$, expected workload $W_i$;
end
for chunks $j \in \mathcal{A}$, starting from the oldest do
    while some peers miss chunk $j$ do
        a) find an idle peer $src$ with the lowest expected workload that has chunk $j$;
        b) find a peer $dst$ with the lowest expected workload that does not yet have chunk $j$, nor have
        a scheduled delivery of chunk $j$;
        c) let peer $src$ uses all its upload bandwidth to upload chunk $j$ to peer $dst$ starting from this
        round. mark peer $src$ busy, mark peer $dst$ with a scheduled delivery of chunk $j$.
    end
end
```

total expected workload for peer i can be calculated as $W_i = \sum_{k \in \mathcal{B}_i} d_k$. The DSB algorithm calculates the chunk uploading schedule among peers round by round. The DSB algorithm is outlined in Algorithm 1.

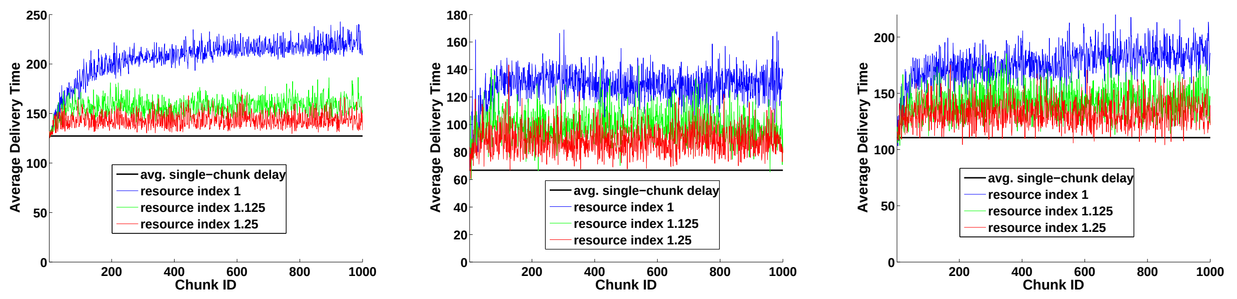
6.2 Performance Study of DSB

We implemented the centralized DSB streaming algorithm and conducted simulations of a P2P video streaming systems with $4,000$ peers. For each simulation, a stream of $1,000$ continuous chunks are disseminated to all peers. We introduce random variations in peer upload bandwidth and propagation delays between peers. More specifically, for each chunk transmission between peers, the transmission time follows a truncated exponential distribution. The propagation delays between two peers follow a truncated normal distribution. We record how long it takes for each chunk to be received by each peer. Then we calculate the average and worst-case streaming delay for each chunk, and compare them with the single-chunk dissemination delays obtained using the simulator described in Section 5 in a system with the same bandwidth and propagation delay settings.

When there is no bandwidth variation and the propagation delays are negligible, the transmission time of a chunk is set to be 8 simulation time-steps. The DSB streaming algorithm achieves the minimum single-chunk delay bound as presented in Section 3. Each of the $1,000$ chunks in the stream is delivered to $4,000$ peers after exactly 97 time-steps and the average delay experienced by peers is 88.81 time-steps. It demonstrated that the dynamic snow-ball streaming is delay-optimal in static homogeneous environment.

Next, we conduct simulations to evaluate the performance of DSB in dynamic network environment. For comparison, we also run the same simulations for the balanced multi-tree streaming with sequential upload and node degree of 5 [2]. We first introduce random propagation delays according to a truncated

[2]The degree is optimized for the transmission and propagation delay ratio of 1 according to Table 3



(a) Random Propagation Delay (b) Random Upload Bandwdith (c) Random Delay & Random Bandwdith

Figure 8: Delay performance of dynamic snow-ball streaming algorithm degrades when there are variations in propagation delays and peer upload bandwidth. The minimum delay bounds can be approached by slightly increasing peer upload bandwidth.

normal distribution with the mean equals to 8 time-steps (the chunk transmission time), and the standard deviation equals to 4 time-steps. The lower and upper bound for the random propagation delay is 1 and 16 time-steps respectively. In Figure 8(a), we compare the delay performance of DSB when the average peer upload bandwidth varies from 1 to 1.125 to 1.25. We use as reference point the single-chunk delays obtained from 100 simulation runs of a single chunk dissemination between $4,000$ peers with the same random propagation delay setting using the simulator described in Section 5. Due to the propagation delay, the average and worst-case single-chunk delays are 127.3 and 151.6 respectively, which are larger than 88.81 and 97 for the zero propagation delay case. Figure 8(a) plots the average streaming delay for each chunk in DSB. The system resource index in the figure is defined as the ratio between the average peer upload bandwidth and the streaming rate [5, 11]. The single-chunk delays can be approached by the DSB algorithm if peers have upload bandwidth slightly higher than the streaming rate. The statistics of average and worst-case delay performance of DSB and Multi-Tree are compared in Table 5. The delay performance of Mulit-Tree is worse than DSB. However, the delay variance is much smaller than DSB. This suggests that Multi-Tree can adapt well to propagation delay variations.

Now we repeat the previous simulation with zero propagation delay and random peer upload bandwidth. Now each chunk transmission time follows a truncated exponential distribution, with the mean equals to 8 time-steps and the lower and upper limit is 1 and 24 respectively. Again, we use the single-chunk dissemination simulation as the reference point. As predicted by the analysis in Section 5, with random chunk transmission time, the average and worst-case single-chunk delays (66.8 and 91.3 respectively) are smaller than those for the zero propagation delay case (88.81 and 97). The streaming delay performance of DSB is plotted in Figure 8(b). When the average peer upload bandwidth equals to the streaming rate, due to conflicts between chunks, the streaming delay performance is much worse than the corresponding single-chunk delay performance. By increasing peer upload bandwidth by 12.5%, the delay performance is reduced by 25%. If we further increase the average peer upload bandwidth to 1.25 times the streaming rate (corresponding to the curve labeled with resource index=1.25 in Figure 8(b)), the delay performance is getting closer to the single-chunk delay bound. As seen in Table 5, the delay performance of Mulit-Tree is much worse than DSB. This is because in Multi-Tree the delivery path of chunks are pre-determined. If one chunk transmission gets delayed on one link, all subsequent chunks have to be queued. This can happen on any link on the path and results in a "chain-effect". To the contrast, DSB can adaptively find peers with available bandwidth to quickly disseminate chunks. Its delay performance is much better than Multi-Tree.

Next, we introduce both random propagation delays and random peer upload bandwidth by combining the random delay and bandwidth variations introduced in the previous two sets of simulations. In Figure 8(c), we compare the delay performance of DSB when the average peer upload bandwidth varies from 1 to 1.125 to 1.25. Again, the minimum single-chunk delay bounds can be approached by the DSB algorithm if peers have upload bandwidth slightly higher than the streaming rate. In Table 5, due to the bandwidth variations, the delay performance of multi-tree is still much worse than DSB.

Table 5: Delay Performance of DSB under Random Propagation Delays and Upload Bandwidth

Algorithms	Random Delay			Random Bandwidth			Random Delay & Bandwidth		
	worst	average	variance	worst	average	variance	worst	average	variance
SB Chunk Bound	151.6	127.3	3.38	91.3	66.8	82.4	147.14	110.59	89.95
DSB, $\rho = 1$	245.4	213.9	74.5	161.7	129.5	90.8	223.34	179.8	152.0
DSB, $\rho = 1.125$	185.5	158.1	60.0	128.0	98.7	108.5	183.8	143.3	161.9
DSB, $\rho = 1.25$	169.8	143.8	51.2	116.5	88.47	93.2	171.4	132.2	127.8
Multi-Tree, $\rho = 1.25$	244.2	151.9	2.49	305.3	149.9	111.2	347.4	185.0	79.7

To study the impact of chunk size on the delay performance improvement of DSB, we fix the average propagation delay at 8 time slots, and vary the size of chunks such that the chunk transmission time ranges from 2 to 16 time slots. As indicated in Table 6, as chunk size decreases, the delay performance of both DSB and mulitree both degrade. The performance gap between them also decreases. When there is random bandwidth variation, DSB still outperforms multi-tree by around 25% even with a small chunk size of 2.

Table 6: Delay Performance Improvement of DSB at Different Chunk Sizes

Algorithms	Random Delay					Random Delay & Bandwidth				
	size=2	size=4	size=6	size=8	size=16	size=2	size=4	size=6	size=8	size=16
DSB, $\rho = 1.25$	64.62	91.53	118.1	143.8	173.1	67.88	87.17	110.5	132.2	217.0
Multi-Tree, $\rho = 1.25$	72.95	97.86	124.5	151.9	264.2	88.51	118.7	152.1	185.0	333.9

More simulation results are presented in Appendix.

Through simulations, we demonstrated that, with a little bit extra peer uploading bandwidth, our dynamic snow-ball streaming algorithm can approach the minimum delay bounds in face of random variations in peer uploading bandwidth and propagation delays on peering connections.

7 Conclusion and Future Work

In this paper, we analytically study the delay performance of P2P live video streaming systems. We derive various delay bounds that can serve as delay performance benchmarks for proposed/deployed P2P streaming systems. Through our analysis, we quantify the impact of the bandwidth distribution among peers on their delay performance. Insights brought forth by our study can be used to guide the design of new P2P streaming systems with shorter start-up delays and playback lags. Static snow-ball streaming algorithms is proposed to achieve the minimum delay bounds in static homogeneous and heterogeneous P2P video systems. A dynamic snow-ball streaming algorithm is also developed to approach the minimum delay bounds with a small peer upload bandwidth overhead. Through analysis and simulation, we show that the snow-ball type of streaming algorithms are robust to network impairments, such as long propagation delays and random bandwidth variations.

The next step is to develop distributed implementation of the proposed snow-ball streaming algorithms in mesh-based P2P video systems. The proposed DSB algorithm can be implemented in a clustered P2P streaming framework, such as HCPS [12]. Within a cluster, peers can employ the DSB chunk scheduling to achieve the minimum delay. For general mesh-based systems, using insights obtained in this study, we will design a distributed chunk scheduling algorithm that will mimic the snow-ball schedule spirit. It will explore the peer bandwidth heterogeneity for shorter delay. It will also schedule uploads of multiple active chunks to achieve a delay performance close to the ideal contention free delay bound. The snow-ball algorithm minimizes the delay by pushing the oldest chunk first. As chunk delays decrease, the number of active chunks in the system decreases. This increases the chance of content bottleneck. *Rarest-first* type of chunk scheduling has proven efficient in eliminating content bottlenecks in P2P file sharing [22]. We will develop chunk scheduling algorithms that efficiently combine oldest-first and rarest-first scheduling rule to achieve a good balance between delay performance and peer bandwidth utilization. We will test its performance in real network environment and compare it with the theoretical bounds predicted by our analysis here. Another direction for future work is to extend the delay performance analysis to take into consideration other factors, such as peer churns, geographic locality of peers and correlations among individual chunk transmissions, etc. More broadly, we are interested in extending our design and analysis of snow-ball type of algorithms to other forms of P2P systems with stringent delay requirements, such as Content Delivery Networks and P2P gaming systems [1].

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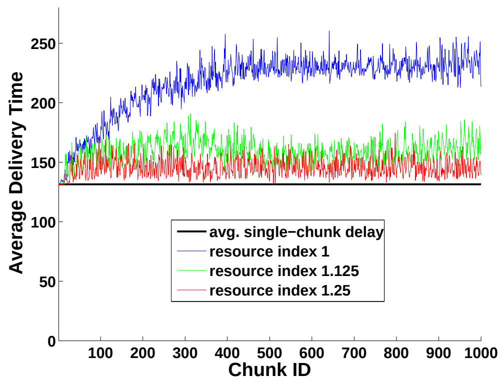
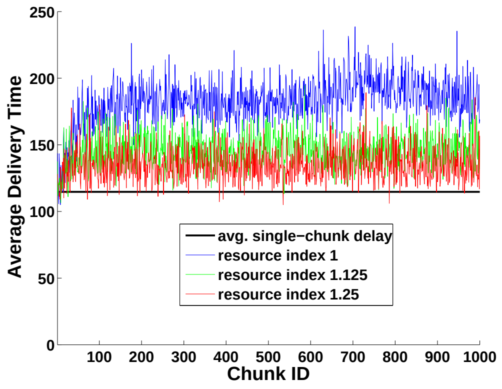
Appendix

A. DSB Performance under Constant Propagation Delay of 8 Timesteps.



(a) Constant Upload Bandwidth



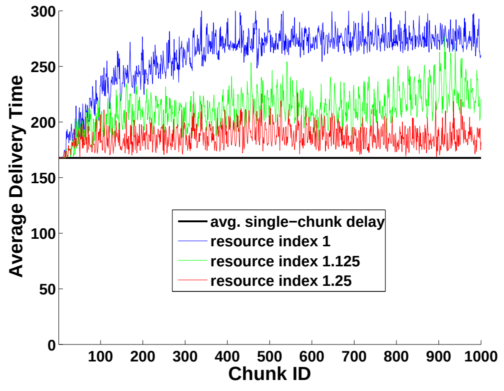
(b) Random Upload Bandwidth

Figure 9: Delay Performance of DSB with Constant Propagation Delay of 8 TimeSteps

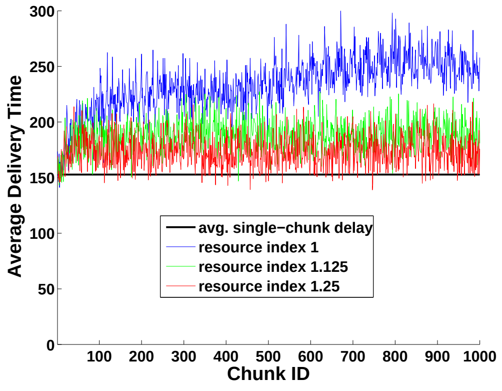
Table 7: Delay Performance of DSB with Constant Propagation Delay of 8 TimeSteps

Algorithms	Constant Bandwidth			Random Bandwidth		
	worst	average	variance	worst	average	variance
SB Chunk Bound	145.0	131.5	-	145.1	114.7	90.7
DSB, $\rho = 1$	251.8	227.2	111.8	224.2	186.3	189.1
DSB, $\rho = 1.125$	179.9	160.5	76.7	180.2	146.0	151.2
DSB, $\rho = 1.25$	163.7	145.8	50.5	168.2	135.2	148.3

B. DSB Performance under Constant Propagation Delay of 16 Timesteps.



(a) Constant Upload Bandwidth



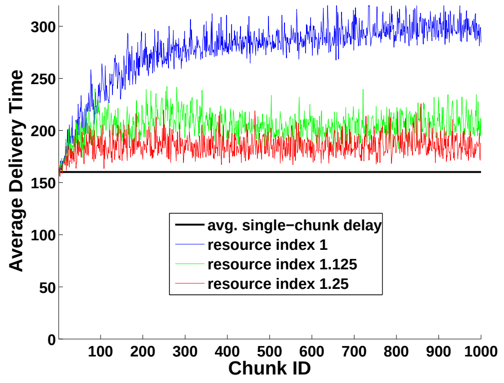
(b) Random Upload Bandwdith

Figure 10: Delay Performance of DSB with Constant Propagation Delay of 16 TimeSteps

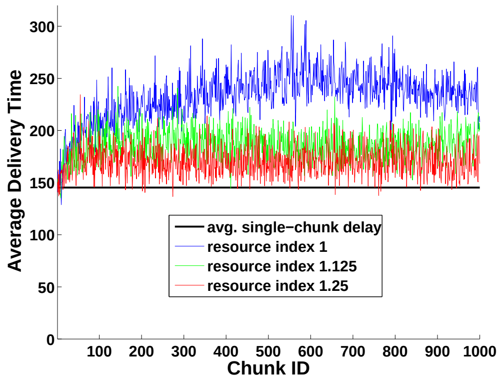
Table 8: Delay Performance of DSB with Constant Propagation Delay of 16 TimeSteps

Algorithms	Constant Bandwidth			Random Bandwidth		
	worst	average	variance	worst	average	variance
SB Chunk Bound	185.0	167.7	-	187.7	152.8	101.2
DSB, $\rho = 1$	299.2	270.8	192.2	283.0	240.2	341.9
DSB, $\rho = 1.125$	240.7	215.5	212.0	227.4	188.9	180.3
DSB, $\rho = 1.25$	210.2	187.8	102.3	210.3	173.3	193.1

C. DSB Performance under Random Propagation Delays with Mean of 16 Timesteps.



(a) Constant Upload Bandwidth



(b) Random Upload Bandwdith

Figure 11: Delay Performance of DSB under Random Propagation Delay with Mean of 16 TimeSteps

Table 9: Delay Performance of DSB under Random Propagation Delay with Mean of 16 TimeSteps

Algorithms	Constant Bandwidth			Random Bandwidth		
	worst	average	variance	worst	average	variance
SB Chunk Bound	196.3	160.2	11.0	193.2	145.3	111.1
DSB, $\rho = 1$	334.5	289.2	159.8	296.5	240.1	290.8
DSB, $\rho = 1.125$	246.5	206.3	117.6	238.8	186.5	195.3
DSB, $\rho = 1.25$	224.5	186.0	90.1	221.2	170.4	192.7