1. (a) The X-ray source is located outside of the body. The photon intensity is measured. The linear attenuation coefficient is revealed.

(b) The source is inside the body. The photon intensity is measured. Radioactive distribution in the tissue is revealed.

2. (a) Maximum spacing \( z < \frac{1}{2f_{\text{max}} \beta} \) based on the Nyquist frequency theory.

(b) \( M = \frac{D}{z} \), \( N = \frac{D}{z} > 2D f_{\text{max}} \)

(c) \( K = \frac{2D}{z} \) (projection need to be zero padded to avoid aliasing)

3. (a) For SPECT, radio-tracer undergo gamma decay. Yes. Collimator is needed for SPECT to reduce the scattering artifact.

(b) For PET, radio-tracer undergo positron decay. No. Collimator is not needed, since PET use annihilation coincidence detection (ACD) to reject scattered gamma rays.

(c) The radio tracers that have short half-life (radio-tracers used in PET) need to be produced at the imaging facility, while the radio tracers that have long half-life (radio-tracers used in SPECT) can be manufactured at off-site locations and shipped to imaging facility.
4. **CT reconstruction:**

Given the photon intensity \( I(\theta_m, \lambda_n) \), \( g(\theta_m, \lambda_n) = -\ln \left( \frac{I(\theta_m, \lambda_n)}{I_0} \right) = \int_{-\infty}^{\infty} \mu(x, y) \, dx \)

CT image is reconstructed by convolution backprojection:

\[
\mu(x, y) = \delta \sum_{\theta_m} \left[ c(\lambda_n) * g(\theta_m, \lambda_n) \right] = \alpha \cos \theta_m + \beta \sin \theta_m
\]

5. **PET reconstruction:**

The data \( I(\theta_m, \lambda_n) \) acquired in CT imaging can be used to correct the PET data:

\[
\Phi_c(\theta_m, \lambda_n) = \frac{\Phi(\theta_m, \lambda_n)}{K \exp - \int_{-\infty}^{\infty} \mu(x, y) \, dx} = \frac{\Phi(\theta_m, \lambda_n)}{K I_c(\theta_m, \lambda_n)} = \int_{-\infty}^{\infty} A(x, y, \lambda_n) \, dx
\]

And then by convolution backprojection:

\[
A(x, y) = \sum_{\theta_m} \left[ c(\lambda_n) * \Phi_c(\theta_m, \lambda_n) \right] \alpha x \cos \theta_m + \beta \sin \theta_m
\]

**Sensitivity:**

\[
\frac{\int_{-\infty}^{\infty} P_0(x)}{\int_{-\infty}^{\infty} P_b(x)} = \int_{-\infty}^{\infty} \frac{1}{\sigma_n \sqrt{2\pi}} \exp \left\{ -\frac{(x - \mu_n)^2}{2\sigma_n^2} \right\}
\]

**Specificity:**

\[
\frac{\int_{-\infty}^{\infty} P_b(x)}{\int_{-\infty}^{\infty} P_n(x)} = \int_{-\infty}^{\infty} \frac{1}{\sigma_n \sqrt{2\pi}} \exp \left\{ -\frac{(x - \mu_n)^2}{2\sigma_n^2} \right\}
\]

When increasing \( t \), sensitivity is decreased while specificity is increased.

One possible way to find a tradeoff is to maximize the diagnostic accuracy, which equals \[
\frac{\text{true positive}}{\text{true negative}}
\]
6. \( I(y) \) when \( D_{1} < y \leq D_{1} + D_{2} + L \):

\[
I(y) = \frac{I_{o}}{4\pi (D_{1} + D_{2} + L)^{2}} \cos^{3} \theta \times e^{-\mu_{o}L/\cos\theta}
\]
where \( \cos\theta = \frac{D_{1} + D_{2} + L}{\sqrt{y^{2} + (D_{1} + D_{2} + L)^{2}}} \)

7. \( g(x, \theta) \):

\( \theta = 0^\circ \):

\[
g(x, \theta) = \max(1 - |x|, 0)
\]

\( \theta = 45^\circ \):

\[
g(x, \theta) = 25
\]

\( \theta = 90^\circ \):

\[
g(x, \theta) = 3
\]
(b) \( b_{90^\circ} \):

\[ b_{90^\circ} + b_{90^\circ} \]

(c) \( F(ρ \cos θ, ρ \sin θ) = G(ρ, θ) \)

\[ g(ρ, θ) = \text{rect}(ρ + \frac{1}{2}) + 2 \text{rect}(ρ - \frac{1}{2}) \]

\[ F(ρ \cos θ, ρ \sin θ) = G(ρ, θ) = \sin c(ρ) e^{jθ} + 2 \sin c(ρ) e^{-jθ} \]

(d) \( g'(x, ω) = g(x, ω) \circ h(x) \)

\[ h(ρ) = \frac{1}{ρ} \]

\[ g'(x, ω) = \]

\[ g'(x, ω) = \]
We know that \( f(x, y) = \mathcal{F}^{-1}\{H(\rho)\} \), where \( H(\rho) = \text{rect}\left(\frac{\rho}{a}\right) \).

Since \( H(\rho) \) is circularly symmetric, \( H(\rho) = H(\rho) = H(\rho) \).

where \( H(\rho) = \mathcal{F}^{-1}\{H(\rho)\} \), \( H(\rho) = 0.2 \sin c(\rho) \)

\[
\frac{2 \text{rect}(\rho)}{\pi \rho^2 - 4\rho^2}
\]

8. (a) \( A_t = A_1 e^{-\lambda t} \), where \( \lambda = \frac{0.693}{T_1} \)

\[
\overline{P_A} = \int_0^1 \frac{A_t}{4\pi (3-s)^2} \exp\{-\mu_1 (3-s) - 2\mu_2 - \mu_3\} ds + \int_1^2 \frac{A_t}{4\pi (3-s)^2} \exp\{-\mu_1 (2-s) - \mu_2 - \mu_3\} ds
\]

\[
\overline{P_B} = \int_0^1 \frac{A_t}{4\pi (3+s)^2} \exp\{-\mu_1 (3+s) - 2\mu_2 - \mu_3\} ds + \int_1^2 \frac{A_t}{4\pi (3+s)^2} \exp\{-\mu_1 - \mu_2 - 2\mu_3\} ds
\]

(b) \( A_t = A_2 e^{-\lambda t} \), where \( \lambda = \frac{0.693}{T_2} \)

\[
\overline{P_A} = \int_0^1 A_t ds \cdot \exp\{-\mu_1 - 2\mu_2 - 3\mu_3\} = 2 A_t \cdot \exp\{-\mu_1 - 2\mu_2 - 3\mu_3\}
\]