Projection Radiography

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Based on Prince and Links, Medical Imaging Signals and Systems and Lecture Notes by Prince. Figures are from the book.
Lecture Outline

• Instrumentation
  – X-ray tube configuration
  – Filtration and restriction of x-ray photons
  – Compensation and Scatter control
  – Film screen detector
  – Digital X-ray

• Image formation
  – Geometric effect
  – Extended source
  – Detector/film response

• Image quality
  – Contrast and SNR
  – Effect of noise and Compton scattering
Overview

- Systems:
  - chest x-rays, mammography
  - dental x-rays
  - fluoroscopy, angiography

- Properties
  - high resolution
  - low dose
  - broad coverage
  - short exposure time
X-ray Tube

[Diagram showing the components of an X-ray tube, including cathode and rotating anode.]
X-Ray Tube Components

- Filament controls tube current (mA)
- Cathode and focussing cup
- Anode is switched to high potential
  - 30–150 kVp
  - Made of tungsten
  - Bremsstrahlung is 1%
  - Heat is 99%
  - Spins at 3,200–3,600 rpm
- Glass housing; vaccum
Exposure Control

- kVp applied for short duration
  - fixed timer (SCR), or
  - automatic exposure control (AEC), 5 mm thick ionization chamber triggers SCR
- Tube current mA controlled by
  - filament current, and
  - kVp
- mA times exposure time yields mAs

\[ I_{\text{tube}} = 1-1000\text{mA} \]

mAs measures x-ray exposure
X-Ray Spectra
Bremsstrahlung

• *Continuous* spectrum of EM radiation is produced by abrupt deceleration of charged particles (“Bremsstrahlung” is German for “braking radiation”).

• Deceleration is caused by deflection of electrons in the Coulomb field of the nuclei.

• Most of the energy is converted into heat, ~0.5% is x-ray.

• The energy of the generated x-ray photon is given by energy conservation:

\[ h\nu = K_e - K'_e \]

• The maximum energy for the produced photon is given by:

\[ E_{p,\text{max}} = h\nu = K_e = eV_{\text{tube}} \]

[From Graber, Lecture Note for BMI1-FS05]
Bremsstrahlung intensity

• Overall Bremsstrahlung intensity $I$:

$$I \propto V_{\text{tube}}^2 I_{\text{tube}}$$

Electrical power consumption of tube: $P_{\text{tube}} = I_{\text{tube}} \times V_{\text{tube}}$ [W]

• The produced x-ray power $P_x$ (in[W]) is given by:

$$P_x = k Z V_{\text{tube}}^2 I_{\text{tube}} = kZ V_{\text{tube}} P_{\text{tube}} = \eta P_{\text{tube}}$$

$$\eta = P_x / P_{\text{tube}} = kZ V_{\text{tube}}$$: x-ray production efficiency

– Material constant $k = 1.1 \times 10^{-9}$ for Tungsten (Z=74).

[From Graber, Lecture Note for BMI1-FS05]
Bremsstrahlung spectrum

- Theoretically, bremsstrahlung from a thick target creates a continuous spectrum from $E = 0$ to $E_{\text{max}}$ with intensity $I_b$:

$$I_b(E) \sim Z(E_{\text{max}} - E)$$

- Actual spectrum deviates from ideal form due to
  - Absorption in window / gas envelope material and absorption in anode
  - Multienergetic electron beam

[From Graber, Lecture Note for BMI1-FS05]
Characteristic radiation

- Narrow lines of intense x-ray at characteristic energies are superimposed on the continuous bremsstrahlung spectrum.
- Caused by removal of inner shell electrons and subsequent filling of hole with electrons from higher shell. The shell-energy difference determines the energy of characteristic rays.
- Lines are named after the lower shell involved in the process; the upper shell involved is denoted by Greek letters:
  \[ \Delta n = 1 \rightarrow \alpha\text{-transitions}, \Delta n = 2 \rightarrow \beta\text{-transitions}, \ldots \]

[From Graber, Lecture Note for BMI1-FS05]
Different types of characteristics rays

From http://hyperphysics.phy-astr.gsu.edu/Hbase/quantum/xterm.html#c1
X-ray spectra

- X-ray for general diagnostic radiology produced at 40 – 150 kVp

- Maximum photon energy: \( E_p[\text{keV}] = h\nu_{\text{max}} = e \times \text{kVp} \)

- Characteristic radiation occurs only for anode voltages 
  \( e \times \text{kVp} > I_{K,L,M,\ldots} \)

[From Graber, Lecture Note for BMI1-FS05]
X-ray tube design

- Cathode w/ focusing cup, 2 filaments (different spot sizes)
- Anode
  - Tungsten, \( Z_w = 74, T_{\text{melt}} = 2250 \, ^\circ\text{C} \)
  - Embedded in copper for heat dissipation
  - Angled (see next slide)
  - Rotating to divert heat

[From Graber, Lecture Note for BMI1-FS05]
Filtration

- Low energy x-ray will be absorbed by the body, without providing diagnostic information
- Filtration: Process of absorbing low-energy x-ray photons before they enter the patient
  - Inherent filtration
    - Within anode
    - Glass housing
  - Added filtration
    - Aluminum
    - Copper/Aluminum
      - Note: Cu has 8keV characteristic xrays
    - Measured in mm Al/Eq
Beam Hardening

Bremmstrahlung
(x-rays within anode)

leaving anode

leaving tube

characteristic radiation

after filter

leaving body

relative intensity

photon energy, keV
Goal: To direct beam toward desired anatomy
Compensation Filters

- Goal: to even out film exposure
Contrast Agents

- Goal: To create contrast where otherwise none.

Iodine (33.2 KeV)

Barium (37.4 KeV)

When the x-ray energy exceeds the Kedge (binding energy of K-shell), the mu coefficient is much higher, providing high contrast.
Contrast Agents in Clinical Use

• Iodine:
  – Can be synthesized into soluble compounds that are safely introduced through intravascular injection or ingestion
  – Used for imaging of
    • Blood vessels, heart chambers, tumors, infections
    • Kidneys, bladder
  – Naturally exist in thyroid, and hence X-ray is very good for thyroid imaging

• Barium
  – Administered as a “chalky milkshake”
  – Used in the gastrointestinal tract,
    • Stomach, bowel

• Air
  – Does not absorb x-ray
  – “opposite” type of contrast
  – By inflating the lungs, air provides contrast for lung tissues
Scatter Control

- Ideal x-ray path: a line!
- Compton scattering causes blurring
- How to reduce scatter?
  - airgap
  - scanning slit
  - grid
Grids

- Effectiveness in scatter reduction?

\[ \text{grid ratio} = \frac{h}{b} \]

- 6:1 to 16:1 (radiography) or 2:1 (mammo)
Problem with Grids

- Radiation is absorbed by grid
  - grid conversion factor
    \[ GCF = \frac{\text{mAs w/ grid}}{\text{mAs w/o grid}} \]
    - Typical range \(3 < GCF < 8\)
- Grid visible on x-ray film
  - move grid during exposure
  - linear or circular motion
Film-Screen Detector

- Film stops only 1–2% of x-rays
- Film stops light really well

Intensifying screen:

Phosphor: convert x-ray photons to light

Reflective layer: Reflect light back to film

- Phosphor = calcium tungstate
- Flash of light lasts $1 \times 10^{-10}$ second
- $\sim 1,000$ light photons per 50 keV x-ray photon
Radiographic Cassette
Digital Radiography

• Replace film by digital detectors
• Earlier system: Computer Radiography (CR)
  – Store latent images in photostimulable imaging plates (PSP)
  – Need to be scanned by laser to form a digital image.
• CCD-based system
  – Small field of view
  – Group multiple CCD arrays
• TFT-based system
• CMOS-based system
  – Low cost: widely used in consumer cameras
  – Small field of view
  – Easier to tie multiple substrates than CCD arrays
• Portable, and reusable, some has wireless transmitter built in to send captured image to servers
TFT-Based Digital Radiology

- Replace the intensifying screen/X-ray film by
  - flat panel detectors (FPD) using thin-film transistor (TFT) arrays
  - A scintillator
    - Consisting of many thin, rod-shaped cesium iodide (CsI) crystals
- When an X-ray is absorbed in a CsI rod, the CsI scintillates and produces light
- The light is converted into an electrical signal by a photodiode in the TFT array
- The electrical signal is amplified and converted to a digital value using an A/D converter
- A typical commercial DR system has flat panel dimensions of 41x41 cm, with an TFT array of 2048x2048 elements
- Ref: Webb, Introduction to biomedical imaging, Sec. 1.5.5
Biological effects of ionizing radiation

• Damage depends on deposited (= absorbed) energy (intensity × time) per tissue volume
• Threshold: No minimum level is known, above which damage occurs
• Exposure time: Because of recovery, a given dose is less harmful if divided
• Exposed area: The larger the exposed area the greater the damage (collimators, shields!)
• Variation in Species / Individuals: LD 50/30 (lethal for 50% of a population over 30 days, humans ~450 rads / whole body irradiation)
• Variation in cell sensitivity: Most sensitive are nonspecialized, rapidly dividing cells (Most sensitive: White blood cells, red blood cells, epithelial cells. Less sensitive: Muscle, nerve cells)
• Short/long term effects: Short term effects for unusually large (> 100 rad) doses (nausea, vomiting, fever, shock, death); long term effects (carcinogenic/genetic effects) even for diagnostic levels ⇒ maximum allowable dose 5 R/yr and 0.2 R/working day [Nat. Counc. on Rad. Prot. and Meas.]

[From Graber, Lecture Note for BMI1-FS05]
Image Formation

• Basic imaging equation
• Geometric effects
• Extended source
• Film blurring
• Impact of noise and scattering
Basic Imaging Equations

\[ I(x, y) = \int_0^{\infty} S_0(E')E' \exp \left\{ - \int_0^{r(x,y)} \mu(s; E', x, y) ds \right\} dE' \]
Example

1) What is the local contrast of the blood vessel?
2) What is the local contrast of the blood vessel when contrast agent is injected?

Blood vessel $\mu=0.2$

w/o contrast:
\[ I_b = I_{\min} = I_0 e^{-(0.4\times2.0)}; \]
\[ I_o = I_{\max} = I_0 e^{-(0.4\times1.5+0.2\times0.5)} \]

Local contrast: \[ C_1 = \frac{I_o-I_b}{I_b}; \]

Global contrast: \[ C = \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} \]

w/ contrast:
\[ I_b = I_{\max} = I_0 e^{-(0.4\times2.0)}; \]
\[ I_o = I_{\min} = I_0 e^{-(0.4\times1.5+20\times0.5)} \]

Local contrast: \[ C_1 = \frac{|I_o-I_b|}{I_b}; \]

Global contrast: \[ C = \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} \]
Geometric Effects

- X-rays are diverging from source
- Undesirable effects:
  - $\cos^3 \theta$ falloff across detector
  - anode heel effect
  - pathlength irregularities
  - magnification
- $I_0$ is intensity at (0, 0)
- $r$ is distance from $(x, y)$ to x-ray origin
- $\theta$ is angle between (0, 0) and $(x, y)$
Inverse Square Law

- Net flux of photons decrease as $1/r^2$.

Therefore

$$I_0 = \frac{I_S}{4\pi d^2}, \quad I_r = \frac{I_S}{4\pi r^2}$$

- Eliminate source intensity $I_S$

$$I_r = I_0 \frac{d^2}{r^2}$$

- Since $\cos \theta = d/r$

$$I_r = I_0 \cos^2 \theta$$

$I_0$ is the detected flux at the origin of the detector plane
$I_r$ is the detected flux at an arbitrary point of the detector plane
with angle $\theta$ w/o considering the oblique effect discussed in the
next page
Obliquity

\[ I_d = I_0 \cos \theta \]

- Intensity is
- \( I_0 \) should be replaced by \( I_r \)
Overall Effect of Beam Divergence

- Inverse square law and obliquity combine

\[ I_d(x_d, y_d) = I_0 \cos^3 \theta \]

- Can usually be ignored. Why?
  - Detector is far away
  - Field of view (FOV) is often small

\[ \Rightarrow \theta \text{ is small} \]
Anode Heel Effect

- Intensity within the x-ray cone
  - Not uniform
  - stronger in the cathode direction
  - 45% variation is typical
- Compensate, use to advantage, or ignore
- We will ignore in math
Imaging of a Uniform Slab

- Uniform slab yields different intensities

Diagram showing a slab with an X-ray origin, line, and angles involving $\theta$ and $z_0$. The distance $d$ is also indicated.
Intensity on detector

\[ I_d(x, y) = I_0 \exp\left\{ -\mu L / \cos \theta \right\} \]

Including inverse square law and obliquity:

\[ I_d(x, y) = I_i \cos^3 \theta \exp\left\{ -\mu L / \cos \theta \right\} \]

If \( d \approx r \) all effects can be ignored

\[ I_i = I_s / (4 \pi d^2) \]

Illustrate the received intensity as function of \( y \) or \( x \) or \( \theta \)
Received Signal as a Function of Theta

\[ \cos(q)^3 \exp(-1/\cos(q)) \]

This plot assumes \( \mu L = 1 \), e.g. \( \mu = 1/cm \), \( L = 1/cm \).
• How does it vary as a function of Y when x=0 (vertical axis of the detector plain)?
• \( \cos(q) = \frac{d}{r} = \frac{d}{\sqrt{d^2+y^2}}/ \)
• Assuming \( d=5\text{m}, y= -10\text{cm} \) to 10cm (q from 0 to 1.14 degree)
• Vary small relative change in the range of y

![intensity as function of y](image)
Imaging of Two Layer Slab

Consider the x-ray imaging of a two-layer slab, illustrated below. Determine the intensity of detected photons along the y axis on the detector plane. Express your solution in terms of the y-coordinate. Sketch this function. You should consider the inverse square law and the oblique effect. Assume the x-ray source is an ideal point source with intensity $I_0$. For simplicity, assume the slab is infinitely long in the y direction.
Solution

- Sketch on the board
Example: Image of a prism due to a point source

Consider the x-ray imaging of a cube. Determine the intensity of detected photons along the y axis on the detector plane. Express your solution in terms of the angle $q$. Sketch this function. You should consider the inverse square law and the oblique effect. Assume the x-ray source is an ideal point source with intensity $I_0$, and the object has a constant linear attenuation coefficient $m$. (Example 5.4 in textbook)
Solution

Sketch over in class. Also see textbook.
Complete as homework
Must consider different regions separately
Note the edge blurring effect.
Objects Magnification

- Size on detector depends on distance from source

\[
w_z = w \frac{d}{z}
\]

Magnification factor:

\[
M(z) = \frac{d}{z}
\]
Imaging of a Thin Non-Uniform Slab

• Assume a very thin slab at z
  – the linear absorption coefficient at \((x', y')\) is \(\mu(x', y')\)
  – Detector position \((x, y)\) -> slab position \((x', y')\)

\[
x/d = x'/z \rightarrow x' = x \cdot z/d = x/M(z)
\]
• Let "transmittivity" be

\[ t_z(x, y) = \exp\{-\mu(x, y)\Delta z\} \]

• On detector, intensity is

\[ I_d(x, y) = I_0 \cos^3 \theta t_z \left( \frac{x}{M(z)}, \frac{y}{M(z)} \right) \]

• After substitution

\[ I_d(x, y) = I_0 \left( \frac{d}{\sqrt{d^2 + x^2 + y^2}} \right)^3 t_z \left( \frac{xz}{d}, \frac{yz}{d} \right) \]

\[ I_0 = I_s / (4 \pi d^2) \]
Blurring Due to Extended Source

First study the image through a pinhole
- Impulse response

Image through an arbitrary objects
- Impulse response * object attenuation profile

Note: blurring depends on the object distance! and source size. Less bluring when the object is closer to the detector plane!
Image of source through a pinhole

Pinhole: a infinitesimal hole (area=0) that passes the X-ray source w/o attenuation. Everywhere else the X-ray is completely absorbed.

Reversed image

\[
D' / (d-z) = -D/z \rightarrow D' = -D (d-z)/z = Dm
\]

• Source magnification:

\[
m(z) = -\frac{d-z}{z} = 1 - M(z)
\]

Call the following \(h(x,y)\) (response due to a pinhole at (0,0))

• Image of source through pinhole at \(z\)

\[
I_d(x, y) = \frac{\cos^3 \theta}{4\pi d^2 m^2} s\left(\frac{x}{m}, \frac{y}{m}\right)
\]

Loss of source intensity due to inverse square law

(a) Loss of source intensity due to inverse square law

(b) Scale factor due to pinhole

(See textbook)
Image of an Arbitrary Slice

- An arbitrary slab at \( z \) can be thought of as many pinholes at different locations \((x',y')\), each with transmittivity \( t_z(x',y') \)
  
  - The received signal due to transmittivity at \((x',y')\) can be written as \( h(x-x',y-y') \ t_z(x',y') \) assuming the system is translation invariant.

- The image of the slab is a sum of individual images of the source through all the pinholes multiplied by the respective transmittivity.
  
  \[ I_d(x,y) = \int_{x',y'} h(x-x',y-y') t_z(x',y') \, dx' \, dy' \]

- The overall effect can be captured through linear convolution.

\[
I_d(x, y) = \frac{\cos^3 \theta}{4\pi d^2 m^2} t_z \left( \frac{x}{M}, \frac{y}{M} \right) * s \left( \frac{x}{m}, \frac{y}{m} \right)
\]

Note: \( m \) and \( M \) depends on \( z \), distance of slab to the source. When the object is next to the detector \( M=1 \), less blurring.
Example

- Source is a circular disk with diameter D
- Object is square plane with dimension W at distance z
- Detector plane at distance d from source
- How does the detected image look for d=2Z and d=3Z
- Note that the blurring of the edge depends on z

- What is $t_z(x,y)$ and $s(x,y)$?
- What is $I_d(x,y)$?
- How is $I_d(x,y)$ related with $t_z(x,y)$?
- How does the image of $I_d(x,y)$ look?
Solution

• Sketch on the board
Example: solution

\( Tz(x,y) \): a square with width \( W \)
\( S(x,y) \): a disk with diameter \( D \). Assuming \( D << W \),

- For \( d=2z \), \( M=d/z=2 \), \( m=1-M=-1 \)
  \( Tz(x/M,y/M) \): a square with width \( 2W \)
  \( S(x/m,y/m) \): a disk with diameter \( D \)
  The resulting detector image is a square with width \( 2W \) but with a blurred edge with blurring width \( D \)

- For \( d=3z \), \( M=d/Z=3 \), \( m=1-M=-2 \)
  \( Tz(x/M,y/M) \): a square with width \( 3W \)
  \( S(x/m,y/m) \): a disk with diameter \( 2D \)
  The resulting detector image is a square with width \( 3W \) but with a blurred edge with blurring width \( 2D \)
Film Screen Blurring

A single x-ray photon causes a blurry spot on the film which is effectively the “impulse response” to the x-ray impulse $h(x,y)$

Typical MTF for a film-screen detector
Digital Detector Blurring

- Digital detectors also lead to blurring in the detected image, due to the thickness of the scintilators.
- Such effect can still be modeled by an impulse response, also denoted by $h(x,y)$. 
Overall Imaging Equation

- Including all effects (geometric, extended source, film-screen blurring), the image corresponding to a slab at \( z \) with transmittivity function \( t_z(x,y) \) is

\[
I_d(x, y) = \cos^3 \theta \frac{1}{4\pi d^2 m^2} s \left( \frac{x}{m}, \frac{y}{m} \right) \times t_z \left( \frac{x}{M}, \frac{y}{M} \right) \ast h(x, y)
\]

- For an object with a certain thickness, the transmittivity function must be modified to reflect the overall attenuation along the \( z \)-axis
- When the source is polyenergetic, integration over photon energy is additionally needed
What does it mean?

• Output image = scaled version of the object * \( g(x,y) \)

• Scaling depends on the object position \( z \)
  - Less magnification when object is closer to the detector

• \( g(x,y) = ? \)

• \( g(x,y) \) depends on both source and detector blurring and position \( z \)!
Example

• In the previous example, how would the image look if the film blurring is a box function of width $h$?
Film Characteristics

- Film darkening (after development) depends on incident light (which depends on the incident x-ray)
- Optical density

\[ D = \log_{10} \frac{I_i}{I_t} \]

- Usable densities \( 0.25 < D < 2.25 \)
- Best densities \( 1.0 < D < 1.5 \)
Optical Density vs. Exposure

- X-ray exposure yields optical density

\[ D = \Gamma \log_{10} \frac{X}{X_0} \]

- \( \Gamma \) is film gamma
- Typical ranges: \( 0.5 < \Gamma < 3.0 \)
- Latitude is range exposures where relationship is linear
- Speed is inverse of exposure at which

\[ D = 1 + \text{fog level} \]
The H&D Curve

Optical density

Exposure, mR

(a) High speed film with CaWO₄ screens

(b) Direct x-ray film

(m) High-speed film without screens

Shoulder

Linear region

Toe

Fog level
Effect of Noise

- **Source of noise:**
  - Detector does not faithfully reproduce the incident intensity
  - X-rays arrive in discrete packets of energy. This discrete nature can lead to fluctuations in the image

- **Local contrast**
  \[ C = \frac{I_t - I_b}{I_b} \]

- **Signal is** \( I_t - I_b \)
- **Noise is due to Poisson behavior**
- **Variance of noise in background:** \( \sigma_b^2 \)
- **Signal to noise**
  \[ \text{SNR} = \frac{I_t - I_b}{\sigma_b} = \frac{CI_b}{\sigma_b} \]
How is noise related to signal?

• Assuming the number of photons in each burst follows the Poisson distribution
  – $P(N=k) = \left(\frac{a^k}{k!}\right) e^{-a}$
  – Variance = mean = $a$

• Let $N_b$ denotes the average number of photons per burst per area
• Let $hv$ denotes the effective energy for the X-ray source
• The average background intensity is

$$I_b = \frac{N_b hv}{A \Delta t}$$

• The variance of photon intensity is

$$\sigma_b^2 = N_b \left(\frac{hv}{A \Delta t}\right)^2$$

• The SNR is

$$\text{SNR} = C \sqrt{N_b}$$

• SNR can be improved by
  – Increasing incident photon count
  – Improving contrast
• Recall $\text{SNR} = (I_t - I_b) / s_b = C \frac{I_b}{s_b}$, $C = (I_t - I_b) / I_b$
Detective Quantum Efficiency

- How good is a detector?
- Consider:
  - Potential SNR before detection
  - Actual SNR upon detection
- Detective Quantum Efficiency

\[
DQE = \left( \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}} \right)^2
\]

- Degradation of SNR during detection

When a x-ray source has mean intensity \( m = N_b \), and variance \( s^2 = N_b \),
\( \text{SNR} = \frac{m}{s} = \sqrt{N_b} \)
Example

• Suppose an X-ray tube is set up to fire bursts of photons each with \( N = 10000 \) photons and the detector’s output (\# of detected photons per burst) \( x \) has a mean =8000, variance=40000. What is its DQE?

• Solution:

The actual # of photons fired at the x-ray tube follows the Poisson process (mean = variance = 10000)

\[
\text{SNR}_{\text{in}} = \frac{\text{mean}}{\sqrt{\text{variance}}} = \sqrt{10000} = 100
\]

The # of detected photons has mean = 8000, variance = 40000

\[
\text{SNR}_{\text{out}} = \frac{\text{mean}}{\sqrt{\text{variance}}} = \frac{8000}{\sqrt{40000}} = \frac{8000}{200} = 40
\]

\[
\text{DQE} = \left( \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}} \right)^2 = 0.16
\]

This means that only about 16% of photons are detected correctly.
Effect of Compton Scattering

- Compton scattering causes the incident photons to be deflected from their straight line path
  - Add a constant intensity $I_s$ in both target and background intensity ("fog")
  - Decrease in image contrast
  - Decrease in SNR

W/o scattering:
- Target intensity: $I_t$
- Background intensity: $I_b$
- Contrast $C = \frac{I_t - I_b}{I_b}$
- SNR $= C \frac{I_b}{\sigma_b} = C \sqrt{N_b}$

W/ scattering:
- Target intensity: $I_t + I_s$
- Background intensity: $I_b + I_s$
- Contrast $C' = \frac{I_t - I_b}{I_b + I_s} = \frac{I_b}{I_b + I_s} C = \frac{C}{1 + \frac{I_s}{I_b}}$
- SNR $'= C \frac{I_b}{\sigma_b} = C \frac{N_b}{\sqrt{N_b + N_s}} = C \frac{\sqrt{N_b}}{\sqrt{1 + N_s / N_b}} = \text{SNR} \frac{1}{\sqrt{1 + \frac{I_s}{I_b}}}$
Medical Applications

- Orthopedic
- Chest
- Abdomen
- Mammography
- Angiography

[From Graber, Lecture Note for BMI1-FS05]
Mammography

- Detection and diagnosis (symptomatic and screening) of breast cancer
- Pre-surgical localization of suspicious areas
- Guidance of needle biopsies.

- Breast cancer is detected on the basis of four types of signs on the mammogram:
  - Characteristic morphology of a tumor mass
  - Presentation of mineral deposits called microcalcifications
  - Architectural distortions of normal tissue patterns
  - Asymmetry between corresponding regions of images on the left and right breast

⇒ Need for good image contrast of various tissue types.
- Simple x-ray shadowgram from a quasi-point source.

[From Graber, Lecture Note for BMI1-FS05]
Mammography contrast

- Image contrast is due to varying linear attenuation coefficient of different types of tissue in the breast (adipose tissue (fat), fibroglanular, tumor).
- Ideal energy distribution of X-ray should be below 20KeV for average size breast, slightly higher for denser breast.

[From Graber, Lecture Note for BMI1-FS05]
Mammography source

- Voltage ~ 25-30 kVp
- Anode material Mo (Molybdenum), Rh (Rhodium) (characteristic peaks at 17.9 and 19 for Mo, and slightly higher for Rh)
- Filtering: use Mo or Rh to absorb energy above 20 or 25 Kev

[From Graber, Lecture Note for BMI1-FS05]
Anti-scatter grid

- Significant Compton interaction for low $E_p$ (37-50% of all photons).
- Linear grid: Lead septa + interspace material. Septa focused toward source. Grid ratio ~ 3.5-5:1. Only scatter correction in one dimension. Scatter-to-primary (SPR) reduction factor ~5
- Recently crossed grid introduced
- Grids are moved during exposure
- Longer exposure

[From Graber, Lecture Note for BMI1-FS05]
X-ray projection angiography

- Imaging the circulatory system. Contrast agent: Iodine (Z=53) compound; maximum iodine concentration ~ 350 mg/cm³
- Monitoring of therapeutic manipulations (angioplasty, atherectomy, intraluminal stents, catheter placement).
- Short intense x-ray pulses to produce clear images of moving vessels. Pulse duration: 5-10 ms for cardiac studies … 100-200 ms for cerebral studies

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Summary

• Projection radiography system consists of an x-ray tube, devices for beam filtration and restriction, compensation filters, grids, and a film-screen detector (or digital detector, filmless)

• The detector reading (or image gray level) is proportional to the number of unabsorbed x-ray photons arriving at the detector, which depends on the overall attenuation in the path from the source to the detector.

• The degree of film darkening is nonlinearly related to the film exposure (detected x-ray) by the H&D curve

• Both detector noise and Compton scattering reduce contrast and SNR of the formed image.
Imaging Equations

- Basic imaging equation (relating detector reading with attenuation coefficient)
  \[ S(x; E) = S_0(E) \exp\left\{ -\int_0^x \mu(x'; E) dx' \right\} \]

- Modified to consider inverse square law, obliquity, anode heel effect
  \[ I_d(x, y) = I_i \cos^3 \theta \exp\left\{ -\frac{\mu L}{\cos \theta} \right\} \]

- Further modified to consider extended source and detector impulse response
  \[ I_d(x, y) = \cos^3 \theta \frac{1}{4\pi d^2 m^2} s \left( \frac{x}{m}, \frac{y}{m} \right) \]
  \[ \ast \ t_z \left( \frac{x}{M}, \frac{y}{M} \right) \ast h(x, y) \]
Reference

• Prince and Links, Medical Imaging Signals and Systems, Chap 5.
• Webb, Introduction to biomedical imaging, Chap 1.
Homework

• Reading:
  – Prince and Links, Medical Imaging Signals and Systems, Chap 5.

• Problems for Chap 5 of the text book:
  – P5.2
  – P5.4
  – P5.5
  – P5.9
  – P5.12 (Hint: you can assume \( h_1(x/m) = \exp(-a x^2/m^2) \) for arbitrary \( m(z) \), and find parameter “a” based on the given information). Also you may use the following Fourier transform pair:
    • \( \exp{-ax^2} \leftrightarrow \sqrt{\pi/a} \exp(- \pi^2 u^2/a) \)
  – P5.18 (this problem was not in 1st ed)
  – P5.22(P5.20 in 1st ed)
  – P.5.23 (P5.21 in 1st ed) (You could use the general imaging equation involving convolution. But you could also derive the solution just by geometric sketching!)
  – P5.25(P5.23 in 1st ed)
  – P5.27 only parts (a) and (b) (P5.25 in 1st ed)