Computed Tomography (Part 2)

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Based on Prince and Links, Medical Imaging Signals and Systems and Lecture Notes by Prince. Figures are from the book.
Last Lecture

- Instrumentation
  - CT Generations
  - X-ray source and collimation
  - CT detectors

- Image Formation
  - Line integrals
  - Parallel Ray Reconstruction
    - Radon transform
    - Back projection
    - Filtered backprojection
    - Convolution backprojection
    - Implementation issues
This Lecture

- Review of Parallel Ray Projection and Reconstruction
- Practical implementation with samples
- Fan Beam Reconstruction
- Signal to Noise in CT
Review: Projection Slice Theorem

- Projection Slice theorem
  - The 1D Fourier Transform of a projection at angle $\theta$ is a line in the 2D Fourier transform of the image at the same angle.

$$G(\rho, \theta) = F(\rho \cos \theta, \rho \sin \theta)$$
Reconstruction Algorithm for Parallel Projections

• Backprojection Sum:
  – Backprojection of each projection
  – Sum
  \[ f_b(x, y) = \int_0^\pi [g(\ell, \theta)]_{\ell = x \cos \theta + y \sin \theta} d\theta \]

• Filtered backprojection:
  – 1D FT of each projection
  – Filtering each projection in frequency domain
  – Inverse 1D FT
  – Backprojection
  – Sum
  \[ f(x, y) = \int_0^\pi \left[ \int_{-\infty}^{\infty} |G(\varphi, \theta)| e^{j2\pi \varphi \ell} d\varphi \right]_{\ell = x \cos \theta + y \sin \theta} d\theta \]

• Convolution backprojection
  – Convolve each projection with the ramp filter
  – Backprojection
  – Sum
  \[ f(x, y) = \int_0^\pi [c(\ell) \ast g(\ell, \theta)]_{\ell = x \cos \theta + y \sin \theta} d\theta \]

• Which one gives more accurate results? Which one takes less computation?
Practical Implementation

- Projections $g(l, \theta)$ are only measured at finite intervals
  - $l = n\tau$;
  - $\tau$ chosen based on maximum frequency in $G(\rho, \theta)$, $W$
    - $1/\tau \geq 2W$ or $\tau \leq 1/2W$ (Nyquist Sampling Theorem)
    - $W$ can be estimated by the number of cycles/cm in the projection direction in the most detailed area in the slice to be scanned

- For filtered backprojection:
  - Fourier transform $G(\rho, \theta)$ is obtained via FFT using samples $g(n\tau, \theta)$
  - If N sample are taken, ideally 2N point FFT should be taken by zero padding $g(n\tau, \theta)$
    - Recall convolving two signals of length N leads to a single signal of length 2N-1

- For convolution backprojection
  - The ramp-filter is sampled at $l = n\tau$
  - Sampled Ram-Lak Filter (limiting the rho-filter to within $(-1/2\tau, -1/2\tau)$)
    
    $$c(n) = \begin{cases} 
    1/4\tau^2; & n = 0 \\
    -1/(n\pi\tau)^2; & n = odd \\
    0; & n = even 
    \end{cases}$$

See paper at http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3708767/
Practical Implementation of Filtered Backprojection

\( P_{\theta_j}(l) \): Projection data along \( \theta_j \); \( s_{\theta_j}(k) \): DFT of \( P_{\theta_j}(l) \);

\( B_{\theta_j}(m,n) \): Backprojected image from filtered projection

\( R(m,n) \): Sum of backprojections.

\( L \): number of samples along each projection, assuming \( L \) is odd

\( K \): number of projection angles.

\[
    s_{\theta_j}(k) = \sum_{l=-L_1}^{L_1} P_{\theta_j}(l) \exp\left(-i2\pi \frac{lk}{L}\right), \quad k = -L_1, \ldots, L_1, L_1 = \text{floor}\left(\frac{L}{2}\right)
\]

\[
    B_{\theta_j}(m,n) = \sum_{k=-L_1}^{L_1} s_{\theta_j}(k) \left| \frac{k}{L} \right| \exp\left(i2\pi \frac{m \cos \theta_j + n \theta_j}{L} \right)
\]

\[
    R(m,n) = \frac{\pi}{K} \sum_{j=1}^{K} B_{\theta_j}(m,n)
\]

From paper at [http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3708767/pdf/1475-925X-12-50.pdf](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3708767/pdf/1475-925X-12-50.pdf)

Note this implementation perform \( L \) point DFT assuming the projection data has \( L \) points. \( L \) is odd. The reconstructed values near the boundary may have ringing artifacts.

Note that backprojection is implemented within the evaluation of inverse FT.
Practical Implementation of Convolution Backprojection

\[ c(l): \text{predesigned filter} \]

\[ B_{\theta_j}(m,n) = \sum_{l=-L_1}^{L_1} P_{\phi_j}(l)c\left(m\cos\theta_j + n\sin\theta_j - l\right) \]

\[ R(m,n) = \frac{\pi}{K} \sum_{j=1}^{K} B_{\theta_j}(m,n) \]

Here again: Back projection is implemented within convolution!
The Ram-Lak Filter (from [Kak&Slaney])

\[
H(w) = |w| b_w(w)
\]

\[
b_w(w) = \begin{cases} 
1 & |w| < W \\
0 & \text{otherwise}. 
\end{cases}
\]

\[
W = \frac{1}{2\pi} \text{ cycles/cm.}
\]

\[
h(t) = \int_{-\infty}^{\infty} H(w) e^{j2\pi wt} dw
\]

\[
= \frac{1}{2\pi^2} \frac{\sin 2\pi t/2\pi}{2\pi t/2\pi} - \frac{1}{4\pi^2} \left(\frac{\sin \pi t/2\pi}{\pi t/2\pi}\right)^2
\]

\[
h(n\pi) = \begin{cases} 
1/4\pi^2, & n = 0 \\
0, & n \text{ even} \\
-\frac{1}{n^2\pi^2}, & n \text{ odd.}
\end{cases}
\]
1st Generation CT: Parallel Projections
3G: Fan Beam

Much faster than 2G
Fan Beam: Equiangular Ray

We will focus on the equiangular detector setting on the right in this lecture.
Equiangular Ray Projection

Source location is described by \((\beta, D)\)

\(D\) is typically fixed, \(\beta\) varies to provide a large view angle.

To provide complete view, \(\beta \in (0, 2\pi)\)

For a given source with angle \(\beta\), \(\gamma\) specifies the detector position or the projection line.

For each \(\beta, \gamma\) varies over a range \((-\gamma_m, \gamma_m)\)

\((D, \beta, \gamma)\) completely specifies the line of projection:

\[
\begin{align*}
\theta &= \beta + \gamma, \quad l = D\sin(\gamma) \\
\theta + \alpha &= \pi / 2 \\
\Rightarrow \beta + \gamma &= \theta
\end{align*}
\]

Instead of \(g(l, \theta)\), we can use \(p(\gamma, \beta)\) to represent a projection
Equiangular Ray Reconstruction

Reconstructed image is represented in the polar coordinate using \((r, \varphi)\).

The relative position of a pixel at \((r, \varphi)\) to the source at \((D, \beta)\) is specified by \((D', \gamma)\):

\[
D'^2(r, \varphi) = (D + r \sin(\beta - \varphi))^2 + (r \cos(\beta - \varphi))^2
\]

\[
\tan \gamma(r, \varphi) = \frac{r \cos(\beta - \varphi)}{D + r \sin(\beta - \varphi)}
\]

Reconstruction formula:

\[
f(r, \varphi) = \frac{1}{2} \int_{0}^{2\pi} q(\gamma, \beta) d\beta
\]

Convolution weighted backprojection

\[
q(\gamma, \beta) = p'(\gamma, \beta) * c_f(\gamma)
\]

\[
p'(\gamma, \beta) = p(\gamma, \beta) \cos(\gamma)
\]

\[
c_f(\gamma) = \frac{1}{2} D \left( \frac{\gamma}{\sin \gamma} \right)^2 c(\gamma)
\]

\(c(\gamma)\) is the ramp filter used in parallel projection.

\(c_f(\gamma)\) is weighted based on the angle \(\gamma\).

Derivation not required for this class. Detail can be found at [Kak&Slaney]. Note typos in [Prince&Links], 1st ed.
Typos in [Prince&Links, 1st Ed]

- P. 207, Eq. (6.38), change to
  \[
  c(D'\sin \gamma) = \left( \frac{\gamma}{D'\sin \gamma} \right)^2 c(\gamma)
  \]

- Eq. (6.39) change to
  \[
  c_f(\gamma) = \frac{1}{2} D \left( \frac{\gamma}{\sin \gamma} \right)^2 c(\gamma)
  \]

- Eq. (6.40),(6.41)
  \[
  p(\gamma, \beta) \rightarrow p'(\gamma, \beta)
  
  p'(\gamma, \beta) = p(\gamma, \beta) \cos(\gamma)
  
  q(\gamma, \beta) = p'(\gamma, \beta) \ast c_f(\gamma)
  \]
Practical Implementation

- Projections $P(\gamma, \beta)$ are only measured at finite intervals
  - $\gamma = n\alpha$;
  - $\alpha$ chosen based on maximum frequency in $\gamma$ direction, $W$
    - $1/\alpha \geq 2W$ or $\alpha < 1/2W$
- For convolution backprojection
  - The filter $c_f(\gamma)$ is sampled at $\gamma = n\alpha$
  - Sampled Filter $g(n\alpha)$
  - $q(n\alpha, \beta) = p'(n\alpha, \beta) \ast g(n\alpha)$
- For backprojection
  - For given $(r, \phi)$, for a given $\beta$, determine $(D', \gamma)$
  - Use interpolation to determine $q(\gamma, \beta)$ from known values at $\gamma = n\alpha$
  - Modify the projection based on $D'$ (weighted backprojection)

Back projection and sum

\[
D'^2(r, \phi) = (D + r \sin(\beta - \phi))^2 + (r \cos(\beta - \phi))^2
\]

\[
\tan \gamma(r, \phi) = \frac{r \cos(\beta - \phi)}{(D + r \sin(\beta - \phi))}
\]

\[
f(r, \phi) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{(D')^2} q(\gamma, \beta) d\beta
\]

\[
q(\gamma, \beta) = p'(\gamma, \beta) \ast c_f(\gamma)
\]

\[
p'(\gamma, \beta) = p(\gamma, \beta) \cos(\gamma)
\]
Matlab Functions for Fan Beam CT

- Relevant functions:
  - `fanbeam()`, `ifanbeam()`
Fan Beam Reconstruction Through Rebinning [Smith&Webb]

The process of data rebinning to produce parallel projections from fan beam data. Data from four different projection angles, $\theta_1 \ldots \theta_4$ are shown. One projection, shown as the solid line, is taken from each of the datasets to produce a synthetic data set (bottom) with four parallel projections, which can then use a simple filtered backprojection algorithm for image reconstruction.
Constrained Optimization Formulation  
(Algebraic Reconstruction Techniques or ART)

The measurement $g(l, \theta)$ in general can be related to the unknown $f(m,n)$ as

$$g(l, \theta) = \sum_{m,n} f(m,n) A(m,n; l, \theta)$$

$$A(m,n; l, \theta) = \begin{cases} 
1 & (m,n) \text{ is on the line } (l, \theta) \\
0 & \text{otherwise}
\end{cases}$$

More generally, we can order all possible $(l, \theta)$ pairs into 1D indices, and represent $g(l, \theta)$ as a vector $\mathbf{g}$, and order all pixel positions $(m,n)$ also into 1D indices, so that all possible $f(m,n)$ can be put into a vector $\mathbf{f}$, and all possible $A(m,n; l, \theta)$ can be put into a matrix $A$. Then the above equations can generally be written as

$$A \mathbf{f} = \mathbf{g}$$

So we can reconstruct the image $\mathbf{f}$ by solving the above matrix equation.

If there are more measurements than the number of pixels, then the problem can be solved using the least squares formulation, i.e.,

$$\min \| A \mathbf{f} - \mathbf{g} \|_2^2 \text{, with a unique solution } \mathbf{f} = \left( A^T A \right)^{-1} A^T \mathbf{g}$$

When there are fewer measurements than the number of pixels, we have an under determined problem, and we need to put additional constraints on $\mathbf{f}$.

For example, we may require that the image $\mathbf{f}$ is sparse in some transform domain, i.e. $\mathbf{T} \mathbf{f}$ has only few non-zero coefficients, or $L_0$ norm of $\mathbf{T} \mathbf{f}$ is small, and solve the following problem

$$\min \| A \mathbf{f} - \mathbf{g} \|_2^2 + \lambda \| \mathbf{T} \mathbf{f} \|_0$$

This is the classical compressive sensing problem.

Instead of using the $L_0$ norm, we can relax to the $L_1$ norm, and solve the following convex optimization problem

$$\min \| A \mathbf{f} - \mathbf{g} \|_2^2 + \lambda \| \mathbf{T} \mathbf{f} \|_1$$

There are many fast algorithms to solve the above problem.
CT Quality Evaluation

- Blurring Effect
- SNR
Effect of Area Detector

- Practical detector integrates the detected photons over an area.
- Mathematically, the detector can be characterized by an indicator function $s(l)$ (aka impulse response).
- The measured projection $g'(l, \theta)$ is related to the “real” projection $g(l, \theta)$ by:
  - $g'(l, \theta) = g(l, \theta) * s(l)$
  - $G'(\rho, \theta) = G(\rho, \theta) S(\rho)$

\[
S(\rho) = \mathcal{F}\{s(\ell)\}
\]
Windowing Function

• Recall that the ideal filter $c(\rho)$ is typically modified by a window function $W(\rho)$

• Overall Effect

$$f(x, y) = \int_{0}^{\pi} \left[ \int_{-\infty}^{\infty} G(\varrho, \theta) S(\varrho) W(\varrho) |\varrho| e^{i2\pi \varrho \ell} d\varrho \right]_{\ell = x \cos \theta + y \sin \theta} d\theta$$

$f(x, y)$ can be thought of as the reconstructed image from the projection $\hat{g}(l, \theta)$, whose Fourier transform is

$\hat{G}(\rho, \theta) = G(\rho, \theta) S(\rho) W(\rho) \iff \hat{g}(l, \theta) = g(l, \theta) * s(l) * w(l)$
Blurred Projection

- Blurry projection:
  \[ \hat{g}(\ell, \theta) = g(\ell, \theta) * s(\ell) * w(\ell) = g(\ell, \theta) * \tilde{h}(\ell) \]

- Radon transform convolution theorem
  \[ \mathcal{R}\{f *_2 h\} = \mathcal{R}\{f\} *_1 \mathcal{R}\{h\} \]

- Leads to
  \[ \hat{f}(x, y) = f(x, y) * (\mathcal{R}^{-1}\{\tilde{h}(\ell)\}) \]

h(x,y): PSF of the blurring
Circular Symmetry of Blurring

- CT image blurred by convolution kernel

\[ h(x, y) = \mathcal{R}^{-1}\{\tilde{h}(\ell)\} \]

- Fourier transform of \( \tilde{h}(\ell) \)

\[ \tilde{H}(\varphi) = \mathcal{F}_1\{\tilde{h}(\ell)\} = S(\varphi)W(\varphi) \]

which is independent of \( \theta \).

- Therefore, \( H(u, v) \) is circularly symmetric

\[ H(q) = \mathcal{F}_2\{h(x, y)\} = S(q)W(q) \]
PSF given by Hankel Transform

- PSF is circularly symmetric and given by
  \[ h(r) = \mathcal{H}^{-1}\{S(\varrho)W(\varrho)\} \]
- Reconstructed image given by
  \[ \hat{f}(x, y) = f(x, y) \ast h(r) \]
  \[ r^2 = x^2 + y^2 \]
Circularly Symmetric Functions and Hankel Transform

• Circularly symmetric:
  – \( f(x,y) = f(r) \), only depends on the distance to the origin, not angle
• Fourier transform of circularly symmetric function is also circularly symmetric
  – \( F(u,v)=F(\rho) \)

\[
F(\rho, \theta) = \int \int f(r, \phi) \exp\{-j2\pi (r\rho \cos \phi \cos \theta + r\rho \sin \phi \sin \theta)\} r\,dr\,d\phi
\]

\[
= \int \int f(r, \phi) \exp\{-j2\pi \rho \cos(\phi - \theta)\} r\,dr\,d\phi
\]

If \( f(x, y) = f(r) \)

\[
F(\rho, \theta) = \int \left\{ \int \exp\{-j2\pi \rho \cos(\phi - \theta)\} d\phi \right\} f(r) r\,dr = 2\pi \int_0^\infty f(r) J_0(2\pi \rho r) r\,dr = F(\rho)
\]

Hankel Transform

\[
F(\rho) = 2\pi \int_0^\infty f(r) J_0(2\pi \rho r) r\,dr; \quad J_0(r) = \frac{1}{\pi} \int_0^\pi \cos(r \sin \phi) d\phi
\]
Common Transform pairs

- See Table 2.3

\[
\text{Fourier}\left\{ e^{-\pi(x^2+y^2)} \right\} = e^{-\pi(u^2+v^2)}
\]

\[
\Rightarrow \text{Hankel}\left\{ e^{-\rho^2} \right\} = e^{-\pi\rho^2}
\]

\[
\text{Hankel}\{\sin c(r)\} = \frac{2\text{rect}(q)}{\pi \sqrt{1 - 4q^2}}
\]

- Scaling property

\[
\text{Hankel}\{f(ar)\} = \frac{1}{a^2} F(q/a)
\]

- Duality: If \( h(r) \leftrightarrow H(\rho) \), then \( H(r) \leftrightarrow h(\rho) \)

Derivation of Hankel transform pairs are not required. But you should be able to use given transform pairs, to determine the blur function.
Example

Example 6.5 in [Prince&Links]
- Detector: rectangular detector with width d
  - $S(l)=\text{rect}(l/d)$
- Rectangular window function
  - $W(\rho)=\text{rect}(\rho/2\rho_o)$; $\rho_o >> 1/d$

Solution
- $S(l)=\text{rect}(l/d) \leftrightarrow S(\rho)=d \text{sinc}(d\rho)$
- $\rho_o >> 1/d \rightarrow$
- $H(\rho)=S(\rho) W(\rho) \approx S(\rho) = d \text{sinc}(d\rho)$ (Hankel transform of $h(r)$)

Illustrate and explain $h(r)$
Noise in CT Measurement

- Basic measurement is:
  \[ g_{ij} = -\ln \left( \frac{N_{ij}}{N_0} \right) \]
  - line \( L_{ij} \)
  - angle \( i \)
  - position \( j \)

- Noise is “in” Poisson random variable \( N_{ij} \)
  - mean \( \bar{N}_{ij} \)
  - variance \( \bar{N}_{ij} \)

  \[
  \Pr\{N_{ij} = k\} = \frac{a^k}{k!} e^{-a}; \quad k = 0,1,\ldots
  \]

  \[
  E\{N_{ij} = k\} = a
  \]

  \[
  Var\{N_{ij} = k\} = a
  \]
What about the measured projection

- It follows that $g_{ij}$ is a random variable

$$
\bar{g}_{ij} \approx \ln \left( \frac{N_0}{\bar{N}_{ij}} \right)
$$

$$
\text{Var}(g_{ij}) \approx \frac{1}{\bar{N}_{ij}}
$$

- $\hat{\mu}(x, y)$ is approximate reconstruction
- It follows that $\hat{\mu}(x, y)$ is a random variable
- What are the mean and variance of $\hat{\mu}$?
CBP Approximation

- Convolution backprojection (CBP):
  \[
  \mu(x, y) = \int_0^\pi \int_{-\infty}^{\infty} g(\ell, \theta) c(x \cos \theta + y \sin \theta - \ell) \, d\ell \, d\theta
  \]

- Approximations:
  - \( M \) angles; \( \Delta \theta = \pi / M \)
  - \( N + 1 \) detectors; \( \Delta \ell = T \)
  - \( \tilde{c}(\ell) \approx c(\ell) \)

- Discrete CBP:
  \[
  \hat{\mu}(x, y) = \left( \frac{\pi}{M} \right) \sum_{j=1}^{M} T \sum_{i=-N/2}^{N/2} g(iT, j \pi / M) \tilde{c}(x \cos \theta_j + y \sin \theta_j - iT)
  \]
Definitions and Assumptions

- $\tilde{N}_{ij}$ is mean for i-th detector and j-th angle
- $N_{ij}$ is independent for different measurements
- $\tilde{N}_{ij} = \bar{N}$, an “object uniformity” assumption
- $\tilde{c}(\ell)$ is created using rectangular window $W(\varphi)$ with cutoff $\varphi_0$.

$g_{ij}$ are independent because $N_{ij}$ are independent
Deriving mean and variance of $\mu(x,y)$ based on the independence assumption
See [Prince&Links] for derivation
Mean(\(\hat{\mu}\)) is desired result

\(<\text{Var}(\hat{\mu}) = \sigma_{\hat{\mu}}^2\) is inaccuracy

\[
\sigma_{\hat{\mu}}^2 \approx \frac{2\pi^2}{3} \varrho_0^3 \frac{1}{M} \frac{1}{\bar{N}/T}
\]

Be cautious on conclusions: not all variables are independent in a real physical system

- Variance increases with \(\varrho_0\) (cut-off freq. of filter), and \(T\) (detector spacing), decreases with \(M\) (number of angles), \(\bar{N}\) (or \(N_0\)) (x-ray intensity)
SNR of the Reconstructed Image

- **Definition (usual)**
  \[
  \text{SNR} = \frac{C\bar{\mu}}{\sigma\hat{\mu}}
  \]

- **After substitution:**
  \[
  \text{SNR} = \frac{C\bar{\mu}}{\pi} \sqrt{\frac{3M\bar{N}}{2g_0^2T}}
  \]

C: fractional change of \( \mu \) from \( \bar{\mu} \)
SNR in a good design

- What should $\varrho_0$ be?
- Let detector width $= w$
- $\varrho_0$ should be anti-aliasing filter:
  $$\varrho_0 = \frac{k}{w} \quad \text{where } k \approx 1$$
- In 3G scanner $w = T$
- Then
  $$\text{SNR} \approx 0.4k\bar{C}\bar{\mu}w\sqrt{N}\sqrt{M}$$
SNR in Fan Beam

- Definitions:
  - $\bar{N}_f$ is mean photon count per fan
  - $D$ is number of detectors
  - $L$ is length of detector array

- Then

$$\text{SNR} \approx 0.4kC\mu L\sqrt{\frac{\bar{N}_f M}{D^3}}$$

SNR decreases as $D$ increases.
Reason: Convolution of the projection reading with the ramp filter couples the noise between detectors, and effectively increases the noise as the number of detector increases. But larger $D$ is desired to obtain a good resolution.

N=Nf/D
w=L/D
Rule of Thumb

- Variables:
  - $D$ is number of detectors
  - $M$ is number of angles
  - $J^2$ is number of pixels in image
- Very approximate “rule”:
  \[ D \approx M \approx J \]
- Typical numbers:
  \[ \text{Lo: } D \approx 700 \quad M \approx 1,000 \quad J \approx 512 \]
  \[ \text{Hi: } D \approx 900 \quad M \approx 1,600 \quad J \approx 1,024 \]
Aliasing Artifacts

- Nyquist Sampling theorem:
  - If the maximum freq of a signal is $f_{\text{max}}$, it should be sampled with a freq $f_s \geq 2f_{\text{max}}$, or sampling interval $T \leq 1/2f_{\text{max}}$
  - If sampled at a lower freq. without pre-filtering, aliasing will occur
    - High freq. content fold over to low freq
    - Prefilter to lower $f_{\text{max}}$, and then sample

- If the number of samples in each projection (D) or the number of projection angles (M) are not sufficiently dense, the reconstructed image will have streak artifacts
  - Caused by aliasing
  - Practical detectors are area detectors and perform pre-filtering implicitly
Summary

- Parallel projection reconstruction
  - Backprojection summation
  - Fourier method (projection slice theorem)
  - Filtered backprojection
  - Convolution backprojection
  - Practical implementation: using finite samples
- Fan beam projection and reconstruction
  - Weighted backprojection
- Blurring due to non-ideal filters and detectors
  - Approximate the overall effect by a filter:
    - \( h(l) = w(l) \ast s(l) \); \( H(\rho) = W(\rho) S(\rho) \)
  - Circularly symmetric functions and Hankel transform
    - Equivalent spatial domain filter \( h(r) = \text{inverse Hankel } \{H(q)\} \)
- Noise in measurement and reconstructed image
  - Factors influencing the SNR of reconstructed image
    - Average X-ray intensity, Number of angles (M), number of samples per angle (D), filter cut-off \( \rho_0 \)
- Impact of number of projection angles and samples on reconstruction image quality
  - Nyquist sampling theorem
  - Streak artifacts
Reference

• Prince and Links, Medical Imaging Signals and Systems, Chap 6.
  – Chap 3 Contain detailed derivation of reconstruction algorithms both for parallel and fan beam projections. Have discussions both in continuous domain and implementation with sampled discrete signals.
  – Chap 5 discusses noise in measurement and reconstructed image.
  – Chap 5 also covers aliasing effect with more mathematical interpretations
• A useful lecture note from Prof. Fessler
• Lecture note by Prof. Parra
• For more discussion on aliasing due to under sampling
  – https://engineering.purdue.edu/~malcolm/pct/CTI_Ch05.pdf
Homework

• Reading:
  – Prince and Links, Medical Imaging Signals and Systems, Chap 6, Sec.6.3.4-6.5

• Note down all the corrections for Ch. 6 on your copy of the textbook based on the provided errata.

• Problems for Chap 6 of the text book:
  – P.6.9
  – P.6.10 (part e is not required)
    • Hint: solution for part (a) should be
      \[
      g(l,60) = \begin{cases} 
      \sqrt{3} \mu(a/2 + l) & -a/2 \leq l \leq 0 \\
      \sqrt{3} \mu(a/2 - l) & 0 \leq l \leq a/2 \\
      0 & \text{otherwise}
      \end{cases}
      \]
  – P.6.19
  – P.6.20
Computer Assignment
Due: 10 am on Friday October 20

Will send assignment via NYU Classes today.