(Closed book, 1 sheets of notes double sided allowed, no calculator or other electronic devices. Please leave all electronics in your backpack and leave your backpack in the front of the classroom.)
1) (20 pts + 5 pts bonus) The mamougram is a new imaging technology to diagnose suspicious masses in the breast. It uses ultrasound technology and returns a Mamou Index (MI) between 0 and 8. In a study of 1000 women undergoing mamougram imaging, the following two probability density functions were found for non-cancerous (NC) and cancerous (C) diagnoses of the suspicious mass as a function of MI:

\[ P_{NC}(x) = \frac{1}{3} \text{tri}((x-3)/3) \]

\[ P_{C}(x) = \frac{1}{2} \text{tri}((x-6)/2) \]

Where \( \text{tri}(x) = 1-|x| \) if \( |x|<1 \) and 0 if \( |x|\geq 1 \) and \( x \) is the MI

These functions are plotted in Figure 1:

Following mamougram imaging, the suspicious mass was diagnosed with gold standard histology. It was found that in this population of 1000 women, 200 women had a cancerous mass and 800 had a non-cancerous mass.

a) Define sensitivity, specificity, accuracy, and prevalence. (5 pts)

b) Compute the sensitivity, specificity, accuracy, and prevalence when an MI threshold of 5 is used for diagnosis (i.e., masses having an MI<5 are said to be non-cancerous and those having an MI\geq5 are said to be cancerous) (5 pts)

c) Repeat b) for thresholds of 4 and 6. (5 pts)

d) Discussion with breast cancer experts led to the determination that the best operating point would be at a sensitivity of 81%. Determine the corresponding MI threshold and the specificity. (5 pts)

e) Plot the ROC curve associated with the mamougram diagnostic test. (Hint: The x axis of the ROC curve is 1-specificity and the y axis is sensitivity.) (bonus 5 pts)
2) (15 pts) X-ray CT:
   a) Write the equation linking the 1D Fourier transform of the projection of an object defined by
      \( f(x,y) \) at angle \( \theta \) and a specific part of the 2D Fourier transform of \( f(x,y) \).
      Illustrate this relationship through a simple drawing. What is the name of the underlying theorem defining that
      relationship? (3 pts)
   b) Describe briefly the four main methods for image reconstruction in x-ray computed tomography
      and provide the key reconstruction equation for each method. (3 pts)
   c) Compare each method’s accuracy and briefly justify your response. (3 pts)
   d) Compare each method’s computational efficiency and briefly justify your response. (3 pts)
   e) In x-ray CT what is the advantage of taking data at two distinct energy levels? (3 pts)

3) (15 pts) X-ray generation
   a) Describe briefly how X-rays are created in an X-ray tube, including the two primary form
      radiation. (4 pts)
   b) If the voltage applied between the anode and the cathode is 140 KV, and the binding energy of K-shell
      electrons in the tungsten anode is 70 KeV, and that of L-shell and M-shell are respectively
      11 KeV and 3 KeV. Sketch the shape of the resulting x-ray energy spectrum. Properly label your
      horizontal axis. You only need to consider characteristic rays generated when ejected K-shell
      electrons are replaced by L-shell and an M-shell electrons. (5 pts)
   c) Sketch the energy spectrum created when the voltage is changed to 45 KV, on the same figure
      you generated for part b). Would there be characteristic rays in this case and why? (3 pts)
   d) Why are patients asked to drink a “chalky milkshake” containing barium when undergoing
      gastrointestinal tract imaging? (3 pts)

4) (15 pts) SPECT and PET
   a) In the forward model of SPECT and PET, what is the key difference regarding the influence of
      the attenuation on the measurement? Explain and justify for which modality it is easier to
      compensate attenuation effects. (5 pts)
   b) In a PET-CT system, why attenuation compensation of the PET measurements cannot typically
      be done directly from CT measurements? (5 pts)
   c) How can CT measurements be used for PET correction? (5 pts)
5) (15 pts) A 2-D slice to be imaged is shown in Fig. 2, which consists of two regions with linear attenuation coefficients $\mu_1$ and $\mu_2$. The background has a linear attenuation coefficient of $\mu_3$.

a) This medium is first imaged using x-ray CT. Assuming parallel beams propagating from the left of the figure. What are the measured signals at B and C? (5 pts)

b) Next, suppose a solution containing a gamma ray emitting radio tracer with an initial radioactivity of $A_1$ and half-life of $T_1$ is injected into region 1 and another solution with initial radioactivity $A_2$ and half-life $T_2$ is injected into region 2. The radioactivity distribution in this slice is imaged using a rotating SPECT camera. Compute the measured signal by the camera at positions A and B, respectively, at time $t = T_3$ after the injection of the radionuclide solutions. (5 pts)

Note: you can just express your solutions in terms of properly written integrals, in terms of given variables.

c) Assuming both imaging were performed at the same energy level and that it is known that $\mu_3 = 1 \text{ cm}^{-1}$, $T_1 = T_3/2$, $T_2 = T_3/4$, express the solution of b) using only $A_1$ and $A_2$ and the measured CT values at B and C. (Hint: First express $\mu_1$ and $\mu_2$ as a function of the CT measurements at B and C.) (5 pts)

![Figure 2.](image-url)
6) (20 pts) In the medium shown in Figure 3, Suppose the tissue slice (with dimension 6cm x 6cm) being imaged by a parallel beam x-ray CT scanner has a distribution of the linear attenuation coefficients as shown in Fig. 3, with \( \mu_1 = 1, \mu_2 = 2, \mu_3 = 3, \) and \( \mu_0 = 0 \).

a) Assume the detector is a point detector. Sketch the projection \( g(l, \theta) \) as a function of \( l \), for \( \theta = 90^\circ \) (projecting horizontally) and \( 45^\circ \) (projecting along diagonal line from top left to bottom right, hint: Figure 4 may be useful), respectively. You should indicate the magnitudes of the projected values where necessary on your sketch. Also clearly specify any transition points in the \( l \)-axis. (5 pts)

b) Sketch the image obtained by back projections from \( 90^\circ \) and \( 45^\circ \) projections, individually and together. (5 pts)

c) Determine the Fourier transform of the original image along the vertical axis in the frequency domain. (5 pts)

d) What will be the projected function for \( \theta = 90^\circ \) if the detector is an area detector with width 0.3 cm. Sketch the projection function. (5 pts)

Hint: you could make use of the following Fourier transform pair and property:

\[
g(x) = \begin{cases} 
1, & \text{if } |x| < \Delta/2 \\
0, & \text{otherwise}
\end{cases} \quad \Leftrightarrow \quad G(f) = \Delta \frac{\sin(\pi \Delta f)}{\pi \Delta f}
\]

\[
g(x - x_0) \leftrightarrow G(f) e^{-j2\pi f x_0}
\]

Figure 3
Figure 4.