**Problem 9.13 Show that the optimal linear estimator solution for OBMC is as given in Equation (9.3.4).**

**Answer:**

The optimization problem can be formulated as:

To minimize

$$E\{|\psi(\mathbf{x}) - \sum_{k\in\mathcal{K}} h_k(\mathbf{x})\psi_r(\mathbf{x}+\mathbf{d}_{m,k})|^2\}.$$

subject to

$$\sum_{k\in\mathcal{K}} h_k(\mathbf{x}) = 1.$$

Using Lagrangian Multiplier Method,

$$J(h_k(\mathbf{x}), k\in\mathcal{K}) = E\{|\psi(\mathbf{x}) - \sum_{k\in\mathcal{K}} h_k(\mathbf{x})\psi_r(\mathbf{x}+\mathbf{d}_{m,k})|^2\} + \lambda(\sum_{k\in\mathcal{K}} h_k(\mathbf{x}) - 1).$$

**Problem 11.3 How does the FGS method described in Section 11.1.6 work? What is the difference between FGS and quality scalability?**

**Answer:**

In FGS method, the conventional block-based hybrid coding method is employed to produce a base-layer stream at a given frame rate, using a relatively large QP. Then, for every coded frame, the differences between the original DCT coefficients and the quantized coefficients in the base layer are coded into a fine-granularity stream. This is accomplished by quantizing the refinement coefficients using a very small QP and then representing the quantized indices through successive bit plane encoding.

Specifically, the absolute values of quantized refinement coefficients in each block are specified in binary representations. Starting from the highest bit plane that contains nonzero bits, each bit plane is successively coded using run-length coding, block by block. The run lengths can be coded using either Huffman or arithmetic coding.

For FGS, when only a partial set of the enhancement-layer stream is decoded, depending on how many bit planes were included in the retained bits, the reconstructed video will have a quality between that obtainable from the base layer only, to that obtainable with the QP used on the refinement DCT coefficients. The granularity of the stream is at the bit-plane level: every additional complete bit plane will yield an improvement in the accuracy of the DCT coefficients by a factore of two.

For qulaity scalable, the quality improvements are obtained with rate increases in large discrete steps.

**Problem 11.4 Consider i.i.d. Gaussian processes. Suppose that one can design a one-layer coder that reaches the RD bound; that is, the distortion and rate are related by $D=\sigma_x^2 2^{-2R}$, where $\sigma_x^2$ is the source signal variance. Show that a quality scalable coder with two quantization layers will reach the same RD bound as the one-layer coder. Extend the proof to an arbitrary number of layers.**

**Answer:** For first layer,

$$D_1 = \sigma_x^2 2^{-2R_1} \tag{1}$$

For second layer, quantization is done on the error image from the first quantizer, with a variance of $D_1$.

$$D_2 = \sigma_{x,2}^2 2^{-2R_2} = D_1 2^{-2R_2} = \sigma_x^2 2^{-2R_1} 2^{-2R_2} = \sigma_x^2 2^{-2R} \tag{2}$$

where $R = R_1 + R_2$. This result can be extended to arbitrary number of layers,

$$D_n = D_{n-1} 2^{-2R_n} = \ldots = \sigma_x^2 2^{-2R} \tag{3}$$

where $R = \sum_{i=1}^{n} R_i$.