1. (a) $f_\ell = 400 \times 25 = 10000$ lines / sec

(b) The maximum temporal freq. the system can handle is half of the frame rate $f_{t,max} = \frac{25}{2} = 12.5$ Hz.

(c) The maximum vertical freq is half of the lines/frame

$$f_{v,max} = \frac{400}{2} = 200 \text{ cycles/picture-height}$$

(d) Assuming the image signal is symmetric in terms of its maximum # of cycles over the same distance horizontally & vertically, the maximum number of cycles/picture-width is

$$f_{h,max} = \frac{4}{3} \cdot f_{v,max} = \frac{800}{3} \text{ cycles/picture-width}$$

Because there are f_ℓ lines/sec, the freq in terms of cycles/sec in the raster signal is

$$f_y = f_{h,max} \cdot f_\ell = \frac{8}{3} E6 = \frac{8}{3} \text{ MHz}$$

(e) See figure below. f_c should satisfy

i) $f_c \le f_y - \frac{1}{10} f_y = \frac{9}{10} f_y$

ii) f_c must fall on the middle of multiples of f_ℓ, ie

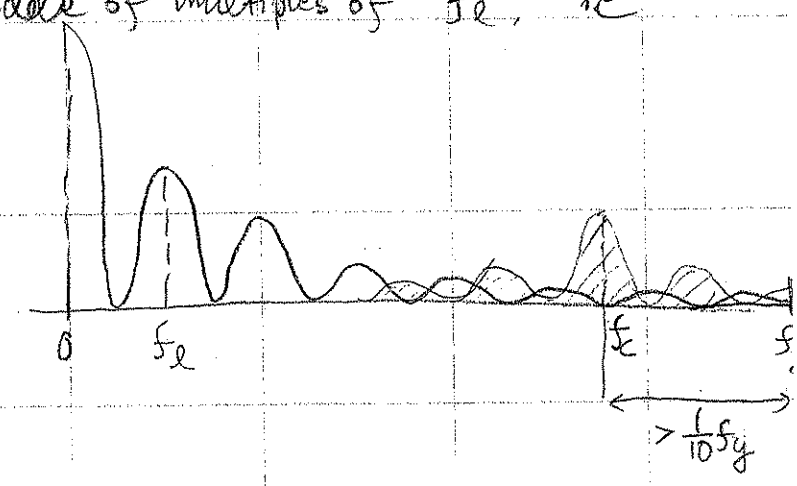
$$f_c = (K + \tfrac{1}{2}) f_\ell$$

(where K is an integer

Let $(K+\frac{1}{2}) f_\ell \approx \frac{9}{10} f_y$

$K \le \frac{9}{10} \frac{f_y}{f_\ell} - \frac{1}{2} = 239$

$f_c = 2.395$ MHz



2. a). When the scene consists of vertical bars moving horizontally, vertical freq $f_v = 0$, there is high temporal freq. using interlaced scan would be better as within each field, the human eye can interpolate vertical bars easily, and higher field rate enables better rendition of bars horizontal motion.

b) When the scene consists of horizontal bars moving horizontally, there are no temporal change, hence a low temporal sampling rate is acceptable. On the other hand, to separate closely spaced horizontal bars, the vertical sampling rate should be as high as possible. Therefore progressive scan is better.

(3) To solve this problem, we need to determine the maximum horizontal & vertical angular frequency the viewer can see if the screen of size $w \times h$ are represented by $N \times M$ pixels.

To determine vertical angular freq, we need to express vertical viewing angle θ in terms of h and d.

From the given figure

$$\tan\left(\frac{\theta}{2}\right) = \frac{\frac{h}{2}}{d} \qquad \theta = 2\left[\tan^{-1}\frac{h}{2d}\right] \approx \frac{2h}{2d} = \frac{h}{d} \text{ radian} = \frac{h}{d}\frac{180}{\pi}_{\text{degree}}$$

With M rows, there are at most $M/2$ cycles/picture hight

So the maximum angular freq is

$$f_{v,max} = \frac{M}{2}/\theta = \frac{M d \pi}{2h \cdot 180} \quad (\text{cycles/degree})$$

Let $f_{v,max} \geq 10$, $\quad M \geq \frac{3600 \cdot h}{d \cdot \pi}$

Similarly,

$$f_{h,max} = \frac{N \cdot d \pi}{2 w 180},$$

Let $f_{h,max} \geq 10$, $\quad N \geq \frac{3600 \cdot w}{d \cdot \pi}$

For example, for a SDTV monitor, $w = \frac{4}{3}h$. Assume $d = 3h$

we have $M \geq \frac{3600 \cancel{h}}{3 \cancel{h} \pi} \approx 383 \qquad N \geq \frac{4}{3} M \approx 511$

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4. $x = F\frac{X}{Z}$, $y = F\frac{Y}{Z}$, $x' = F\frac{X'}{Z'}$, $y' = F\frac{Y'}{Z'}$

$\therefore$ $X = x\frac{Z}{F}$, $Y = y\frac{Z}{F}$, $X' = x'\frac{Z'}{F}$, $Y' = y'\frac{Z'}{F}$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} x'\frac{Z'}{F} \\ y'\frac{Z'}{F} \\ Z' \end{bmatrix} = \frac{Z'}{F}\begin{bmatrix} x' \\ y' \\ F \end{bmatrix} = \begin{bmatrix} 1 & 0 & \theta_y \\ 0 & 1 & 0 \\ -\theta_y & 0 & 1 \end{bmatrix}\begin{bmatrix} x\frac{Z}{F} + T_x \\ y\frac{Z}{F} \\ Z \end{bmatrix}$$

$$\therefore \begin{bmatrix} x' \\ y' \\ F \end{bmatrix} = \begin{bmatrix} 1 & 0 & \theta_y \\ 0 & 1 & 0 \\ -\theta_y & 0 & 1 \end{bmatrix}\begin{bmatrix} x + \frac{T_x F}{Z} \\ y \\ F \end{bmatrix} \cdot \frac{Z}{F} \cdot \frac{F}{Z'}$$

$$x' = \left[x + \frac{T_x F}{Z} + F\theta_y\right]\frac{Z}{Z'} \qquad ①$$

$$y' = y \cdot \frac{Z}{Z'} \qquad ②$$

$$F = \left[-\theta_y\left(x + T_x\frac{F}{Z}\right) + F\right]\frac{Z}{Z'} \qquad ③$$

from ③, we know

$$\frac{Z}{Z'} = F \Big/ \left[-\theta_y x - \theta_y \cdot \frac{T_x F}{Z} + F\right]$$

$$\therefore x' = \left[x + \frac{T_x F}{Z} + F\theta_y\right] \cdot \frac{F}{\left[-\theta_y x - \theta_y \cdot \frac{T_x F}{Z} + F\right]}$$

$$y' = y \cdot \frac{F}{\left[-\theta_y x - \theta_y \frac{T_x F}{Z} + F\right]}$$

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5. a. The number of operations is

$$MN \cdot (2R+1)^2 \cdot f_s$$

b. If use half-pel, the number is

$$4MN(2R+1)^2 \cdot f_s$$

c. The operation count for EBMA increases with search range. increases with search accuracy (for example, factor of 4 for half-pel). It doesn't depend on block size. linearly increase with frame size and frame rate.

The block size, search range and search stepsize affect the prediction accuracy. Smaller block size, larger search range, and smaller stepsize will increase the accuracy.

d. Advantage: less computation, yields smoother motion field which is usually more accurate physically (avoiding local minima)

Disadvantage: may not minimize the matching error as does EBMA. more complicated software and hardware implementation.

6. For each block B, we would like to determine $\bar{d}_k$, $k=1,2,3,4$, to minimize the following error

$$E = \sum_{x \in B} \left| f_1(\bar{x}) - f_2(\bar{x} + d(\bar{x})) \right|^2$$

$$d(\bar{x}) = \sum_{k=1}^{4} \phi_k(\bar{x})\, \bar{d}_k = \begin{bmatrix} \sum \phi_k(\bar{x})\, d_{x,k} \\ \sum \phi_k(\bar{x})\, d_{y,k} \end{bmatrix} = \begin{bmatrix} d_x(\bar{x}) \\ d_y(\bar{x}) \end{bmatrix}$$

To use gradient descent method, we need to find the gradient

$$\begin{bmatrix} \frac{\partial E}{\partial d_{x,k}} \\ \frac{\partial E}{\partial d_{y,k}} \end{bmatrix} = \frac{\partial E}{\partial \bar{d}_k} = -\sum_{x \in B} 2\left(f_1(\bar{x}) - f_2(\bar{x} + d(\bar{x})) \right) \begin{bmatrix} \frac{\partial f_2}{\partial x} \cdot \frac{\partial d_x(\bar{x})}{\partial d_{x,k}} \\ \frac{\partial f_2}{\partial y} \quad \frac{\partial d_y(\bar{x})}{\partial d_{y,k}} \end{bmatrix}$$

Because $\frac{\partial d_x(\bar{x})}{\partial d_{x,k}} = \phi_k(\bar{x})$, $\frac{\partial d_y(\bar{x})}{\partial d_{y,k}} = \phi_k(\bar{x})$

We have

$$\frac{\partial E}{\partial \bar{d}_k} = -\sum_{x \in B} 2\left(f_1(\bar{x}) - f_2(\bar{x} + d(\bar{x})) \right) \phi_k(\bar{x}) \begin{bmatrix} \frac{\partial f_2}{\partial x} \\ \frac{\partial f_2}{\partial y} \end{bmatrix}$$

Gradient descent algorithm: $E_{old} = E_{max}$

~~Set~~ set $n=0$, $\bar{d}_k^{(0)} = \bar{d}_{k,0}$ (chosen initial solution, eg $\bar{d}_{k,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Compute: $d^{(n)}(x) = \sum \phi_k(\bar{x})\, d_k^{(n)}$, $e(\bar{x}) = f_1(\bar{x}) - f_2(\bar{x} + \bar{d}^{(n)}(\bar{x}))$

$E_{new} = \sum_{\bar{x} \in B} [e(\bar{x})]^2$, If $\frac{|E^{old} - E^{new}|}{E^{old}} > \varepsilon$, continue

$$\bar{g}^{(n)} = \frac{\partial E}{\partial \bar{d}_k}\Big|_{\bar{d}_k^{(n)}} = -2 \sum e(\bar{x})\, \phi_k(\bar{x}) \begin{bmatrix} \frac{\partial f_2}{\partial x} \\ \frac{\partial f_2}{\partial y} \end{bmatrix}\Bigg|_{x + d^{(n)}(x)}$$

$\bar{d}_k^{(n+1)} = \bar{d}_k^{(n)} - \alpha\, \bar{g}^{(n)}$, $E_{old} = E_{new}$, $n = n+1$

7. a). First-order Entropy

$H_1(X_k) = -\sum_{k=1}^{L} P_1(x_k)\log_2 P_1(x_k)$. $H_1(X_k)$ is the lower bound.

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b) Joint Entropy

$$H(X_{2k}, X_{2k+1}) = -\sum_{m=1}^{L}\sum_{n=1}^{L} P_2(x_m, x_n)\log_2 P_2(x_m, x_n)$$

$H(X_{2k}, X_{2k+1})/2$ is the lower bound

c) First-order conditional Entropy

H(Prediction Error) $= H_{c,1}$ below

$$H_{c,1}(X_k|X_{k-1}) = \sum_{m=1}^{L} P_1(x_m) H(x_n|x_m)$$

$$H(x_n|x_m) = -\sum_{n=1}^{L} P_{2|1}(x_n|x_m)\log_2 P_{2|1}(x_n|x_m)$$

$H_{c,1}$ is the lower bound.

$$\because H_{c,1}(X_k|X_{k-1}) \le \frac{H(X_{2k}, X_{2k+1})}{2} \le H_1(X_k)$$

$\therefore$ efficiency. * 3rd Method $\ge$ 2nd method $\ge$ 1st method