**Medical Imaging (EL582/BE620/GA4426)**

**Ultrasound Imaging**

**Reference**
Prince and Links, Medical Imaging Signals and Systems, Chap. 10 (Sec. 10.5 not required), 11.2, 11.3

**Acknowledgement**
Thanks to Professor Yao Wang for use of her course materials!

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**Ultrasound Imaging**

- Measure the reflectivity of tissue to sound waves
- Can also measure velocity of moving objects, e.g. blood flow (Doppler imaging)
- No radiation exposure, completely non-invasive and safe (*)
- Fast
- Inexpensive (relatively)

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**Medical Ultrasound Imaging**

- Medical applications: fetus, heart, abdominal, ...
- 3-10 MHz
- ≤ 1 mm resolution (limited contrast)
- ≥ 60 images per second

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**In vivo microimaging in mice**

Ultrasound Biomicroscopy (UBM)
- 40-50 MHz ultrasound
- 50-80 µm lateral resolution, better axial
- up to 1000 images/s

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**UBM of Mouse Embryos**

**APPLICATIONS**

- Brain imaging
- Image-guided injections
- Cardiac imaging
- (Image-guided injections)
- Vascular imaging
- Blood flow (Doppler)
- Tumors

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**Ultrasound biomicroscopy enables in utero volumetric brain analysis**

E13.5 Mouse Embryo

In utero UBM  Ventricle Segmentation  3D rendering

Aristizábal et al., Ultrasound Med Biol, 2006
UBM-Doppler provides sensitive analysis of cardiovascular function

Phoon et al, Circ Res, 2004

Molecular imaging with ultrasound: targeted microbubbles

Microbubbles targeted to vascular endothelial cells in E11.5 mouse embryos


Acoustic Waves

- Pressure waves that propagate through matter via compression and expansion of the material
  - Generated by compressing and releasing a small volume of tissue
- Longitudinal wave
  - Particles in the medium move back and forth in the same direction that the wave is traveling
- Shear Wave
  - Particles move at right angles to the direction of the wave
  - Not used for medical ultrasound imaging

Longitudinal Wave

EM vs Acoustic Waves

- Electromagnetic
  - Self-propagating, consisting of electric and magnetic components oscillating at right angles to each other, and in propagation direction
  - Does not require a material medium through which to propagate
  - Classification (increasing in frequency, decreasing in wavelength):
    - radio, microwave, infrared, visible light, ultraviolet, x-ray, gamma ray
- Acoustic
  - Pressure waves that propagate through matter via compression and expansion of the material
  - Requires a material medium through which to propagate
  - Classification (increasing in frequency, decreasing in wavelength):
    - Infrasonic sound, audible sound, ultrasound

Transfer / Transformation of Energy

- Light becomes sound — photoacoustic phenomena
- Sound becomes light — sonoluminescence
- Absorbed electromagnetic (EM) and acoustic energy both become heat
- Nevertheless, EM and acoustic energy are fundamentally distinct phenomena!
Acoustic Wave Energy Ranges

<table>
<thead>
<tr>
<th>Infrasound</th>
<th>Audible</th>
<th>Ultrasound</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 Hz</td>
<td>20 kHz</td>
<td></td>
</tr>
</tbody>
</table>

- Just as there are infrared, visible, and ultraviolet ranges in the EM spectrum, so there are infrasound ("infra" = "below," "beneath"), audible (i.e., sound) and ultrasound ("ultra" = "beyond," "above") ranges of acoustic wave frequencies.

- Note that the ratio of the highest to the lowest audible frequencies is $10^3$, while the ratio of the highest to the lowest frequencies of visible light is a bit less than 2!

### Speed of Sound

- Each medium has a characteristic speed:
  - $c \text{ [m/s]} = \lambda \text{ [m]} \times f \text{ [MHz]}$
  - $= \text{wavelength} \times \text{frequency}$

- Approximate ultrasound speeds:
  - Air: 330 m/s
  - Water: 1500 m/s
  - Muscle: 1600 m/s
  - Fat: 1480 m/s
  - Bone: 3000 m/s

### Pulse-Echo Ultrasound Imaging

- **Transducer**
- **Object**
- **Scan**
  - Beam builds up image

- **Excitation Pulse**
- **RF Amplitude**
- **Envelope Amplitude**

- **Time = Depth**
  - 1 line of image data

### Image frame rate is determined by sound speed

- Sound speed = 1540 m/s = 1.54 mm/μs
- 256 line image / Depth = 10 mm

  - Propagation length = 20 mm (2-way)
  - Time per line = 20/1.54 ~ 13 μs

  - Time per image = 13 × 256 = 3300 μs = 3.3 ms

  - Frame rate = 1/3.3 ms ~ 300 images/s

### 3D Wave Equation

- **Acoustic pressure:** $p(x, y, z, t)$
- **3-D wave equation**
  $$\nabla^2 p(x, y, z, t) = \frac{1}{c^2} \frac{\partial^2 p(x, y, z, t)}{\partial t^2}$$

  where
  $$\nabla^2 p = p_{xx} + p_{yy} + p_{zz}$$

  and $c$ is the speed of sound

- **General solution is very complicated**

### Plane Wave

- **Plane wave in z-direction:**
  $$p(z, t) = p(x, y, z, t)$$

- **Plane wave equation:**
  $$p_{zz}(z, t) = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}(z, t)$$

- **General solution:**
  - **Forward traveling wave**
    $$p(z, t) = \phi(t - c^{-1}z) + \phi(t + c^{-1}z)$$

  where $\phi(t)$ and $\phi(t)$ are arbitrary
Harmonic Waves

- Harmonic plane wave:
  \( p(z,t) = \cos(k[z - ct]) \)
- Viewed at a fixed particle, the pressure changes in time with frequency \( f = k/c \) (cycles/s)
- Viewed at a fixed time, the pressure changes in \( z \) with frequency \( f_z = k/2\pi \)
  - \( k \) is called wavenumber
- Wavelength is the spacing between peak or valleys of the wave at any time
  \( \lambda = 1/f_z = \frac{2\pi}{k} = \frac{c}{f} \)
- (approximately) Harmonic waves are widely used in ultrasound imaging
- Given \( f \), the wavelength depends on material properties!
  - Wavelength determines the resolution of ultrasound imaging
  - Example: \( f = 3.5 \text{ MHz}, c = 1540 \text{ m/s (most tissue)}, \lambda \approx 0.44 \text{ mm} \)

Reflection/refraction: Snell’s Law

- Reflection / refraction at interfaces follows Snell’s law:
  \[
  \frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{c_1}{c_2}
  \]
  \( \theta_i = \theta_r \)

Acoustic Impedance

- Acoustic Impedance, \( Z \) [MRayl = \( 10^6 \text{ kg/m}^2/\text{s} \)]
  \( Z = \text{density} \times \text{sound speed} \)
  - Determines the amplitude of the reflected / transmitted waves at interface
  - Complex scattering properties of tissues are due to acoustic impedance interfaces in microstructure of tissues

Reflection at Interfaces

Reflection Coefficients:

<table>
<thead>
<tr>
<th>Reflection</th>
<th>Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_1 )</td>
<td>( Z_2 )</td>
</tr>
<tr>
<td>( I )</td>
<td>( R )</td>
</tr>
<tr>
<td>( T )</td>
<td></td>
</tr>
</tbody>
</table>

\( R = |(Z_2 - Z_1)/(Z_2 + Z_1)| \)
\( T = 1 - R \)
\( = 2Z_2/(Z_2 + Z_1) \)

Reflection at Interfaces: Example

<table>
<thead>
<tr>
<th>Reflected Sound Pressure:</th>
<th>Muscle</th>
<th>Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{\text{mus}} = 1.7 \text{ MRayl} )</td>
<td>( Z_{\text{fat}} = 1.4 \text{ MRayl} )</td>
<td></td>
</tr>
</tbody>
</table>

\( R/I = |(Z_{\text{mus}} - Z_{\text{fat}})/(Z_{\text{mus}} + Z_{\text{fat}})| \)
\( = 0.3/3.1 \)
\( \approx 0.1 (10% = -20 \text{ dB}) \)

Attenuation of Ultrasound

- Attenuation = Energy lost through interactions between ultrasound waves and soft tissues:
  - Absorption:
    \( \text{Power deposited in tissue (Heat)} \)
  - Scattering:
    \( \text{Ultrasound radiated away from transducer} \)
Attenuation of Ultrasound

- Attenuation is frequency dependent:
  \[ a(f) = a_o f^n \]
  - \( a_o \) is the attenuation coefficient at 1 MHz
  - \( n \approx 1 \) for most soft tissues
- Attenuation leads to a decrease in amplitude of the ultrasound signal:
  \[ \text{Attenuation} \approx 1 \text{ dB} / \text{cm} / \text{MHz} \]

Attenuation: An Example

What relative amplitude of a 60 MHz ultrasound signal do you expect to receive from a depth of 5 mm?

- Attenuation \( \approx 1 \text{ dB} / \text{cm} / \text{MHz} \)
- @ 60 MHz: Attenuation \( \approx 60 \text{ dB/cm} \)
- Depth = 5 mm: Ultrasound propagates through 1 cm
- Attenuation \( \approx 60 \times 1 = 60 \text{ dB} \)
- 1/1000 of the transmitted signal is received!

Attenuation: Consequences

- Consequences of frequency dependent attenuation for imaging:
  - Penetration of ultrasound is limited by frequency
  - Frequency of ultrasound decreases with increasing depth of imaging

Resolution in Ultrasound Imaging

- Axial Resolution:
  - Resolution in propagation direction
  - Determined by length of pulse propagating in tissue
- Lateral Resolution:
  - Resolution orthogonal to propagation direction
  - Determined by focusing properties of transducer

Axial Resolution

- Axial Resolution:
  \[ \text{Axial Resolution} = \frac{\text{pulse width (s)} \times \text{speed of sound (m/s)}}{2} = \frac{2N\lambda}{2} \]

Lateral Resolution

- Lateral Resolution:
  \[ \text{Lateral Resolution} = \frac{\text{wavelength} \times \text{f-number}}{2a} \]
**Resolution vs Penetration**
- Resolution (axial and lateral) with frequency
- Penetration with frequency

Compromise between resolution and penetration

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**Doppler Ultrasound: Basic Concepts**
- Ultrasound wave reflected from moving targets (Blood cells)
- Frequency shift in received ultrasound wave compared to transmitted wave:
  
  Doppler Shift Frequency, $f_d$

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**Doppler Ultrasound: Basic Concepts**

Transducer $\square \square \square \square \square \bullet$ Target (stationary): $f_d = 0$

- Target moves towards transducer:
  - More compressions per unit time: $f_d > 0$

- Target moves way from transducer:
  - Fewer compressions per unit time: $f_d < 0$

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**Doppler Ultrasound: Concepts**
- Doppler Equation:
  
  $f_d = 2f_o \cdot v \cdot \cos \theta / c$

  - $f_o$ is the frequency transmitted
  - $v$ is the velocity of the moving blood
  - $c$ is the sound speed in the medium (blood, ~1600 m/s)

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**Doppler Equation: Consequences**
- Shift frequency is proportional to blood velocity
- $f_o = 2-10$ MHz, $v = 0-5$ m/s $\rightarrow$ $f_d = 0-15$ kHz
  
  (Audio frequencies)
- $f_d$ is maximized when blood flow is in-line with ultrasound beam ($\theta = 0$)
- $f_d = 0$ when flow is perpendicular to the beam

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**Doppler Data Processing**

(Reprinted from Ultrasound in Medicine & Biology 1998)
UBM-Doppler analysis is sensitive to blood flow abnormalities

Flow reversal in mouse mutants with defective cardiac valves

E13.5

Wildtype

NFATc1-/-

Schematic: Ultrasound Imaging System

Functions of the transducer

- Used both as Transmitter And Receiver
- Transmission mode: converts an oscillating voltage into mechanical vibrations, which causes a series of pressure waves into the body
- Receiving mode: converts backscattered pressure waves into electrical signals

Piezoelectric Material

- Converts electrical voltage to mechanical vibration and vice versa
- The thickness of the crystal varies with the applied voltage
- When an AC voltage is applied across the crystal, the thickness oscillates at the same frequency of the voltage
- Examples of piezoelectric Materials:
  - Crystalline (quartz), Ceramic (PZT, lead zirconium titanate), Polymers (PVDF)
  - PZT is the most efficient material
- The crystal vibrates sinusoidally after electrical excitation has ended (resonance)
  - Resonant frequency \( f = c/2d \) (with) thickness
  - The damping material damps the vibration after 3-5 cycles
  - When the diameter \( D \) of the surface is much larger than \( d \), longitudinal waves are transmitted into the body
  - The crystal is shaped into a disk or rectangle, with either flat or concave surface

Matching Layer(s)

- To provide acoustic coupling between transducer and the patient skin to transfer maximum energy into the body
- \( Z \) of PZT \( Z_T \) is \( 15 \) times greater than \( Z \) of tissue \( Z_L \)
- Placing crystal directly on skin results in a large amount of energy being reflected back from the boundary
- \( R = \frac{|(Z_L - Z_T)/(Z_L + Z_T)|}{|Z_L + Z_T|} \approx 1 \)
- Matching layer
  - \( \frac{Z_T}{Z_L} \) thickness = \( \frac{1}{4} \)
  - \( Z_T = (Z_L + Z_T) \)
  - Maximizes energy transfer into the body
  - Show as a homework(*)
- Problem: Finding material with exact \( Z \) value
Flat (Piston) Plate Transducer

- Simple model:
  - Fraunhofer
  - Fresnel

Beam Properties of a Piston Transducer

- At border of the beam width, the signal strength drops by a factor of 2, compared to the strength on the z-axis.
- Beam width determines the imaging resolution (lateral resolution).
- Smaller D is good only before far field
- \(D=1-5\) cm in practice, very poor lateral resolution
- Focused plate is used to produce narrow beam

Focused Transducer

- Beam focusing can be accomplished by
  - Using a crystal with a curved surface
  - Placing a concave lens in front of the crystal

Transducer Array

- With a single crystal, manual or mechanical steering of the beam is needed to produce a two-dimensional image
- Practical systems today use an array of small piezoelectric crystals
- Allow electronic steering of the beam to optimize the lateral resolution

Array types

- Linear Sequential (switched)
  - \(1\) cm \(\times\) 10-15 cm, up to 512 elements
- Curvilinear similar to (a), wider field of view
- Linear Phased
  - up to 128 elements, small footprint = cardiac imaging
- 1.5D Array
  - 5-9 elements in elevation allow for focusing
- 2D Phased
  - Focusing, steering in both dimensions

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap. 10 (Sec. 10.5 not required), 11.2, 11.3
- Problems:
  - P10.1
  - P10.3
  - P10.6
  - P10.8
  - P10.12
  - P10.13
- Considering the \((1/4)\) matching layer in a transducer, show that the transmitted energy into the tissue is maximized with an impedance of \(\sqrt{Z_T Z_L}\)