Physics of MRI

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Based on J. L. Prince and J. M. Links, Medical Imaging Signals and Systems, and lecture notes by Prince. Figures are from the textbook except otherwise noted.
Lecture Outline
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- Overview of MRI
- Nuclear spin properties
- Precession and Larmor Frequency
- RF excitation
- Relaxation
- Contrast mechanism
Magnetic Resonance Imaging

- Provide high resolution anatomic structure (as with X-ray CT)
- Provide high contrast between different soft tissues (X-ray CT cannot)
- No exposure to radiation and hence safe
- More complicated instrumentation
- Takes longer to acquire a scan than CT, more susceptible to patient motion

Figure I.4
X-ray projection

(a)

(b)

MRI

(c)

(d)

Figure V.1

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Basic Principle of MRI

- The hydrogen (1^H) atom inside body possess “spin”
- In the absence of external magnetic field, the spin directions of all atoms are random and cancel each other.
- When placed in an external magnetic field, the spins align with the external field.
- By applying an rotating magnetic field in the direction orthogonal to the static field, the spins can be pulled away from the z-axis with an angle $\alpha$
- The bulk magnetization vector rotates around z at the Larmor frequency (precess)
- The precession relaxes gradually, with the xy-component reduces in time, z-component increases
- The xy component of the magnetization vector produces a voltage signal, which is the NMR signal we measure
What is Spin?

- Spin is a fundamental property of nature like electrical charge or mass. Spin comes in multiples of 1/2 and can be + or -. Protons, electrons, and neutrons possess spin. Individual unpaired electrons, protons, and neutrons each possesses a spin of $\frac{1}{2}$ or $-\frac{1}{2}$.

- Two or more particles with spins having opposite signs can pair up to eliminate the observable manifestations of spin.

- In nuclear magnetic resonance, it is unpaired nuclear spins that are of importance.
Nuclear Spin

- A nucleus consists of protons and neutrons
- When the total number of protons and neutrons (=mass number A) is odd or the total number of protons is odd, a nucleus has an angular momentum ($\phi$) and hence spin
  - Ex. Hydrogen ($^{1}\text{H}$) (1 proton), $^{13}\text{C}$
- The spin of a nucleus generates a magnetic field, which has a magnetic moment ($\mu$)
- The spin causes the nucleus to behave like a tiny magnet with a north and south pole
Angular momentum vs Magnetic Moment

• Microscopic magnetic moment vector:
  \[ \mu = \gamma \Phi \]

• \( \gamma \) is gyromagnetic ratio [radians/s-T]
• \( \gamma \) has more convenient units [Hz/T]
  \[ \gamma = \frac{\gamma}{2\pi} \]

• For \(^1\text{H}\)
  \[ \gamma = 42.58 \text{ MHz/T} \]
Nuclear Spin System

- Collection of identical nuclei in a given sample of material (also known as spin packet, a voxel in the imaged volume)
- In the absence of external magnetic field, the spin orientations of the nuclei are random and cancel each other
- When placed in a magnetic field, the microscopic spins tend to align with the external field, producing a net bulk magnetization aligned with the external field
In the absence of external magnetic field

Hydrogen Nuclei (Protons)

Axis of Angular Momentum (Spin), Magnetic Moment

From Graber, Lecture note for BMI F05
Nuclear Magnetization

- Put sample in external magnetic field
  \[ B_0 = B_0 \hat{z} \]
- Spins align in one of two directions
  - 54° off \( \hat{z} \) “up”
  - 180° - 54° off \( \hat{z} \) “down”
- Slight preference for “up” direction
- Sample becomes magnetized

Magnetization vector:

\[ M = \sum_{n=1}^{N_s} \mu_n \]

\[ \frac{N-}{N+} = e^{-E/kT} \]
Precession

Spins **PRECESS** at a single frequency ($w_0$), but *incoherently* – they are not in phase, so that the sum of x-y components is 0, with net magnetization vector in z direction.

$W_0 = \gamma B_0$: Larmor freq.
Bulk Magnetization at Equilibrium

- Equilibrium value: $M_0$
  - same direction as $B_0$
  - depends on $\mathbf{x} = (x, y, z)$ only

- Magnitude: $M_0$

$$M_0 = \frac{B_0 \gamma^2 \hbar^2}{4kT} P_D$$

- $k$ is Boltzmann’s constant
- $T$ is temperature
- $P_D$ is proton density

Which depends on tissue type
How to make the spins in phase?

Irradiating with a rotating magnetic field $B_1$ of frequency $w_0$, causes spins to precess coherently, or in phase, generating a xy-component.
Process Involved in MRI

- Put patient in a static field $B_0$ (much stronger than the earth’s field)
- (step 1) Wait until the nuclear magnetization reaches an equilibrium (align with $B_0$)
- Applying a rotating magnetic field $B_1$ (much weaker than $B_0$) to bring $M$ to an initial angle $\alpha$ with $B_0$ (rotating freq=Larmor freq.)
- $M(t)$ precess around $B_0$ at Larmor frequency around $B_0$ axis (z direction) with angle $\alpha$
- The component in z increases in time (longitudinal relaxation) with time constant $T_1$
- The component in x-y plane reduces in time (transverse relaxation) with time constant $T_2$
- Measure the transverse component at a certain time after the excitation (NMR signal)
- Go back to step 1
- By using different excitation pulse sequences, the signal amplitude can reflect mainly the proton density, $T_1$ or $T_2$ at a given voxel
Evolution of magnetization when a Time varying magnetic field is applied

- $M = M(x, t)$
- Relation to bulk angular momentum $J$
  \[ M = \gamma J \]
- Focus on small sample $\rightarrow$ voxel
  - $M = M(t)$
  - Equations of motion = Bloch equations
\[
torque = \mathbf{M} \times \mathbf{B}
\]

- Torque is related to angular momentum

\[
\tau = \frac{d\mathbf{J}}{dt}
\]

- Eliminate \( \mathbf{J} \) to yield

\[
\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t)
\]

- Valid for "short" times

Using the right hand rule, \( \mathbf{M} \) will rotate around \( z \) if \( \mathbf{M} \) is not aligned with \( z \).
Cross Product: Review

\[
\mathbf{M} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{vmatrix} = (M_y B_z - M_z B_y) \mathbf{i} + (M_z B_x - M_x B_z) \mathbf{j} + (M_x B_y - M_y B_x) \mathbf{k}
\]

Direction of MxB follows “right hand” rule
Solution under a Static Field with an Initial Angle

- $B(t)=[0,0,B_0]$

- $\mathbf{M} \times \mathbf{B} = M_y B_0 \mathbf{i} - M_x B_0 \mathbf{j} + 0 \mathbf{k}$

- $\frac{dM_x}{dt} = M_y B_0$
- $\frac{dM_y}{dt} = -M_x B_0$

- Solving above yields solution in the next slide
Precession Due to a Static Field with an Initial Angle

- Let $B(t) = B_0$; $M(0)$ angle $\alpha$ with $\hat{z}$
- Then
  
  $M_x(t) = M_0 \sin \alpha \cos (-\gamma B_0 t + \phi)$
  $M_y(t) = M_0 \sin \alpha \sin (-\gamma B_0 t + \phi)$
  $M_z(t) = M_0 \cos \alpha$

  where
  
  $M_0 = |M(0)|$  \hspace{0.5cm} $\phi$ arbitrary

- Precession with Larmor frequency
  
  $\omega_0 = \gamma B_0$ \hspace{0.5cm} or \hspace{0.5cm} $\nu_0 = \gamma B_0$

This is the frequency of the photon which would cause a transition between the two energy levels of the spin.

$B_0=1.5\,\text{T}$, $\gamma=42.58\,\text{MHz/T}$, $\nu_0=63.9\,\text{MHz}$
Longitudinal and Transverse Components

- Magnetization
  \[ \mathbf{M}(t) = (M_x(t), M_y(t), M_z(t)) \]

- Think of \( \mathbf{M}(t) \) with two components
  - Longitudinal magnetization
    \[ M_z(t) \]
  - Transverse magnetization
    \[ M_{xy}(t) = M_x(t) + jM_y(t) \]

Rapidly rotating

No change
Laboratory Frame vs. Rotating Frame

Coordinate system rotated about z axis at the Larmor freq.

The rotating M(t) vector appear stationary in the rotating frame.
• See animation at
• http://www.cis.rit.edu/htbooks/mri/chap-3/c13-1.htm
NMR Signal

- The rapidly rotating transverse magnetization \((M_{xy})\) creates a radio frequency excitation within the sample.
- If we put a coil of wire outside the sample, the RF excitation will induce a voltage signal.
- In MRI, we measure this voltage signal.
- Voltage produced is (Faraday’s Law of Induction)

\[ V(t) = -\frac{\partial}{\partial t} \int_{\text{object}} M(r, t) \cdot \mathbf{B}^r(r) \, dr \]

- \(\mathbf{B}^r(r)\) is field produced at \(r\) by unit direct current in coil around sample.
Simplification

- $B^r(r) = B^r$

- Longitudinal magnetization changes too slow
- **Transverse magnetization** dominates

\[ M_{xy}(t) = M_0 \sin \alpha e^{-j(\omega_0 t - \phi)} \]

- Final expression

\[ V(t) = -\omega_0 V_s M_0 \sin \alpha B^r \sin(-\omega_0 t + \phi - \theta_r) \]

\[ |V| = \omega_0 V_s M_0 \sin \alpha B^r \]

Recall \( \omega_0 = \gamma B_0 \), \( M_0 = \frac{B_0 \gamma^2 h^2}{4kT} P_D \)

Therefore \( |V| \propto B_0^2, P_D \)
How do we tilt M to an initial angle?

- Applying a circularly polarized (rotating) magnetic field $B_1(t)$ in the x-y plane with the same Larmor frequency forces the magnetization vector to tilt down to the x-y plane.

  $B_1(t) = B_{1e}(t)e^{-j(\omega_0 t - \varphi)}$

  - $B_{1e}(t)$ has two orthogonal components, in x and y directions respectively, and is produced by using quadrature RF coil.
  - Simplest envelop $B_{1e}$ is a rectangular pulse.

- Motion of $M(t)$ is spiral.
Animation of spiral motion

- Laboratory frame:  

- Rotating frame:  
Circularly Polarized Magnetic Field

two more magnets, whose fields are orthogonal to $B_0$, that rotate, in opposite directions, at the Larmor frequency.
Tip Angle

- If $M$ is parallel to $z$-axis before the RF excitation pulse, the tip angle after the excitation (with duration $\tau_p$) is

$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t) dt$$

- If $B_1^e(t)$ is rectangular

$$\alpha = \gamma B_1 \tau_p$$

- Pulse that leads to $\alpha = \pi/2$ is called “$\pi$ over 2 pulse”, which elicits the largest transverse component $M_{xy}$, and hence largest NMR signal

- Pulse that leads to $\alpha = \pi$ is called “$\pi$ pulse” or inverse pulse, which is used to induce spin echo (later)

- The excitation pulse (envelope of $B_1(t)$) is also called “an alpha pulse”
Relaxation

- Magnetization cannot precess forever
- Two independent relaxation processes
  - Transverse relaxation
    - ≡ spin-spin relaxation
  - Longitudinal relaxation
    - ≡ spin-lattice relaxation
- Detailed properties differ in tissues
  - Gives rise to tissue contrast
Longitudinal Relaxation

- The magnetization vectors tend to return to equilibrium state (parallel to B_0)

\[ M_z(t) = M_0(1 - e^{-t/T_1}) + M_z(0^+)e^{-t/T_1} \]

- \( M_z(0^+) \) is value after RF excitation pulse
- \( M_0 \) is final (equilibrium) value

\[ = M_0 \cos\alpha \]
\[ = 0 \text{ for } \pi/2 \text{ pulse} \]
In the laboratory frame, $\mathbf{M}$ takes a spiralling path back to its equilibrium orientation. But here in the rotating frame, it simply rotates in the $y'-z'$ plane.

The $z$ component of $\mathbf{M}$, $M_z$, grows back into its equilibrium value, exponentially:

$$M_z = |\mathbf{M}|(1 - e^{-t/T_1})$$
Transverse Relaxation

• The strength of the magnetic field in the immediate environment of a $^1$H nucleus is not homogeneous due to presence of other nucleus (and their interactions)

• Hence the Larmor frequencies of nearby nuclides are slightly different (some spins faster, some slower)
  – Spin-spin interactions

• This causes dephasing of the xy components of the magnetization vector, leading to an exponential decay of $M_{xy}$
• See animation at

  • http://www.cis.rit.edu/htbooks/mri/inside.htm
    – Under T2 processes

• Overall effect of both transverse and longitudinal relaxation:

• $T_2$ is called transverse relaxation time, which is the time for $M_{xy}$ to decrease by $1/e$.
• Also called spin-spin relaxation time
• $T_2$ is much smaller than $T_1$
  – For tissue in body, $T_2$: 25-250ms, $T_1$: 250-2500 ms
Free Induction Decay

- The voltage signal (NMR signal) produced by decaying $M_{xy}$ also decays.

- This is called free induction decay (FID), and is the signal we measure in MRI.
T2 Star Decay

- Received signal actually decays faster than $T_2$ (having a shorter relaxation time $T_2^*$).
- Caused by fixed spatial variation of the static field $B_0$ due to imperfection of the magnet.
  - Accelerates the dephasing of magnetization vectors.
  - Note that $T2$ is caused by spatial variation of the static field due to interactions of nearby spins.
- The initial decay rate is governed by $T_2^*$, but the later decay by $T_2$. 

![Diagram](image)
Formation of Spin Echo

- By applying a 180 degree pulse, the dephased spins can recover their coherence, and form an echo signal
RF Pulse Sequence and Corresponding NMR Signal
Spin echo sequence

- Multiple $\pi$ pulses create “Carr-Purcell-Meiboom-Gill (CPMG)” sequence
- Echo Magnitude Decays with time constant T2
Bloch Equations

- Equation(s) of “motion” for $M(t)$
  \[
  \frac{dM(t)}{dt} = \gamma M(t) \times B(t) - R\{M(t) - M_0\}
  \]
- Includes RF excitation
  \[
  B(t) = B_0 + B_1(t),
  \]
- Includes relaxation
  \[
  R = \begin{pmatrix}
  1/T_2 & 0 & 0 \\
  0 & 1/T_2 & 0 \\
  0 & 0 & 1/T_1
  \end{pmatrix}
  \]
• Solving the previous equation in x, y, z direction will yield the equations representing the transverse and longitudinal relaxations, shown previously
Source of MR Contrast

- Different tissues vary in T1, T2 and PD (proton density)
- The pulse sequence parameters can be designed so that the captured signal magnitude is mainly influenced by one of these parameters
- Pulse sequence parameters
  - Tip angle $\alpha$
  - Echo time $T_E$
  - Pulse repetition time $T_R$
Typical Brain Tissue Parameters

- Table 12.2 in [Prince]

<table>
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<th>P_D</th>
<th>T_2 (ms)</th>
<th>T_1 (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White matter</td>
<td>0.61</td>
<td>67</td>
<td>510</td>
</tr>
<tr>
<td>Gray matter</td>
<td>0.69</td>
<td>77</td>
<td>760</td>
</tr>
<tr>
<td>CSF</td>
<td>1.00</td>
<td>280</td>
<td>2650</td>
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</tr>
</tbody>
</table>

(a) PD weighted

(b) T2-weighted

(c) T1-weighted

White matter

CSF

Gray matter
T1-weighting

- **Short TR:**
  - Maximizes T1 contrast due to different degrees of saturation
  - If TR too long, tissues with different T1 all return equilibrium already

- **Short TE:**
  - Minimizes T2 influence, maximizes signal
Spin density weighting

- Signal at equilibrium proportional to PD
- Long TR:
  - Minimizes effects of different degrees of saturation (T1 contrast)
  - Maximizes signal (all return to equilibrium)
- Short TE:
  - Minimizes T2 contrast
  - Maximizes signal

\[ M_0 = \frac{B_0 \gamma^2 \hbar^2}{4kT} P_D \]
T2 weighting

- Long TR:
  - Minimizes influence of different T1
- Long TE:
  - Maximizes T2 contrast
  - Relatively poor SNR
Summary: Process Involved in MRI

- Put patient in a static field $B_0$ in z-direction
- (step 1) Wait until the bulk magnetization reaches an equilibrium (align with $B_0$)
- Apply a rotating field (alpha pulse) in the xy plane to bring $M$ to an initial angle $\alpha$ with $B_0$. Typically $\alpha=\pi/2$
- $M(t)$ precesses around $B_0$ (z direction) at Larmor freq. with angle $\alpha$
- The component in $z$ increases in time (longitudinal relaxation) with time constant $T_1$
- The component in x-y plane reduces in time (transverse relaxation) with time constant $T_2$
- Apply $\pi$ pulse to induce echo to bring transverse components in phase to increase signal strength
- Measure the transverse component at different times (NMR signal), to deduce $T_1$ or $T_2$
- Go back to step 1
- By using different excitation pulse sequences (differing in TE, TR, $\alpha$), the signal amplitude can reflect mainly the proton density, $T_1$ or $T_2$ at a given voxel
Summary

- What is nuclear spin? What type of nucleus can have spin?
- What is the bulk magnetization vector in the absence of external magnetic field?
- What is the bulk magnetization vector in the presence of an external static magnetic field?
- What is precession? Under what condition will precession occur?
  - Static field, initial angle
  - Larmor frequency = $\gamma B_0$
- What is the function of the rotating field ($\alpha$ pulse)
  - Tilt the magnetization vector to an angle
- What happens after?
  - Longitudinal and transversal relaxation
  - Gradually return to the equilibrium state
- Tissues differ in T1, T2 and PD
  - Using different TR, TE, so that the signal magnitude is mainly influenced by one of the parameters, T1, T2 or PD
Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap. 12
- A. Webb, Introduction to Biomedical Imaging, Chap. 4
- **The Basics of MRI**, A web book by Joseph P. Horn (containing useful animation):
Homework

• Reading:
  – Prince and Links, Medical Imaging Signals and Systems, Chap. 12
  – Note down all the corrections for Ch. 10,11 on your copy of the textbook based on the provided errata (see Course website or book website for update).

• Problems (Due 12/4):
  – P12.1
  – P12.2
  – P12.4
  – P12.5
  – P12.7
  – P12.10
  – P12.11
  – P12.12