EL582/BE620 --- Medical Imaging - I

Physics of Radiography

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Based on J. L. Prince and J. M. Links, Medical Imaging Signals and Systems, and lecture notes by Prince. Figures are from the textbook.

Lecture Outline

- Atomic structure and ionization
- Particulate Radiation
  - Focusing on energetic electron interaction
- EM Radiation
  - Photoelectric
  - Compton scattering
  - Likelihood of each
  - EM radiation measurement
  - Attenuation of radiation
- Radiation Dosimetry
  - Exposure, dose

Atomic Structure

- An atom={a nucleus, electrons}
- nucleons = {protons*;* neutrons}
- mass number *A* = # nucleons
- atomic number *Z* = # protons = # electrons
  - Define an element with a particular symbol: H, C, etc.
  - An element is denoted by its A and Z
  - Ex: $^{12}_{6}C$ or $C\text{-}12$

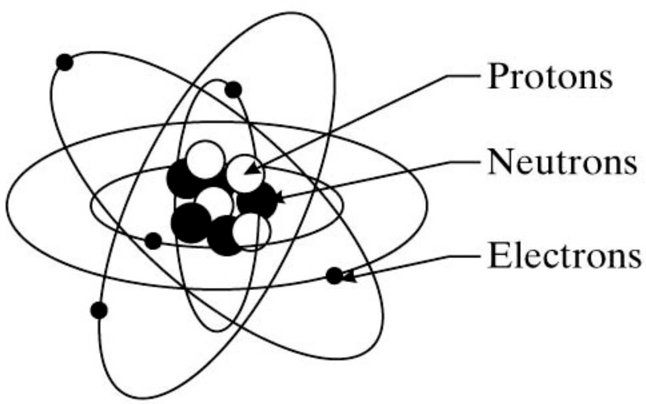


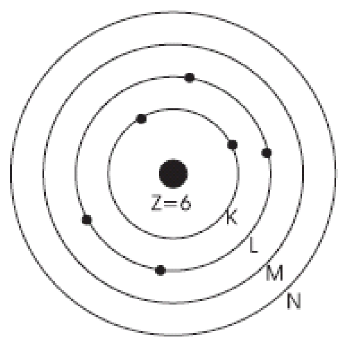
Figure 4.1

Stable vs. Unstable States

- Stable nuclides:
  - # neutrons ~= # protons (A ~= 2Z)
- Unstable nuclides (radionuclides, radioactive atoms)
  - Likely to undergo radioactive decay, which gives off energy and results in a more stable nucleus

Orbits of Electrons

Shell Number n	Shell Label	# Electrons $2n^2$
1	K	≤ 2
2	L	≤ 8
3	M	≤ 18
4	N	≤ 32



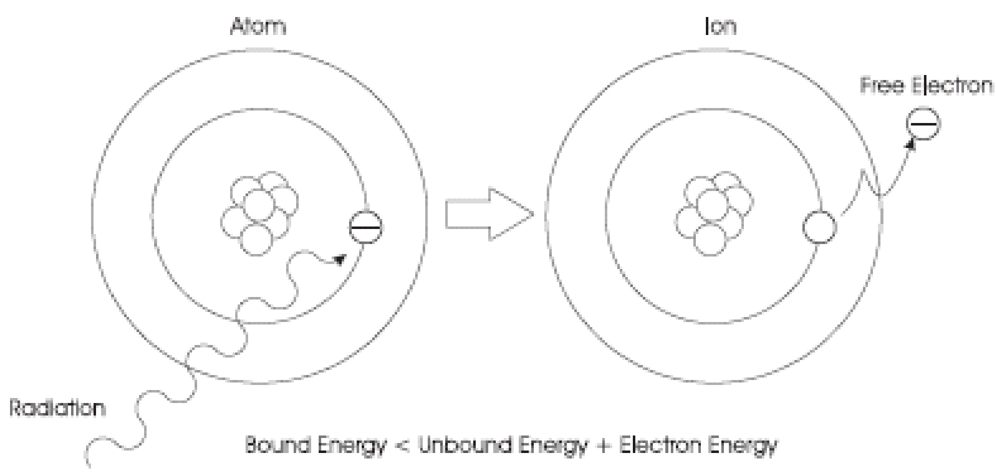
Ground state: electrons are in the lowest orbital shells and within the lowest energy quantum states within each shell

Electron Binding Energy

- A free electron has higher energy than when it is bounded to an nuclei in an atom
- Binding energy = total energy with free electrons – total energy in ground state
  - Depends on the element to which the electron is bound and the shell within which it resides in ground state
  - Sufficient to consider “average” binding energy of a given atom
- One electron volt (eV) = kinetic energy gained by an electron when accelerated across one volt potential
  - 1 eV = 1.6×10^{-19} Joule
- Binding energies of typical elements:
  - hydrogen = 13.6 eV, Smallest among all lighter atoms
  - Air: 29 eV
  - Lead: 1 KeV
  - Tungsten: 4 KeV

Ionization and Excitation

- Ionization is “knocking” an electron out of an atom
  - Creates a free electron + ion (an atom with +1 charge)
  - Occurs when radiated with energy above the electron binding energy
- Excitation is “knocking” an electron to a higher orbit
  - When the radiation energy is lower than the binding energy
- After either ionization or excitation, an atom has higher energy

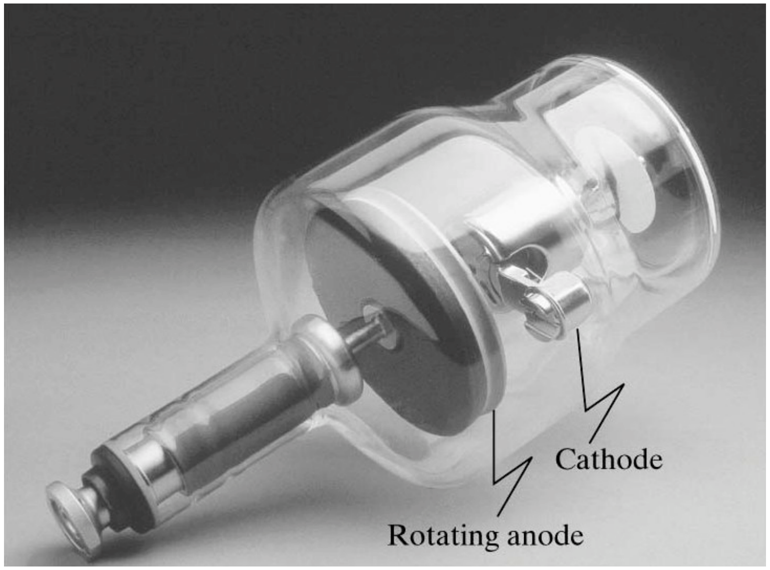


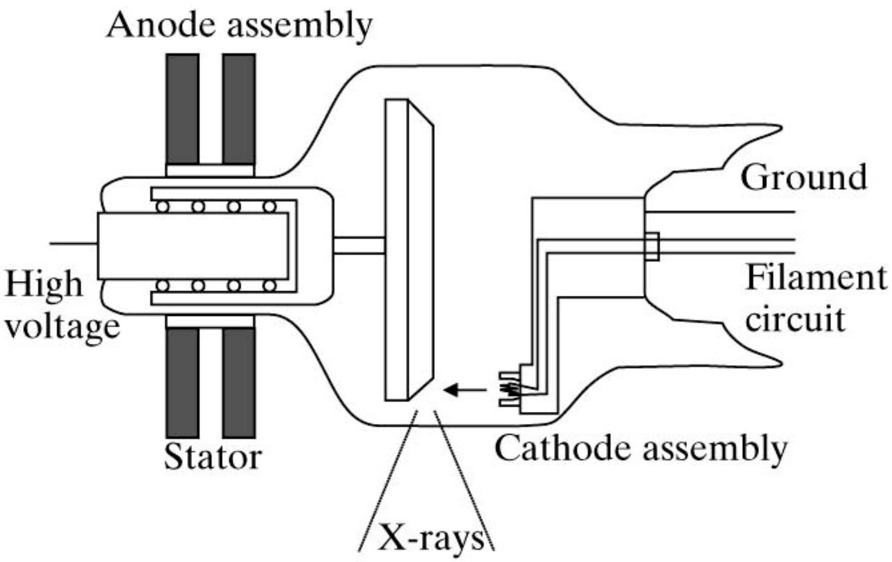
Characteristic Radiation

- What happens to ionized or excited atom?
  – Return to ground state by rearrangement of electrons
  – Causes atom to give off energy
  – Energy given off as characteristic radiation
    - infrared
    - light
    - x-rays

Example

- Consider an electron accelerated through an X-ray tube where the anode if made of tungsten. If the anode is held at 120 KV, what is the maximum number of tungsten atoms that can be ionized?
- Solution:
  - The electron will have 120 KeV kinetic energy when reaching the anode, by definition of eV
  - The average binding energy of tungsten = 4 KeV
  - # ionized atoms = 120/4=20





Ionizing Radiation

- Radiation with energy > 13.6 eV is ionizing
- Energy required to ionize:
  - air ≈ 34 eV
  - lead ≈ 1 keV
  - tungsten ≈ 4 keV

  These are average binding energies.
- Radiation energies in medical imaging

  30 keV–511 keV

  can ionize 10–40,000 atoms

Two Types of Ionizing Radiation

- Particulate
- Electro-magnetic (EM)

Particulate Radiation

- Radiation by any particle (proton, neutron or electron) if it possesses enough kinetic energy to ionize an atom

$$\text{Kinetic Energy} = \text{the energy gained due to motion}$$

$$\text{Mass of a moving particle}: \; m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

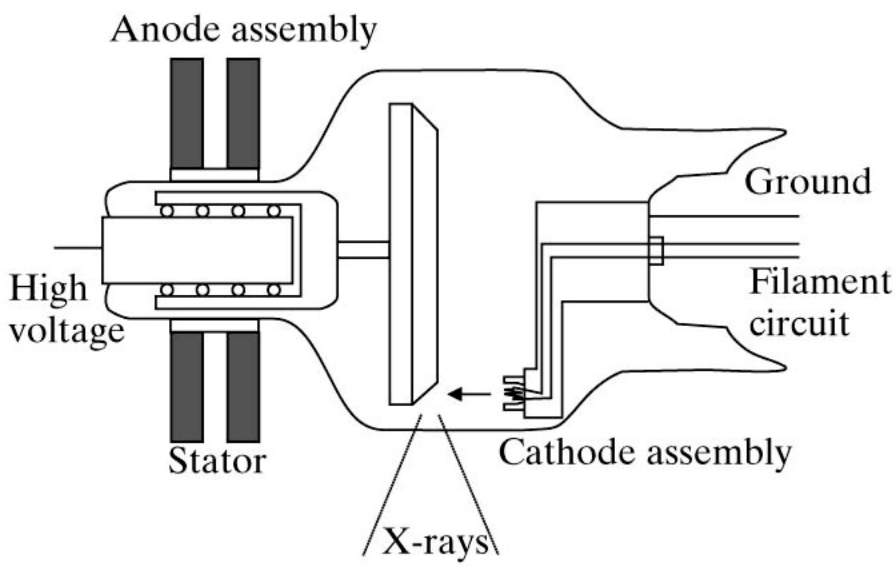
$$\text{Energy vs. mass}: E = mc^2$$

$$\text{Kinetic Energy}: \quad KE = E - E_0 = (m - m_0)c^2$$

$$\text{When } v << c, KE = \frac{1}{2}mv^2$$

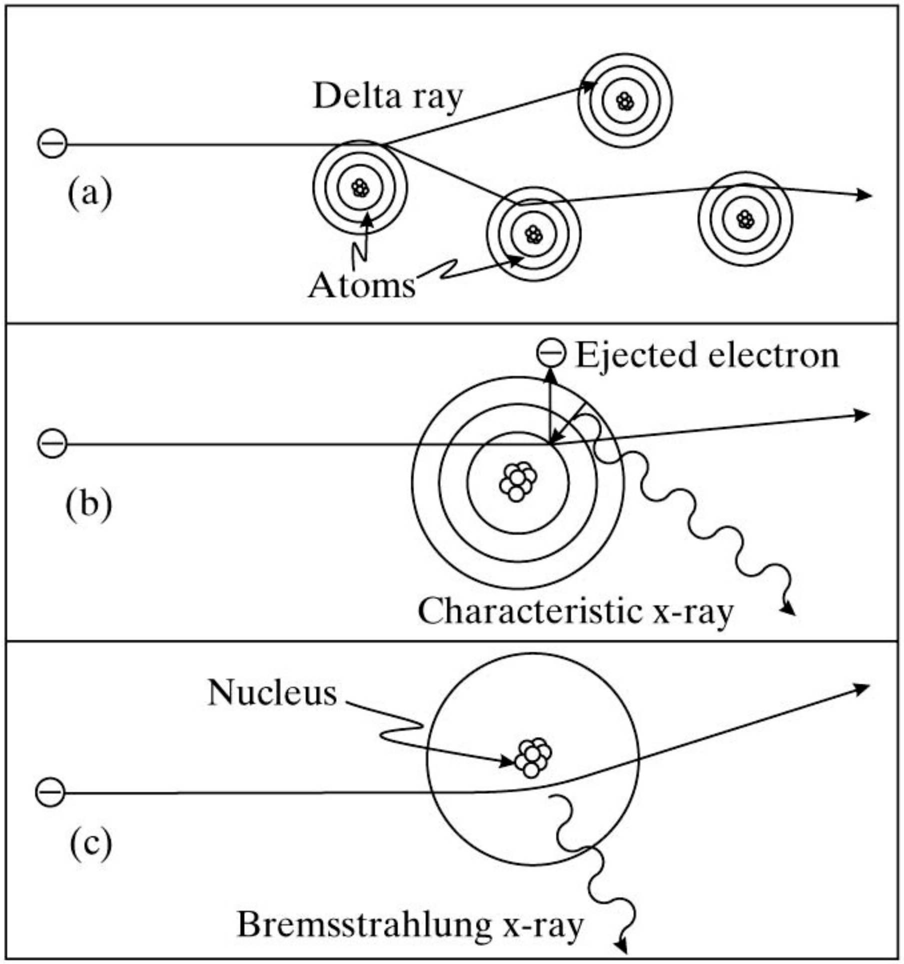
Particulate Radiation by Energetic Electrons

- We are only concerned with the electron accelerated in a X-ray tube here
  - An electron accelerated across a tube with 100 KV potential possesses 100 KeV kinetic energy



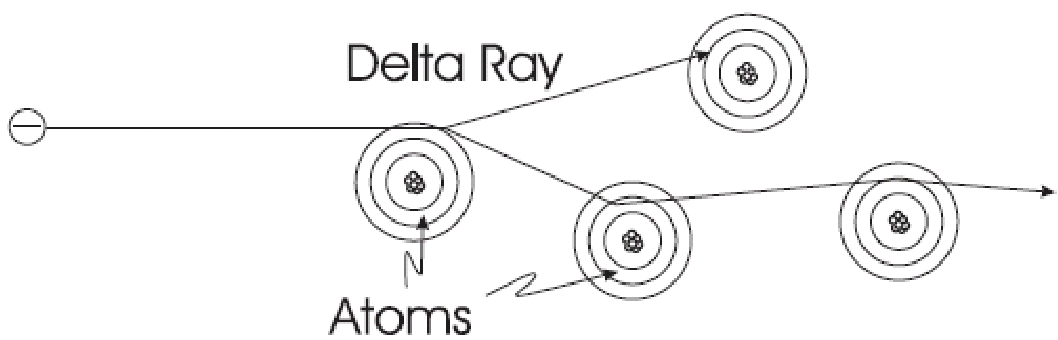
Energetic Electron Interactions

- Two primary interactions
  - Collisional transfer
    - Most common
    - Produces heat
  - Radiative transfer
    - Produces x-ray
    - Characteristic radiation
      - Collide with K-shell
    - Bremsstrahlung radiation
      - Collide with nucleus
      - More common than characteristic radiation



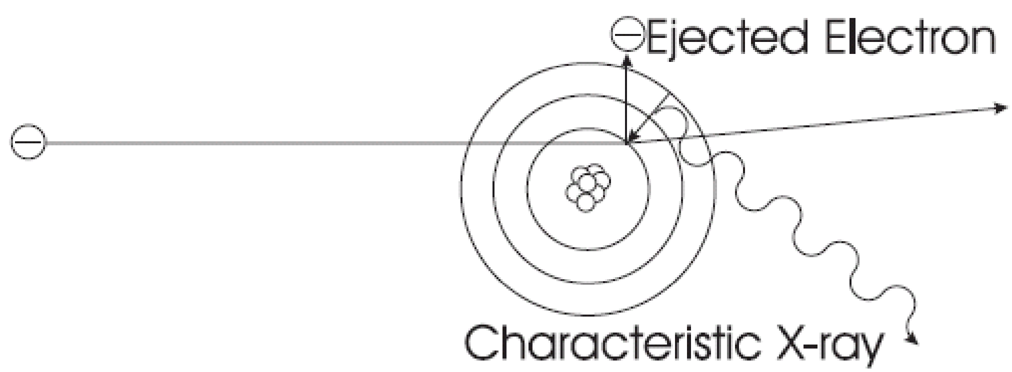
Collisional Transfer

- The energetic electron collides with an atom in the target
- Typically, a small fraction of the kinetic energy of the electron is transferred to another electron in the atom
  - As the affected atom returns to its original state, infrared radiation (heat) is generated
- Occasionally, a large fraction of the incident energy is transferred to another electron, the newly freed electron may form a delta ray
- The incident electron's path may be redirected, and many other subsequent interactions may occur, until the kinetic energy of the incident electron is exhausted



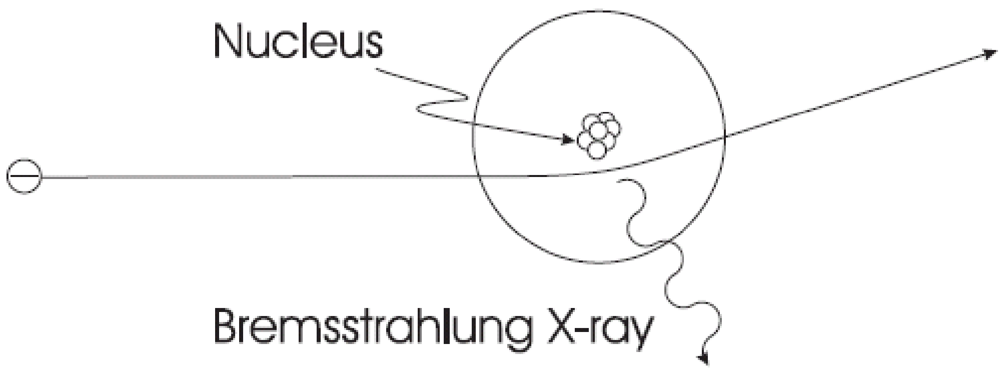
Characteristic X-Ray

- The incident electron collides with a K-shell electron, exciting or ionizing the atom, leaving a hole in that shell.
  - As the atom returns to its ground state, the k-shell hole is filled by a higher shell electron
  - The loss of energy creates an EM photon, known as Characteristic x-ray
  - The energy of the x-ray photon = difference between the binding energy of the two shells (element dependent)

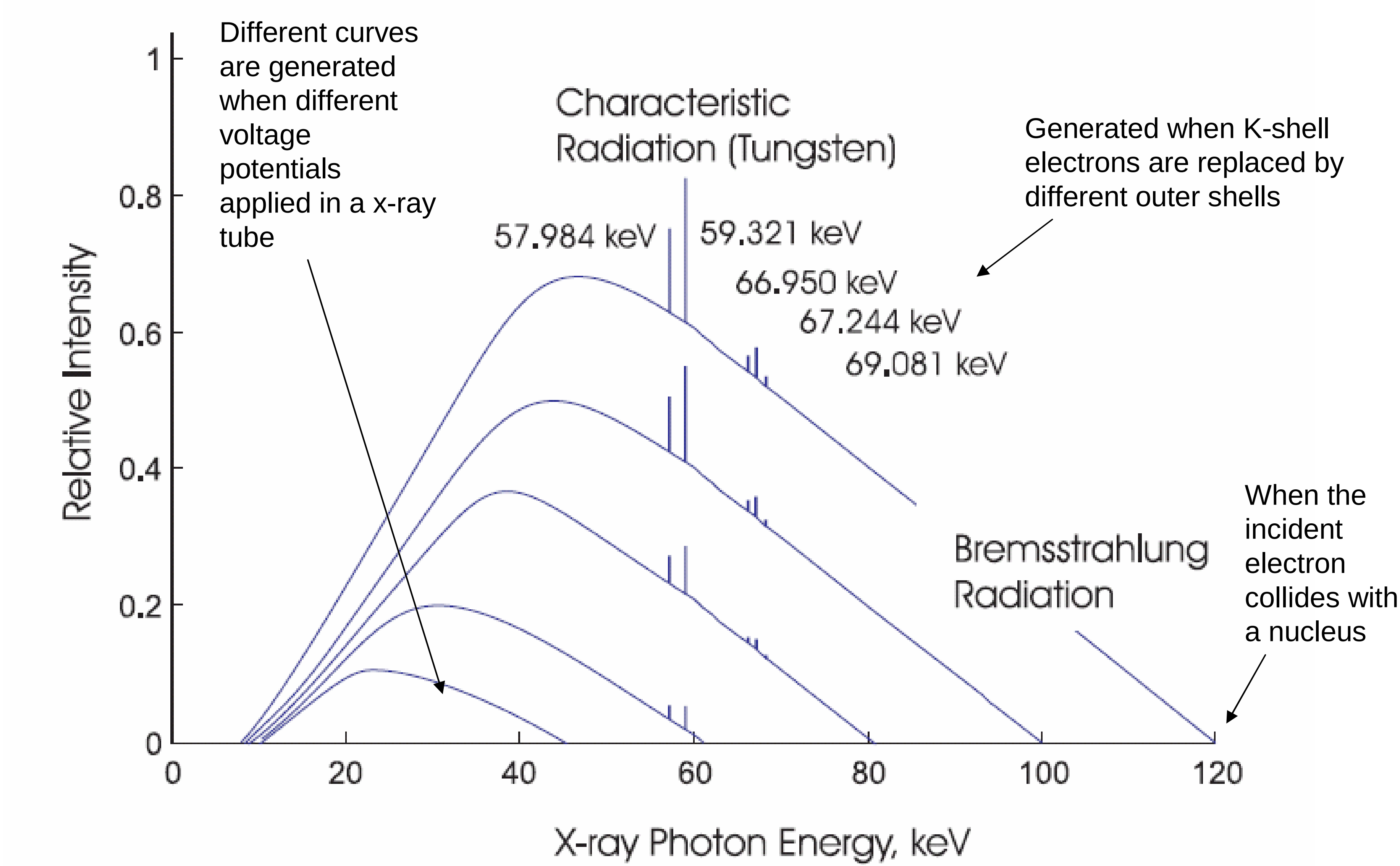


Bremsstrahlung Ray

- As the incident electron approaches the nucleus of an atom, the positive charge of the nucleus causes the incident electron to bend around the nucleus and decelerates
  - The loss of energy leads to the Bremsstrahlung x-ray (energy vary over a continuous range, depending on the speed loss)
- Occasionally when the incident electron collides with the nucleus, the electron is annihilated, emitting a photon with an energy equal to the kinetic energy of the incident electron (highest possible energy)
- Primary source of x-rays from an x-ray tube



Spectrum of X-Ray



EM Radiation

- EM radiation comprises an electric wave and a magnetic wave traveling at right angles to each other
- Typical EM waves:
  - Non ionizing: radio, microwaves, infrared, visible light, ultraviolet
  - Ionizing: X-rays, gamma rays
- Energy of a photon of an EM wave with frequency *v:*

$$E = h\nu$$

$$\text{Planck's constant } h = 4.14 \times 10^{-15} \text{ eV-sec}$$

EM Waves for Medical Imaging

- X-rays and Gamma rays:
  - Have energy in the KeVs to MeVs -> Ionizing Radiation
  - used in X-ray/CT and nuclear medicine respectively
  - X-rays are created in the electron cloud of atoms due to ionizing radiation
  - Gamma rays are created in the nuclei of atoms due to radioactive decay or characteristic radiation
- Radio waves
  - Used to stimulate nuclei in MRI to generate EM radiation
- Visible light
  - Used in radiography to improve the efficiency of photographic film to detect X-rays
- See Table 4.2 for more details

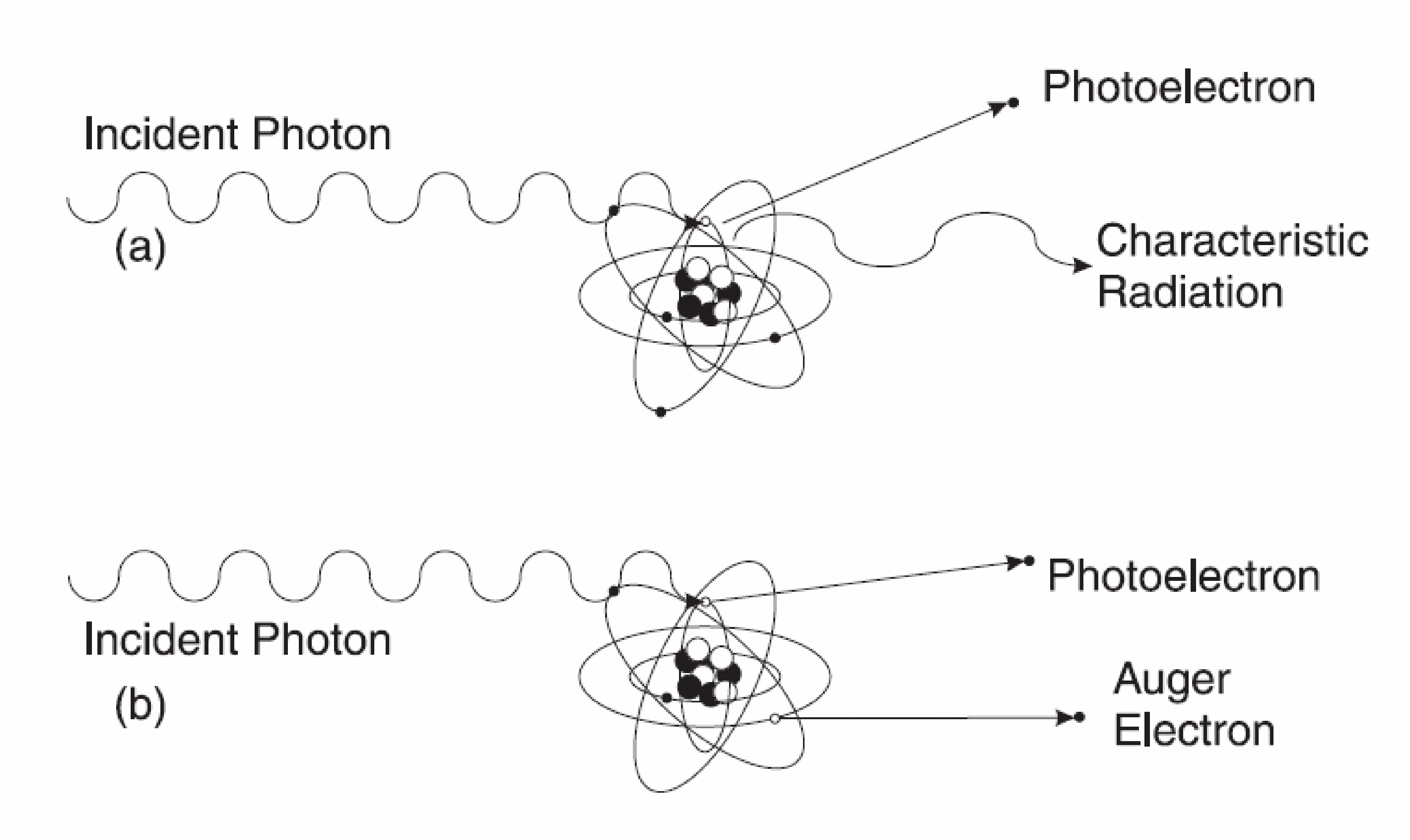
EM Radiation Interactions

- Two main interactions
  - Photoelectric effect
    - The incoming photon is completely absorbed and ejecting K-shell or L-shell electrons, producing characteristic x-ray
  - Compton scattering
    - The incoming photon changes its direction

Photoelectric Effect

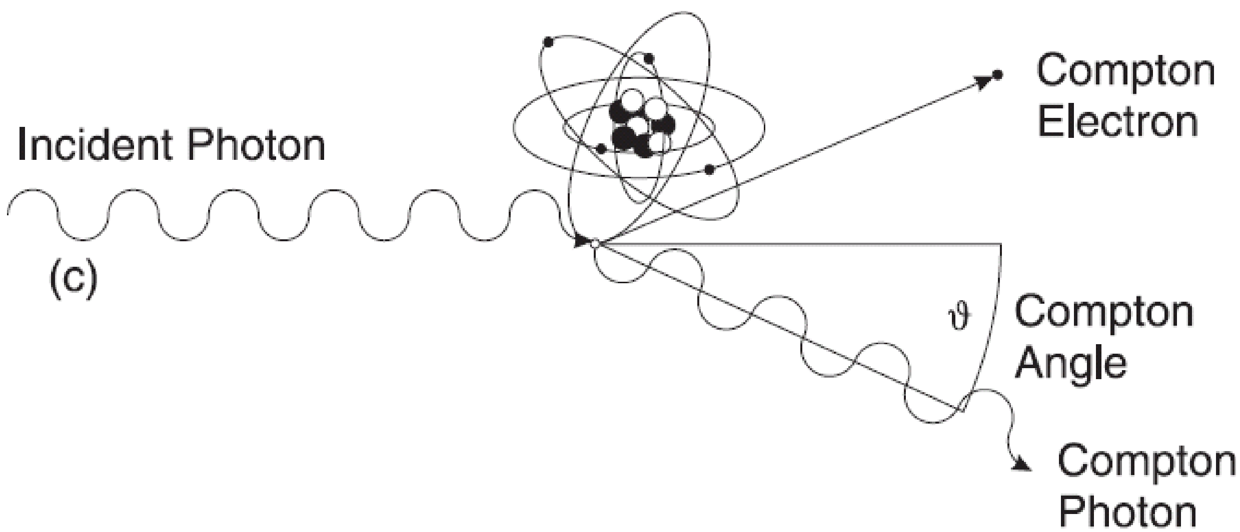
- An incoming photon interacts with the nucleus of an atom, causing ejection of a K-shell or L-shell electron (photoelectron)
  - Atom completely absorbs incident photon and all energy is transferred
  - The photoelectron propagates away with energy $E_{e^-} = h\nu - E_B$
  - The affected atom produces characteristic x-ray, while outer electrons fill the K-shell.
  - Sometimes the characteristic x-ray transfers its energy to an outer electron (called Auger electron)
- Both photo electron and Auger electron are energetic electrons that can interact with the matter as discussed before

Photoelectric Effect



Compton Scattering

- An incoming photon ejects an outer shell electron, yielding a Compton electron
- The incident photon loses its energy and changes its direction
- The scattered photon is called Compton photon



- The energy of the scattered photon depends on the scatter angle

$$E' = \frac{E}{1 + E(1 - \cos\theta)/(m_0c^2)}$$

- m_0 is rest mass of electron
- $m_0c^2 = 511$ keV

  - The maximum energy loss occurs when the photon is deflected backward (\theta=180^o)
  - When E is higher, more photons scatter forward
  - The kinetic energy of the Compton photon = E-E'

Which interaction is better?

- Photoelectric effect helps to differentiate different human tissues/organs
- Compton scattering causes incident photons to deviate from straight path, and causes unnecessary exposure of x-ray to untargeted areas
- In medical imaging, we want to increase the likelihood of photoelectric events, while minimizing Compton scattering

Probability of Photoelectric Effect

- Recall that photoelectric event happens when incident photons interact with the coulomb field of the nucleus of an atom
- More likely when colliding with an atom with more photons (higher Z number)
- Less likely when incident photons have higher energy (higher frequency)

$$\text{Prob[photoelectric event]} \propto \frac{Z_{\text{eff}}^4}{(h\nu)^3}$$

  - The probability increases abruptly when the photon energy rises above the binding energy of L-shell or K-shell electrons (so as to eject the electrons), then begins to diminish
  - Rationale behind the use of “contrast agent”

Probability of Compton Scattering

- Recall that Compton scattering occurs when an incident photon collides with outer shell electrons
- Likelihood proportional to the number of electrons per kilogram of the material (the electron density or ED)
- Relatively independent of incident photon energy in the biological material

$$\text{Prob}[\text{Compton event}] \propto \text{ED}$$

$$ED = \frac{N_A Z}{W_m}$$

N_A : Avogadro's number (atoms/mole)

Z : atomic number (electrons/atom)

W_m : molecular weight (grams/mole

- ED is approximately constant for various biological material, ~ 3E26, except for Hydrogen (6E26)

Relative Likelihood

- Compton scattering is equally likely in various materials and invariant of incident energy
- Photoelectric effect is more likely in high Z material and less likely with high incident energy
- Overall, Compton scattering is more dominant with higher incident energy in the same material
- But the percent of energy deposited due to photoelectric event is larger because all incident energy is absorbed.

Measures of X-ray Beam: Photon Count

- Photon fluence:

$$\Phi = \frac{N}{A}$$

- Photon fluence rate:

$$\phi = \frac{N}{A\Delta t}$$

Measures of X-ray Beam: Energy Flow

- Energy fluence:

$$\Psi = \frac{N h\nu}{A}$$

- Energy fluence rate:

$$\psi = \frac{N h\nu}{A\Delta t}$$

- Intensity: $(= \psi)$

$$I(E) = \frac{NE}{A\Delta t}$$

Spectrum of X-Ray

- The x-ray beam produced by an x-ray tube (mainly Bremsstrahlung) is polyenergetic (consisting photons with different energies or frequencies)
- X-ray spectrum S(E):
  - The number of photons with energy E per unit area per unit time

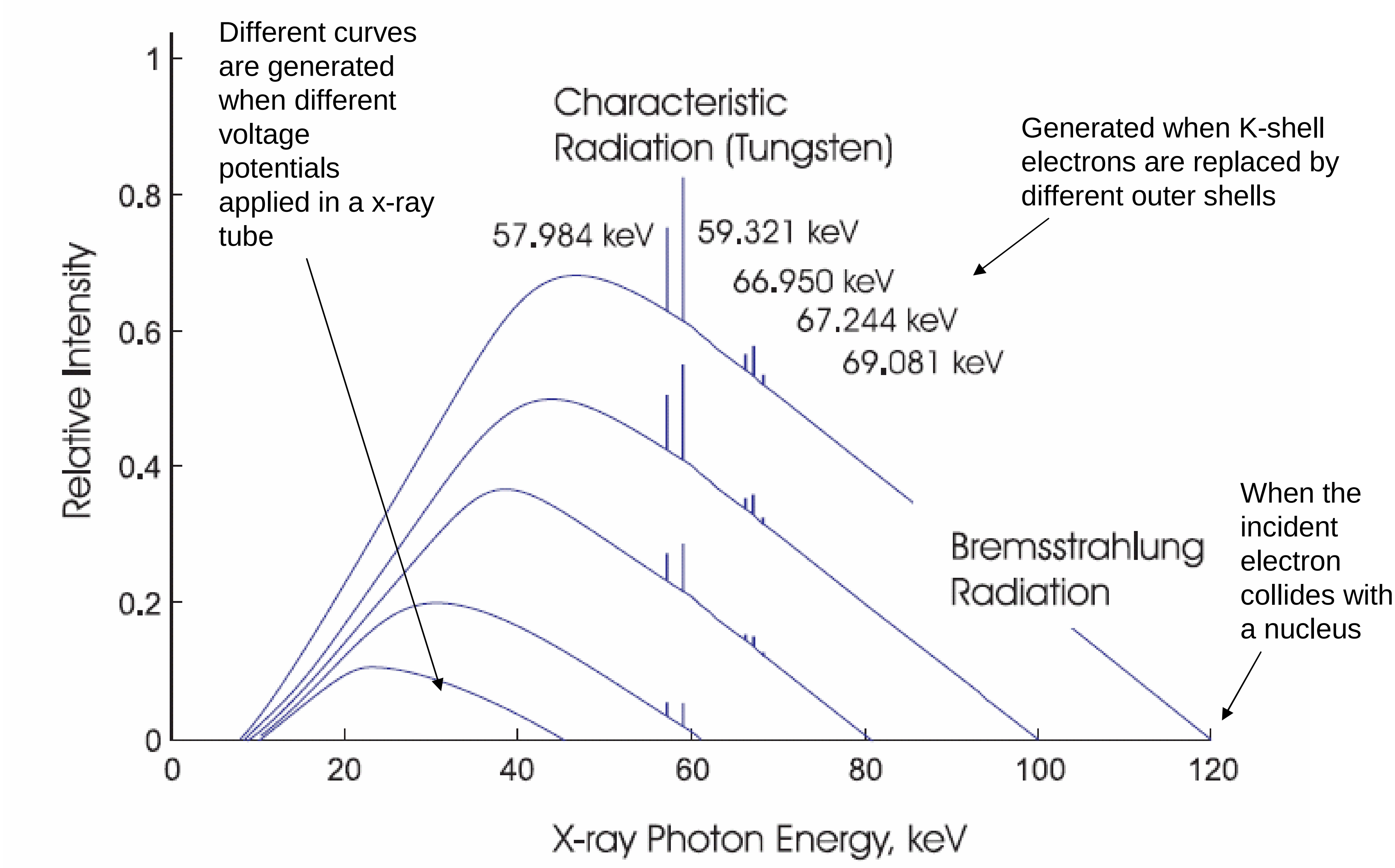
- Photon fluence rate from spectrum:

$$\phi = \int_0^\infty S(E')\, dE'$$

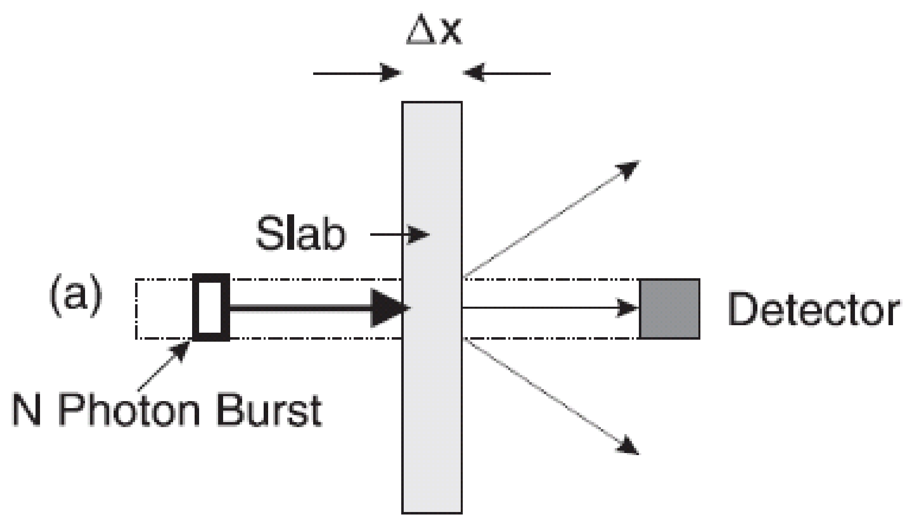
- Intensity from spectrum:

$$I = \int_0^\infty E' S(E')\, dE'$$

Spectrum of X-Ray



Attenuation of X-ray Radiation: Narrow Beam, Monoenergetic Photons



Photons will be absorbed/deflected through the slab

photons lost through the slab (n) ~ N Δx

linear attenuation coefficient: $\mu = n/N / \Delta x$

μ is the fraction of photons that are lost per unit length

# of photons at x = N'(x)

$N'(x) - N = -n = -\mu N \Delta x$

$dN/N = -\mu\, dx$

$N'(x) = N \exp\{-\mu \Delta x\}$ ← The fundamental photon attenuation law

Linear Attenuation Coefficients of Biological Tissues

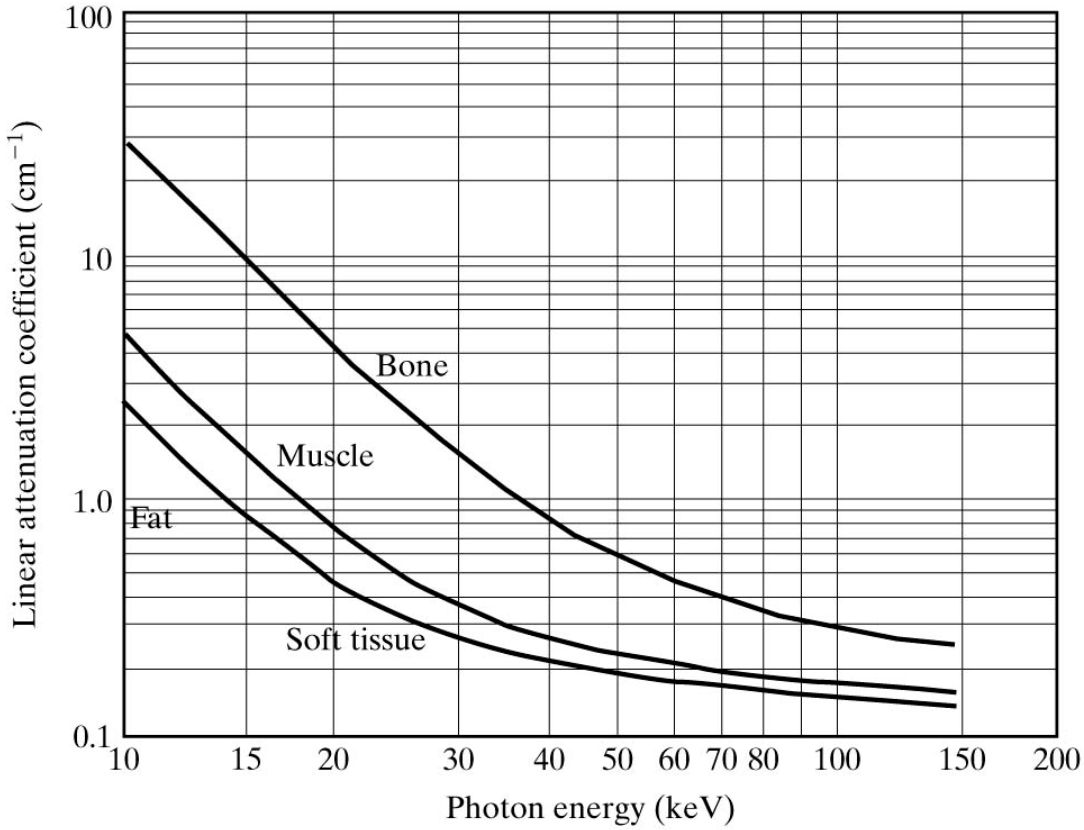


Figure 4.8

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.

Homogeneous Slab

- Homogeneous slab: the attenuation rate is the same over the entire slab

  - Homogeneous slab thickness Δx
  - Fundamental photon attenuation law

$$N = N_0 e^{-\mu \Delta x}$$

  - μ is linear attenuation coefficient
  - In terms of intensity:

$$I = I_0 e^{-\mu \Delta x}$$

This is known as Beer's Law

Half-Value Layer (HVL)

- Homogeneous slab (shielding)
- HVL = thickness that will stop half the photons

$$\frac{1}{2} = \exp\{-\mu \text{ HVL}\}$$

- Relation to μ

$$\text{HVL} = \frac{0.693}{\mu}$$

Example

- Consider the image taken of a bar phantom uniformly irradiated by monoenergetic x-ray photons
  - Assuming the bars are made of material that has a HVL of 0.2cm
  - Assuming x-ray photons pass through the space between bars w/o attenuation
  - Assuming the intensity of the image is proportional to the number of detected photons in a unit area
  - What is the contrast of the resulting image?

- Go through in the class

Non-Homogeneous Slab

- The attenuation coefficient depends on x

  - Non-homogeneous slab:

$$\frac{dN}{N} = -\mu(x)dx$$

  - Integration yields:

$$N(x) = N_0 \exp\{-\int_0^x \mu(x')dx'\}$$

  - For intensity:

$$I(x) = I_0 \exp\{-\int_0^x \mu(x')dx'\}$$

Polyenergetic Photons

- The linear attenuation coefficient depends on the medium property as well the energy of incident photon (E)
- For a given material, μ can be denoted by μ(x;E)
- When the incident photons are polyenergetic, with spectrum S(E), the outgoing photon spectrum is

$$S(x;E) = S_0(E)\exp\left\{-\int_0^x \mu(x';E)\,dx'\right\}$$

- In terms of intensity

$$I = \int_0^\infty E' S(E')\,dE'$$

$$I(x) = \int_0^\infty S_0(E')E'\exp\left\{-\int_0^x \mu(x';E')\,dx'\right\}dE'$$

Radiation Dosimetry

- Previous topics deal with the production of radiation and measurement of radiation wave
- Radiation dosimetry considers the effect of radiation to the interacting media
  - Exposure
  - Dose
  - Kerma
  - Effective dose

Exposure (Creation of Ions)

- Exposure (X) is measured in terms of the number of ions produced in a specific volume of air by EM radiation
- SI unit: C/kg
- Common unit: Roentgen (R)
  - 1 C/kg = 3876 R
- Exposure decreases with distance from source (d) following a inverse square law

$$X(d) = X(0)/d^2$$

Does (the deposition of energy)

- How much energy is deposited into material?
- Dose, D, the energy deposited per unit volume
- SI unit: Gray (Gy) 1 Gy = 1 J/kg
- Common unit: rad

$$1 \text{ Gy} = 100 \text{ rads}$$

- When $X = 1$ R soft tissue incurs 1 rad absorbed dose.

Kerma

- How much energy is deposited into the *electrons*?
- Kerma, K, is the energy deposited into the electrons of a material
- SI units: Gray (Gy) = 1 J/kg = 100 rads
- At diagnostic energies in the body, $K = D$
- (In general, $K \geq D$. Some electrons can cause bremsstrahlung and their energy irradiated away $\rightarrow$ no dose. Not likely in body.)

Dose vs. Exposure

$$D = fX$$

f - factor depends on material:

$$f = 0.87 \frac{\left(\mu/\rho\right)_{material}}{\left(\mu/\rho\right)_{air}}$$

$\left(\mu/\rho\right)$: mass attenuation coefficient

$f = 0.87$ for air

$f \approx 1$ for soft-tissue

See Table 4.6 for the mass attenuation coefficient of typical materials

Equivalent Dose

- Dose equivalent
  - The effect of radiation depends on the source of radiation (energy)
  - Dose equivalent: $H = D\,Q$
  - Q: quality factor
    - $Q = 1$ for x-ray, gamma ray, electron, beta particle (used in medical imaging)
    - $Q = 10$ for neutrons and protons
    - $Q = 20$ for alpha particles
- Effective dose
  - Effect of a dose also depends on the tissue type
  - Effective dose measures the average effect over different tissue types

$$D_{effective} = \sum_{organs} w_j H_j$$

w_j :weighting factor for organ j

Summary

- Ionization: ejection of an orbiting electron from an atom, the affected atom produces radiation in the process of returning to ground state
- Two types of ionizing radiation
  - Particulate
  - EM
- Particulate radiation transfers energy via
  - Collisional transfer: resulting in heat
  - Radioactive transfer: resulting in characteristic x-ray and Bremsstrahlung x-ray
  - X-ray is produced by energetic electrons accelerated in a x-ray tube, consisting primarily Bremsstrahlung x-ray
- EM radiation transfers energy via
  - Photoelectric effect: incident photons completely absorbed
  - Compton scattering: incident photons are deflected

Summary (cntd)

- Attenuation of EM radiation:
  - Linear attenuation coefficient is the fraction of photons that are lost per unit length
    - Depends on material property and the incident photon energy
  - Fundamental photon attenuation law
    - Homogeneous slab $N = N_0 e^{-\mu \Delta x}$
    - Heterogeneous slab $N(x) = N_0 \exp\{-\int_0^x \mu(x') dx'\}$
- Radiation dosimetry
  - Exposure vs. dose: D=fX
  - Equivalent dose: H=DQ
  - Effective dose:

$$D_{effective} = \sum_{organs} w_j H_j$$

w_j :weighting factor for organ j

Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap 4.

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap 4.
- Note down all the corrections for Ch. 4 on your copy of the textbook based on the provided errata.
- Problems for Chap 4 of the text book:
  - P4.4
  - P4.5
  - P4.6
  - P4.8
  - P4.10
  - P4.11
  - P4.12
  - P4.13