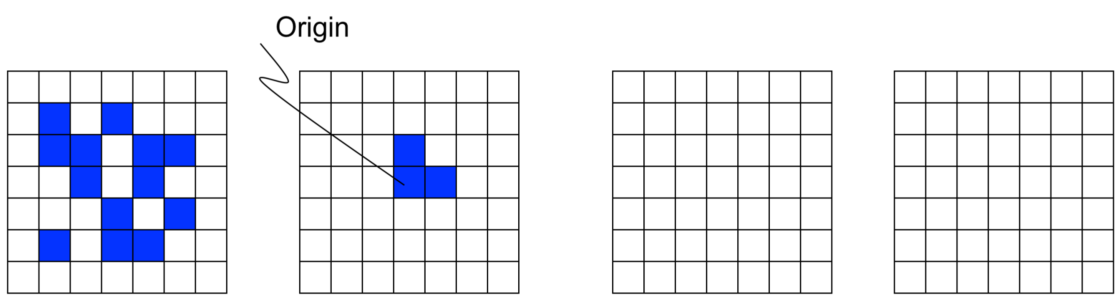
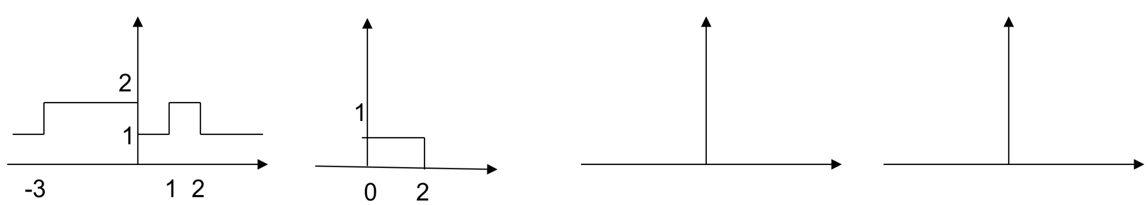
Final Exam (12/16/2013, 10:20AM-12:50PM)
Closed book, 1 sheet of notes (double sided) allowed.
No peeking into neighbors or unauthorized notes. No calculator or other electronics allowed.
Cheating will result in getting an F on the course.
Write your answers on this problem sheet for problems where space is provided. Otherwise write you answer in the blue book. Submit both the blue book and this problem set.

Your name: ______________________________________

1. (20pt) (i) (10pt) Find the closing of the binary image, F, in Fig.(a) by the structuring element H in Fig.(b). You can use the grids in Fig.(c) and Fig.(d) to draw the intermediate and the final results. (ii) (10pt) Find the gray scale opening of the function sketched in Fig.(e) with the structural element in Fig.(f). Sketch the intermediate and final result in Fig.(g) and Fig.(h).



$(a)\ F$ $(b)\ H$ (c) (d)



$(e)\ f$ $(f)\ h$ (g) (d)

2. (15pt) Consider a 2D image f(x,y)=sin(6πx-4πy).

a) (3pt) Determine its Fourier transform F(u,v) and illustrate the spectrum (i.e., the impulses in the transform) in Fig. (a). Indicate the magnitude of each impulse. Please assume that *x* and *u* represent vertical position and frequency, respectively; and *y* and *v* represent horizontal position and frequency, respectively.

b) (3pt) Suppose this signal is sampled uniformly with sampling intervals $\Delta x=\Delta y=\Delta=1/5$. Draw the spectrum of the sampled signal in Fig. (b). You only need to specify inside the frequency region defined by $|u| < f_s, |v| < f_s$, where $f_s=1/\Delta$.

c) (4 pt) Suppose the sampled signal is interpolated by an ideal low-pass filter $h_1(x,y)$ with frequency response

$$H_1(u,v) = \begin{cases} \Delta^2 & -f_s/2 < u,v < f_s/2, \\ 0 & otherwise \end{cases}$$

Draw the spectrum of the reconstructed signal in Fig. (c). Give the spatial representation of the reconstructed signal $f_{r1}(x,y)$.

d) (5pt) Suppose that a sample and hold filter is used instead. Its filter response can be written as

$$h_2(x,y) = h(x)h(y), \text{with } h(x) = \begin{cases} 1 & -\Delta/2 < x < \Delta/2, \\ 0 & otherwise \end{cases}$$

Determine the filter's frequency response, $H_2(u,v)$ (2pt). If the filter is further band-limited to the region described by $|u| < f_s, |v| < f_s$, give the spatial representation of the reconstructed signal $f_{r2}(x,y)$ (3pt). Note that you don't have to evaluate the value of $H_2(u,v)$ for any particular *u* or *v*. For example, for the value of $H(u,v)$ at *u*=1 and *v*=2, just write $H(1,2)$.

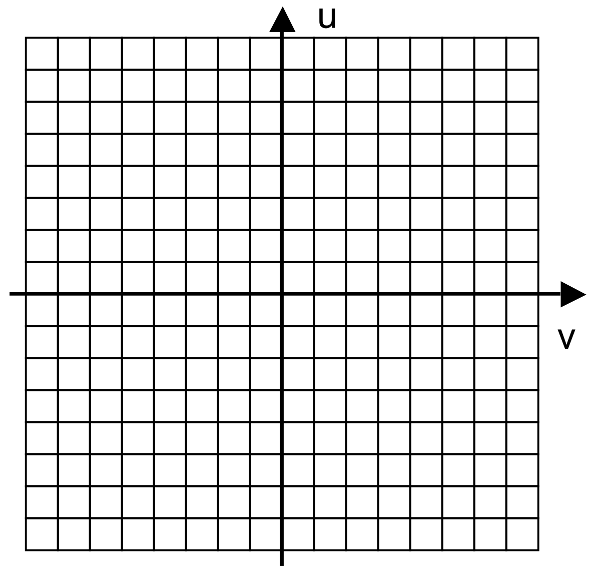


Fig. (a): Original Signal

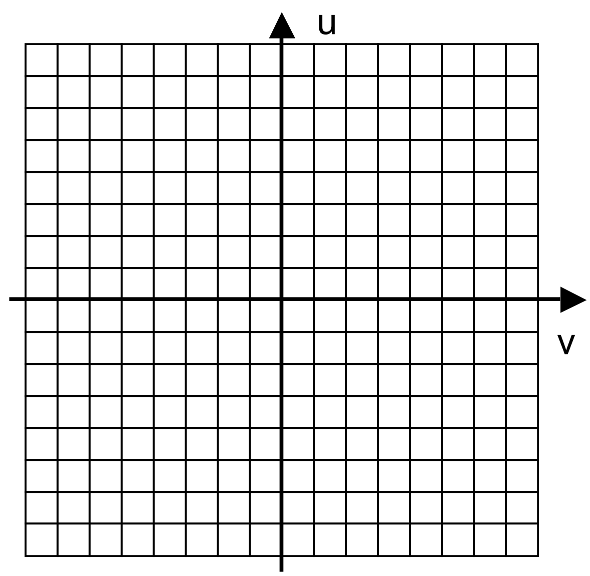


Fig.(b): Sampled Signal

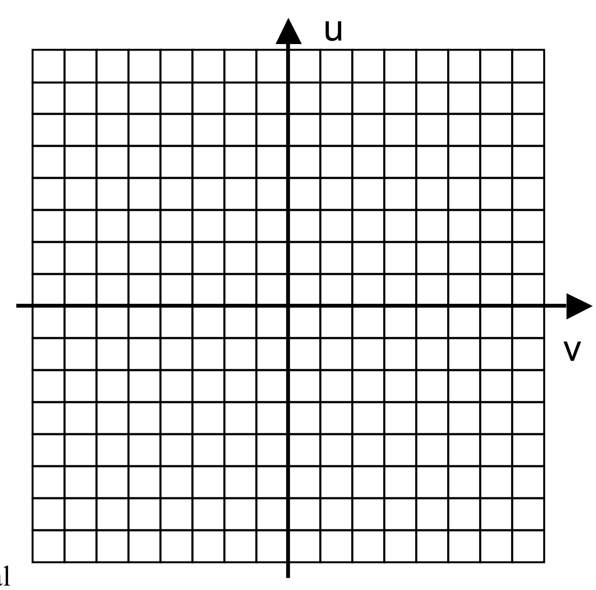


Fig. (c): Reconstructed Signal

3. (15 pt) Figure below shows the quantized DCT coefficients of an image block. Consider the following method to code the DCT coefficients. Using the zig-zag order, starting with the first coefficient, count how many zeros you have before the next non-zero coefficient. Code alternatively the non-zero coefficient and runlength of zeros. Use "EOB" to indicate there are no more non-zero coefficients.
(a) (3 pt) Generate the alternate runlength and nonzero coefficient representation.
(b) (4pt) Generate the distribution of all possible runlengths and the distribution of all possible non-zero values, based on this particular image block; Note that the "EOB" should be considered as one of the runlength symbol.
(c) (6pt) Create a Huffman codebook for the runlengths and a Huffman codebook for the non-zero coefficients.
(d) (2pt) Using your codebooks, generate the binary representation of the DCT coefficients. Determine the bit rate (bits/pixel).

10	5	3	0	1	0	0	0
3	0	3	0	2	0	0	0
0	0	0	0	0	0	0	0
2	1	0	5	0	0	0	0
1	0	3	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

4. (10pt) In transform coding using orthonormal transforms, the sum of squared errors of the reconstructed samples in a block equals to the sum of squared errors of the transform coefficients due to quantization. Prove this result. I suggest that you use the following notation: *f* for the original vector, h_k for the basis vectors, t_k for the transform coefficients, $\hat{t}_k$ for quantized coefficients, and $\hat{f}$ for the reconstructed vector from quantized coefficients.

5. (15 pt) Answer the following questions briefly.
(a) (5pt) Consider image coding using prediction from previously coded pixels. How does prediction help to reduce bit rate in lossless coding? How does prediction help to reduce bit rate in lossy coding? How should you design the predictor?
(b) (5pt) In block transform coding, how should the transform be designed to maximize the coding efficiency?
(c) (5pt) What are some of the advantages of wavelet transform compared to block DCT transform, when used for image compression? What are some of the disadvantages? List 2 advantages and 1 disadvantage.

6. (10 pt) Suppose a doctor took MR brain images of the same patient at two different times, and the patient positioning differs slightly during these two exams. The doctor would like to find out whether there is any change between the two MR images. Consider the following approach for registering the two images and subtract the registered images to show the difference. Step 1) you find *N* feature points $(u_n, v_n), n = 1,2, \dots, N$, in image *F1* and *N* corresponding feature points $(x_n, y_n), n = 1,2, \dots, N$, in image *F2*. Step 2) you determine a bilinear mapping function that allows you to map *F2* to *F3,* so that *F3* is aligned with *F1*. Step 3) You generate *F3* using the mapping function you determined in step 2) and find the difference between *F1* and *F3*.
(a) (4pt) Write down the equations for determining the bilinear mapping function parameters in Step 2. Assume *N>4*.
(b) (2pt) Write down the equation for generating image *F3* at every integer samples *(u,v)* using the mapping function you determined. For this task, assume that the image *F2* has sample values over a continuous domain.
(c) (4pt) Now consider that image *F2* only has samples over an integer grid over a finite range (1<=x<=N, 1<=y<=M), you use bilinear interpolation to find the image value at any non-integer position. Write down a pseudo code for determine the value of *F3(u,v)* for any given integer sample *(u,v)*. You should not use the interp2( ) function in MATLAB.

7) (15 pt) Write a MATLAB script for implementing a simplified wavelet transform coder using the Haar wavelet. Your program should do the following:
1) Read in an image from a file;
2) Perform one stage Haar wavelet decomposition to create 4 subimages;
3) Quantize the wavelet coefficients in each subband using a uniform quantizer with a specified stepsize Q. For the LL band, you should assume that all coefficients are non-negative and use a quantizer than assumes the minimal value is 0. For the other bands, you should assume that all coefficients follow a symmetric distribution with respect to 0.
4) Count the number of non-zeros in the quantized subimages, and determine the approximate compression ratio defined as total_number_pixels/total_number_nonzero_coefficients.
5) Reconstruct the image using the quantized sub-images. Perform necessary truncation so that the reconstructed pixel values can be represented by 8 bit unsigned integers, and save the results in a file.
8) Determine the MSE and PSNR of the reconstructed image.
9) Your program should report the estimated compression ratio and the PSNR, as well as the decompressed image.

Your matlab function should have the following syntax:

[outfile, compression_ratio, PSNR]=WaveletCoding(infile,Q)

Note that you can use built-in functions in MATLAB for reading an image file and writing an image file. For all other parts, please write your own functions. For simplicity, assuming the image has only the luminance component represented in 8 bit unsigned integer in the infile.