1. Determine and illustrate the partition levels and reconstruction levels of a uniform quantizer in the range of \((-1, 1)\) with 4 levels.

Solution:
The entire data range is \(-1+1=2\)
\(N=4, \ Q=2/4=0.5\)
The partition and reconstruction levels are illustrated below.

![Partition Levels Diagram](image)

2. For the sequence \(\{0.2, -0.3, -0.7, 0.8,\}\),
   a. Determine the quantized sequence using the uniform quantizer of Prob. 1
   b. Determine the binary stream corresponding to the quantized sequence.

Solution:
   a. \(\{0.25, -0.25, -0.75, 0.75\}\)
   b. \(\{10, 01, 00, 11\}\)
There are 4 quantized levels, so we need 2 bits to represent each. The above solution is based on the following mapping of quantized levels to codewords: 
-0.75 -> "00", -0.25 ->"01", 0.25->"10", 0.75->"11".

3. Determine and illustrate the partition levels and reconstruction levels of a \(\mu\)-law quantizer in the range of \((-1, 1)\) with 4 levels, \(\mu =64\).

Solution:
See next page
\( \forall x \in (-1, 1) \)

\( x_{\text{min}} = -1, \ x_{\text{max}} = 1 \)

\( X \Rightarrow Y, \)

\[ y(n) = F[x(n)] = x_{\text{max}} \frac{\log [1 + \frac{\left|x(n)\right|}{x_{\text{max}}}]}{\log [1 + u]} \cdot \text{sign}(x(n)) \quad \text{(1)} \]

\( x(n) = -1 \quad \Rightarrow \quad y(-1) = 1 \cdot \frac{\log [1 + 64 \frac{1}{2}]}{\log [1 + 64]} \cdot x(-1) = -1 \)

\( x(n) = 1 \quad \Rightarrow \quad y(1) = 1 \)

Hence, \( y \in (-1, 1) \)

Using a uniform quantizer to divide entire data range of \( y \) into 4 Partitions:

\( y \in (-1, 0.5) \cup (-0.5, 0) \cup (0, 0.5) \cup (0.5, 1) \)

\( Y \Rightarrow X \)

The inverse formula of (1):

\[ x(n) = \frac{\log [1 + u]}{\log [1 + \frac{\left|y(n)\right|}{x_{\text{max}}}]} \cdot \text{sign}(y(n)) \]

\[ x(n) = -1 \quad \Rightarrow \quad x(-1) = -1 \]

\[ x(n) = 0.5 \quad \Rightarrow \quad x(-0.5) = 0.12 \]

\[ y(n) = 0 \quad \Rightarrow \quad x(0) = 0 \]

\[ x(n) = 0.5 \quad \Rightarrow \quad x(0.5) = 0.12 \]

\[ y(n) = 1 \quad \Rightarrow \quad x(1) = 1 \]

Similarly, for reconstruction levels:

Consider the uniform quantizer we mentioned above:

Note that since the range of \( x \) (and consequently \( y \)) is symmetric, you only need to find \( x \) value for \( y = 0.5 \). It should be clear, that \( x(0) = 0, \ x(1) = 1, \ x(-1) = -x(1), \) and \( x(-0.5) = -x(0.5) \). Also, the inverse formula in (2) assumes the log function in (1) has base 10.
4. For the same sequence as in Problem 2,
a. Determine the quantized sequence using the \( \mu \)-law quantizer of Prob. 3
b. Determine the binary stream corresponding to the quantized sequence.

Solution
a. The original sequence \{0.2, -0.3, -0.7, 0.8,\}
   The quantized sequence \{0.34, -0.34, -0.34, 0.34,\}

b. The binary stream corresponding to the quantized sequence \{11, 00, 00, 11,\}
5. We want to transmit a baseband signal \( x(t) \) with frequency spectrum ranging in \((-10 \, \text{KHz}, 10 \, \text{KHz})\) with a channel operating in \((200 \, \text{KHz}, 220 \, \text{KHz})\).

a. What should be the carrier frequency?

b. Write the equation for modulation and demodulation.

c. Draw the block diagram for modulation and demodulation. Specify the cut-off frequencies of any filter used.

d. Sketch the spectrum of the original baseband signal, modulated signal, and demodulated signal before low pass filtering.

Solution:

a. The bandwidth of the baseband signal is \(10 \, \text{KHz} - (-10 \, \text{KHz}) = 20 \, \text{KHz}\).
The bandwidth of the operating channel is \(220 \, \text{KHz} - 200 \, \text{KHz} = 20 \, \text{KHz}\).
The carrier frequency should be the center of the operating channel, which is \(210 \, \text{KHz}\).

b. Modulation: \( y(t) = x(t) \cos(\omega_c t) = x(t) \cos[2\pi \times (210 \times 10^3) t] \)
Demodulation: \( w(t) = y(t) \cos(w_c t) \) followed by a low pass filter.

c. Block diagram of modulation

\[
\begin{align*}
x(t) & \quad \times \quad \cos[2\pi \times (210 \times 10^3) t] \\
& \quad \downarrow \\
x(t) \cos[2\pi \times (210 \times 10^3) t] & \quad \text{H(\omega)} \\
& \quad \downarrow \\
& \quad \downarrow \\
x(t) & \quad \text{LPF}
\end{align*}
\]

Block diagram of demodulation

\[
\begin{align*}
y(t) & \quad \times \quad \cos[2\pi \times (210 \times 10^3) t] \\
& \quad \downarrow \\
& \quad \downarrow \\
w(t) & \quad \text{H(\omega)} \\
& \quad \downarrow \\
& \quad \downarrow \\
x(t) & \quad \text{LPF}
\end{align*}
\]

Note: The LPF cutoff frequency can be any where between 10 KHz and 410 KHz. But the more practical solution for this problem is to cut off at 10 KHz, given that we know the signal has maximum frequency at 10KHz.

d. See figures on the next page
   (a) The spectrum of the original baseband signal
   (b) The spectrum of the carrier signal
   (c) The spectrum of modulated signal
   (d) Demodulated signal before low pass filtering
6. We want to transmit two sound signals, each with frequency spectrum ranging in (–5 KHz, 5 KHz) over a channel operating in the range of (100 KHz-120 KHz).

a. Draw the block diagram for the transmitter that uses amplitude modulation to multiplex these two signals. Specify the necessary carrier frequencies for the two baseband signals.

b. Draw the block diagram for the receiver to demultiplex the two signals and bring back each signal to its baseband. Specify the cut-off frequencies of any filter used.

Solution:

a. To move the two signals to the range of 100-120 KHz, the carrier frequencies for the two signals should be 105 KHz and 115 KHz, respectively.
b. Receiver

The following block diagram is for recovering $x_a(t)$. For recovering $x_b(t)$, the same block diagram applies, but the bandpass filter in the demultiplexer should have passbands in (-120,-110), (110, 120) KHz. For the demodulator, $w_a$ should be replaced by $w_b$. 