 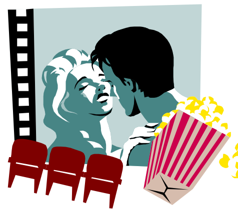 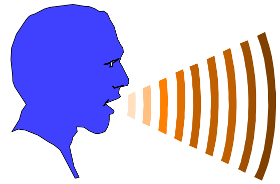 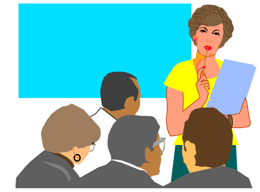

EE3414
Multimedia Communication Systems - I

Frequency Domain Characterization of Signals

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Signal Representation

- What is a signal
- Time-domain description
  - Waveform representation
  - Periodic vs. non-periodic signals
- Frequency-domain description
  - Periodic signals
  - Sinusoidal signals
  - Fourier series for periodic signals
  - Fourier transform for non-periodic signals
  - Concepts of frequency, bandwidth, filtering
  - Numerical calculation: FFT, spectrogram
  - Demo: real sounds and their spectrogram (from *DSP First)*

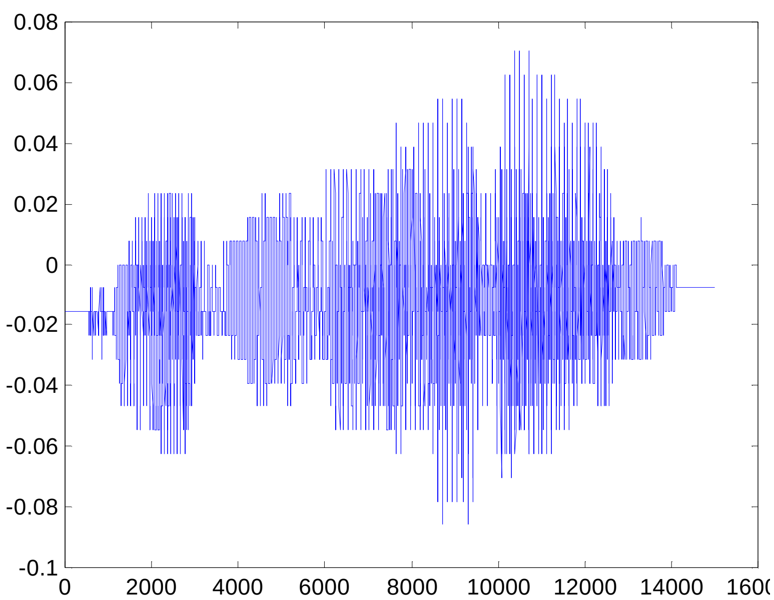
What is a signal

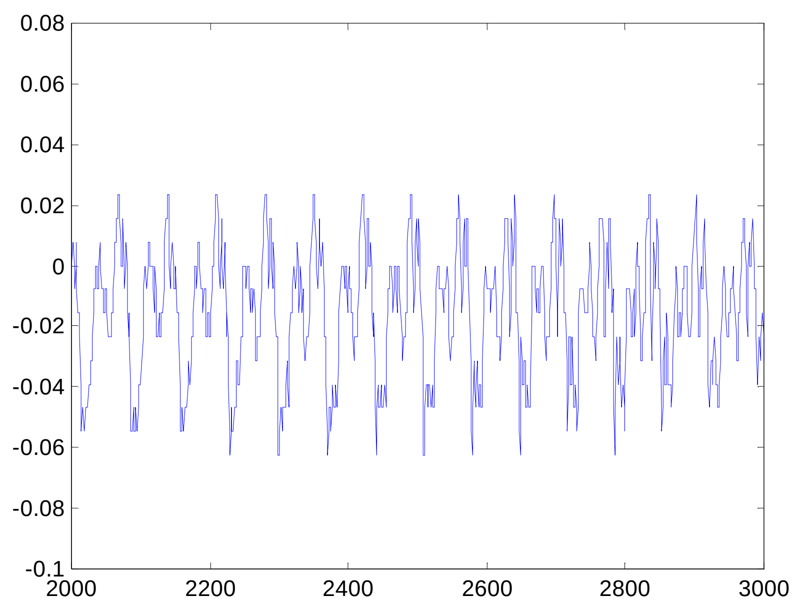
- A variable (or multiple variables) that changes in time
  - Speech or audio signal: A sound amplitude that varies in time
  - Temperature readings at different hours of a day
  - Stock price changes over days
  - Etc …
- More generally, a signal may vary in 2-D space and/or time
  - A picture: the color varies in a 2-D space
  - A video sequence: the color varies in 2-D space and in time
- Continuous vs. Discrete
  - The value can vary continuously or take from a discrete set
  - The time and space can also be continuous or discrete
  - We will look at continuous-time signal only in this lecture

Waveform Representation

- Waveform representation
  - Plot of the variable value (sound amplitude, temperature reading, stock price) vs. time
  - Mathematical representation: $s(t)$

Sample Speech Waveform





Entire waveform

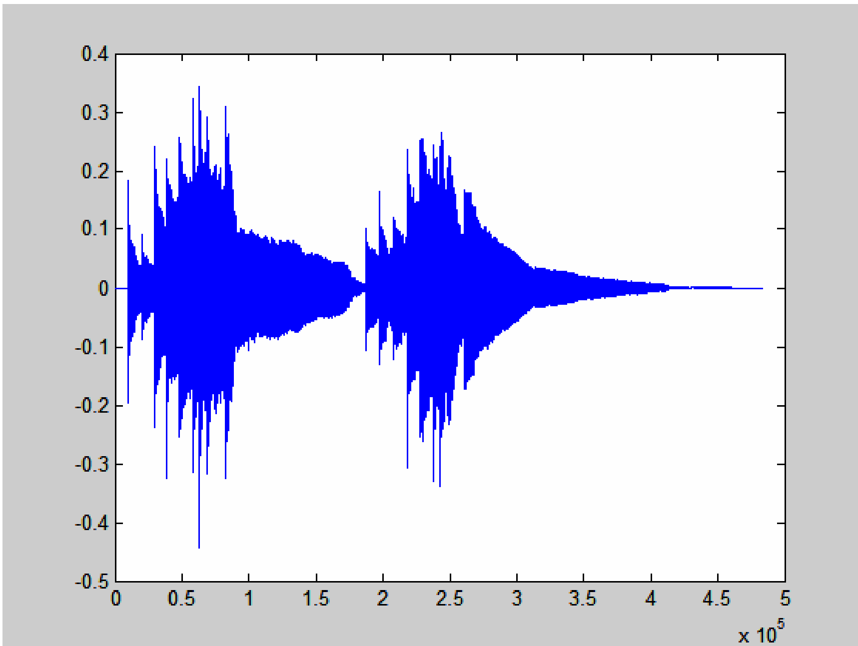
» [y,fs]=wavread('morning.wav');
» sound(y,fs);
» figure; plot(y);
» x=y(10000:25000);plot(x);

Blown-up of a section.

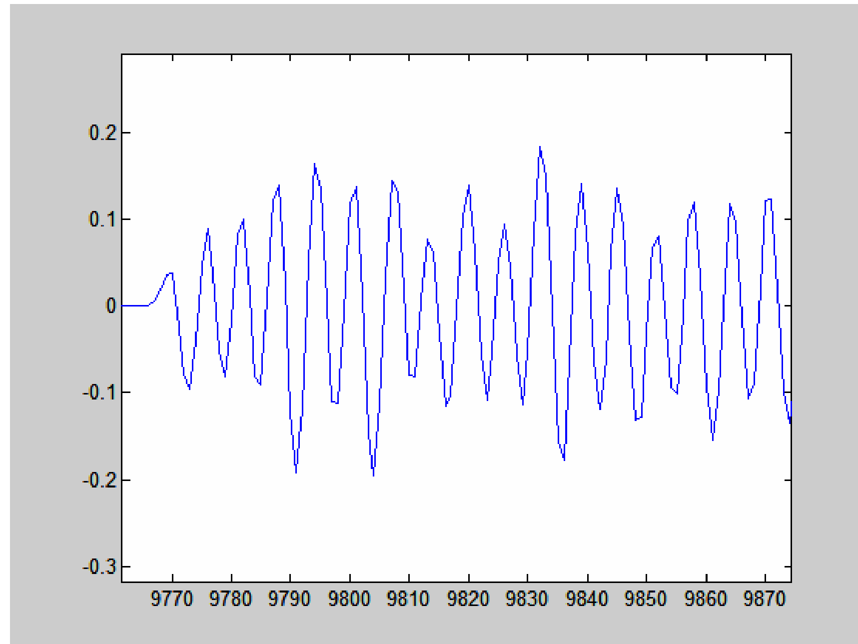
» figure; plot(x);
» axis([2000,3000,-0.1,0.08]);

Signal within each short time interval is periodic
Period depends on the vowel being spoken

Sample Music Waveform



Entire waveform



Blown-up of a section

```
» [y,fs]=wavread('sc01_L.wav');
» sound(y,fs);
» figure; plot(y);
```

```
» v=axis;
» axis([1.1e4,1.2e4,-.2,.2])
```

Music typically has more periodic structure than speech
Structure depends on the note being played

Sinusoidal Signals

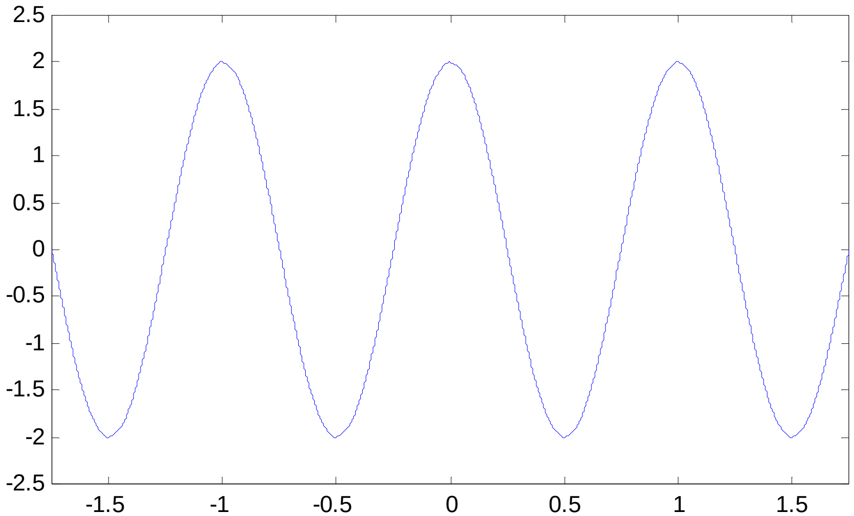
$$s(t) = A\cos(2\pi f_0 t + \phi)$$

f_0 : frequency (cycles/second)

$T_0 = 1/f_0$: period

A : Amplitude

ϕ : Phase (time shift)

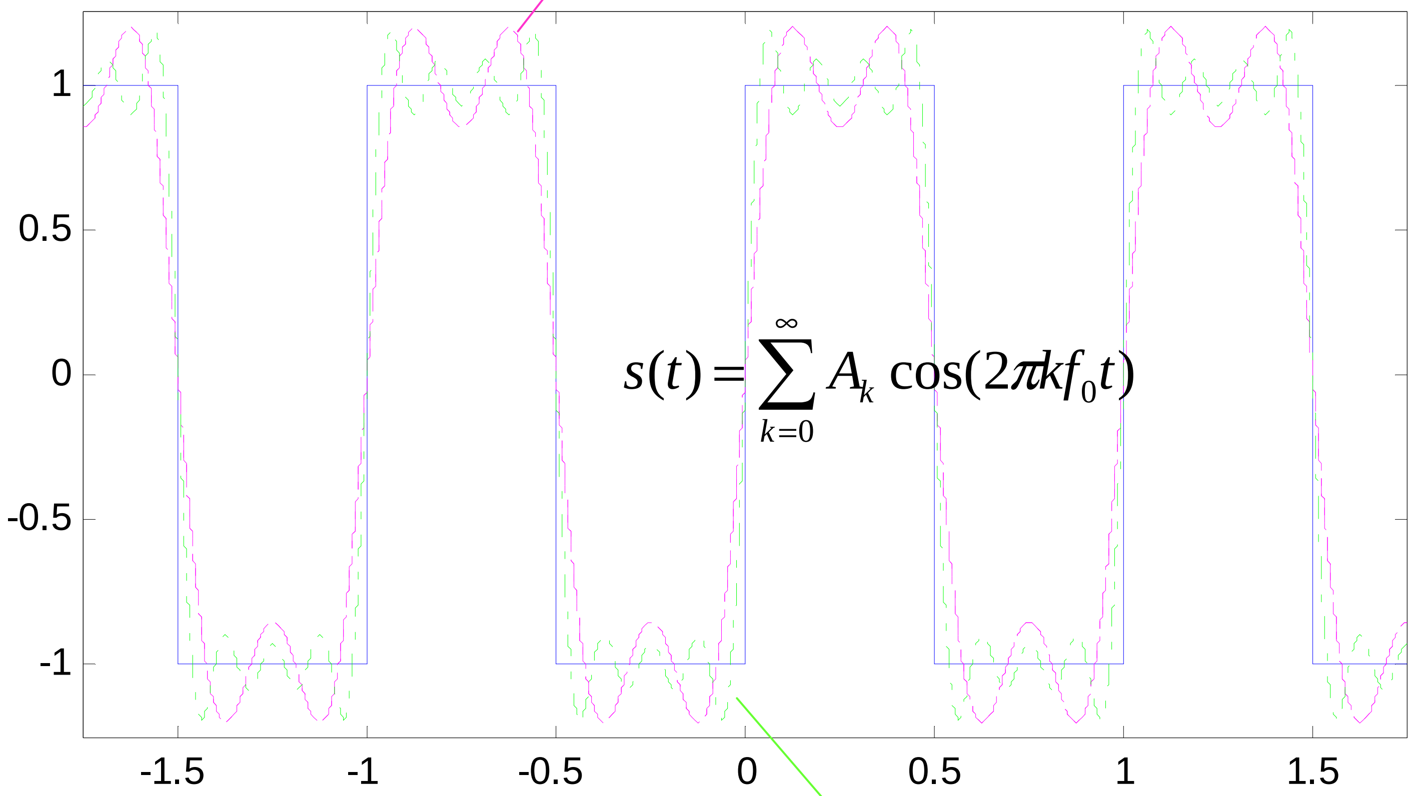


- Sinusoidal signals are important because they can be used to synthesize any signal
  - An arbitrary signal can be expressed as a sum of many sinusoidal signals with different frequencies, amplitudes and phases
- Music notes are essentially sinusoids at different frequencies

What is frequency of an arbitrary signal?

- Sinusoidal signals have a distinct (unique) frequency
- An arbitrary signal does not have a unique frequency, but can be decomposed into many sinusoidal signals with different frequencies, each with different magnitude and phase
- The spectrum of a signal refers to the plot of the magnitudes and phases of different frequency components
- The bandwidth of a signal is the spread of the frequency components with significant energy existing in a signal
- Fourier series and Fourier transform are ways to find spectrums for periodic and aperiodic signals, respectively

Approximation of Periodic Signals by Sum of Sinusoids

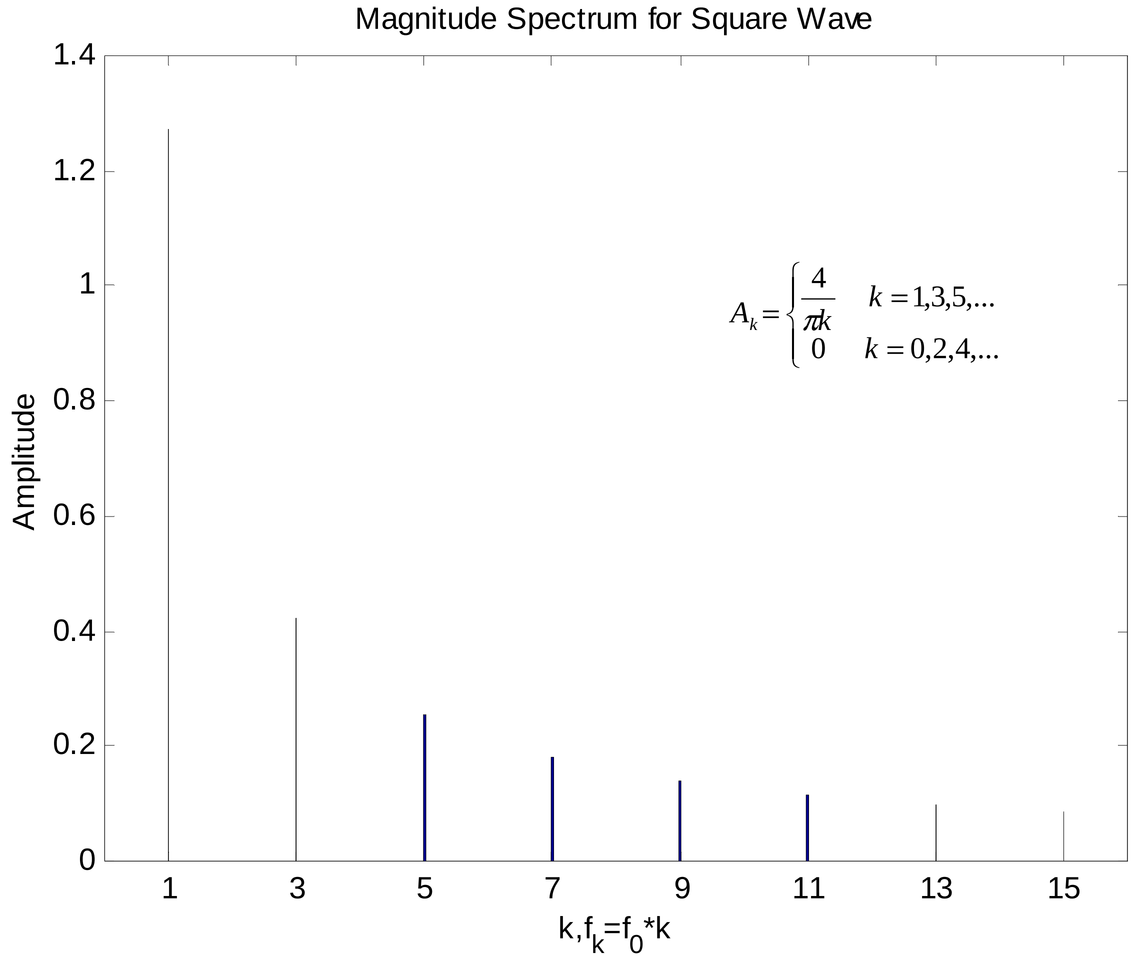


$$s(t)=\sum_{k=0}^{\infty} A_k \cos(2\pi k f_0 t)$$

View note for matlab code

With many more sinusoids with appropriate magnitude, we will get the square wave exactly

Line Spectrum of Square Wave



Each line corresponds to one harmonic frequency. The line magnitude (height) indicates the contribution of that frequency to the signal.

The line magnitude drops exponentially, which is not very fast. The very sharp transition in square waves calls for very high frequency sinusoids to synthesize.

Period Signal

- Period T: The minimum interval on which a signal repeats
  - Sketch on board
- *Fundamental frequency*: $f_0 = 1/T$
- *Harmonic frequencies*: kf_0

Approximation of Periodic Signals by Sinusoids

- Any periodic signal can be approximated by a sum of many sinusoids at harmonic frequencies of the signal (kf_0) with appropriate amplitude and phase.
- The more harmonic components are added, the more accurate the approximation becomes.
- Instead of using sinusoidal signals, mathematically, we can use the complex exponential functions with both positive and negative harmonic frequencies

Complex Exponential Signals

- Complex number:

$$A = |A|\exp(j\phi) = |A|\cos\phi + j|A|\sin\phi = \mathrm{Re} + j\,\mathrm{Im}$$

- Complex exponential signal

$$s(t) = A\exp(j2\pi f_0 t) = |A|\cos(2\pi f_0 t + \phi) + j|A|\sin(2\pi f_0 t + \phi)$$

- Euler formula

$$\exp(j\omega t) + \exp(-j\omega t) = 2\cos(\omega t)$$
$$\exp(j\omega t) - \exp(-j\omega t) = j2\sin(\omega t)$$

Fourier Series Representation of Periodic Signals

Fourier Series Synthesis (inverse transform):

$$s(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(2\pi k f_0 t + \phi_k) \quad \text{(single sided, for real signal only)}$$

$$= \sum_{k=-\infty}^{\infty} S_k \exp(j2\pi k f_0 t) \quad \text{(double sided, for both real and complex)}$$

Fourier series analysis (forward transform):

$$S_k = \frac{1}{T_0} \int_0^{T_0} s(t) \exp(-j2\pi k f_0 t)\, dt; k = 0, \pm 1, 2, \ldots$$

S_k is in general a complex number

For real signals, $S_k = S^*_{-k}$ $|S_k| = |S_{-k}|$ **(Symmetric spectrum)**

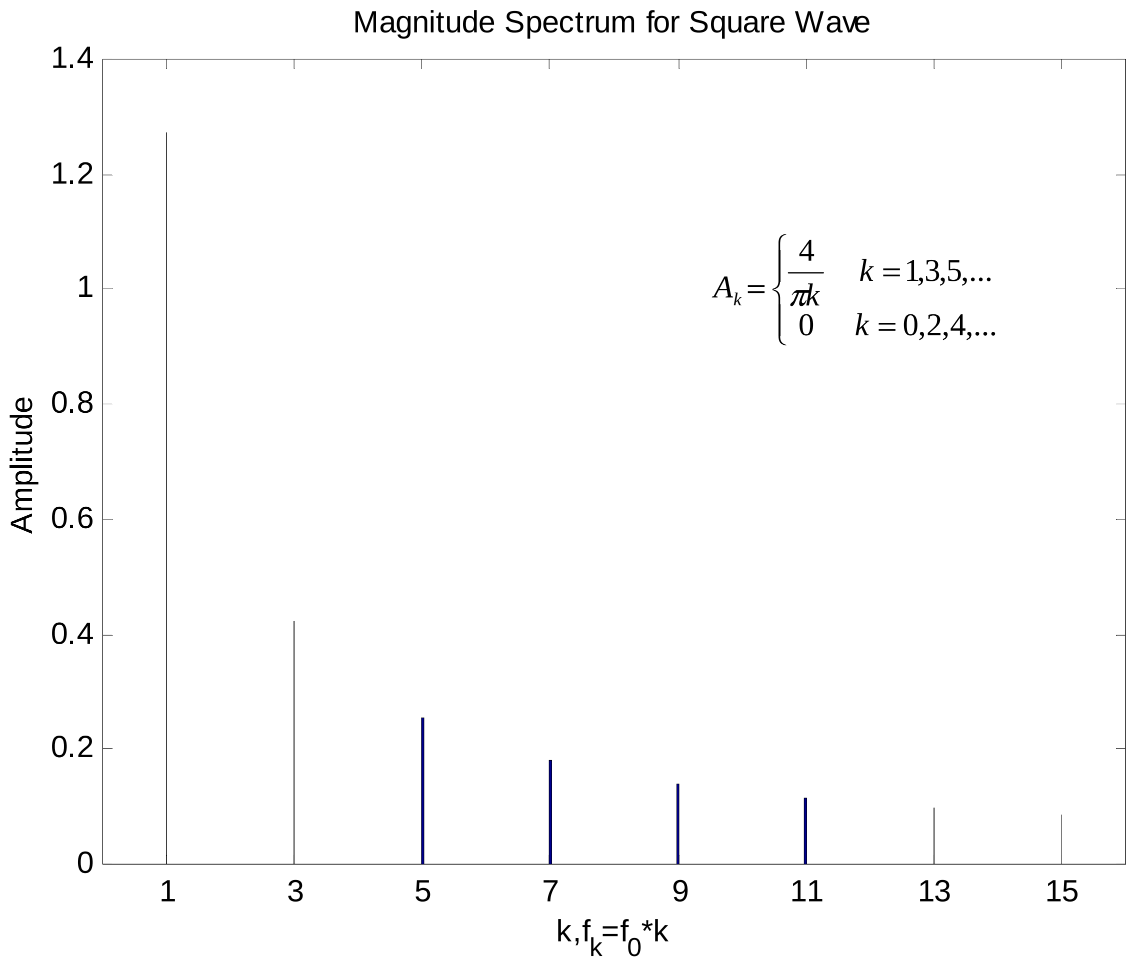
Fourier Series Representation of Square Wave

- Applying the Fourier series analysis formula to the square wave, we get

$$S_k = \begin{cases} \dfrac{2}{j\pi k} & k = \pm 1,3,5,\ldots \\ 0 & k = 0,\pm 2,4,\ldots \end{cases}$$

- Do the derivation on the board

Line Spectrum of Square Wave



Only the positive frequency side is drawn on the left (single sided spectrum), with twice the magnitude of the double sided spectrum.

Fourier Transform for Non-Periodic Signals

Aperiodic signal $\rightarrow T_0 = \infty \rightarrow f_0 = 0 \rightarrow$ uncountable number of harmonics

$\rightarrow$ integral instead of sum

Fourier synthesis (inverse transform):

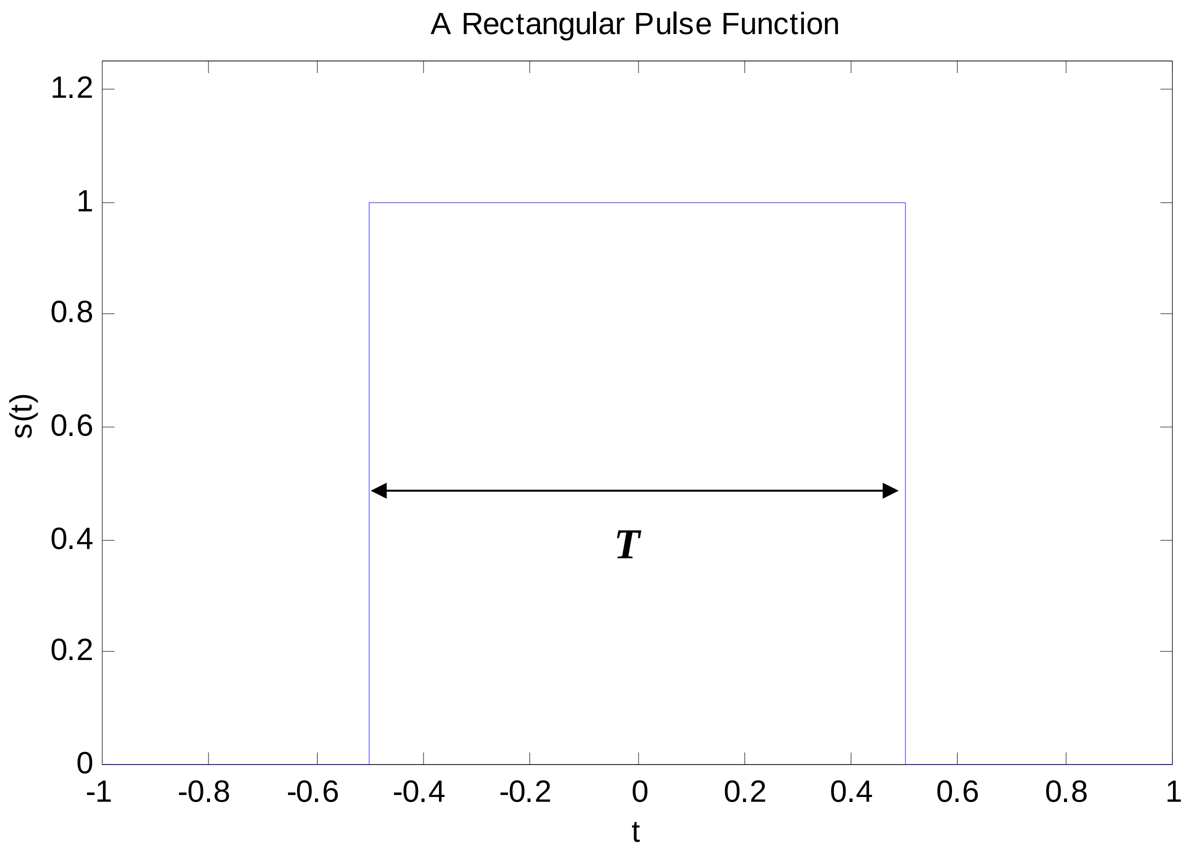
$$s(t) = \int_{-\infty}^{\infty} S(f) \exp(j2\pi ft) df$$

Fourier analysis (forward transform):

$$S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi ft) dt$$

For real signals, $|S(f)| = |S(-f)|$ **(Symmetric magnitude spectrum)**

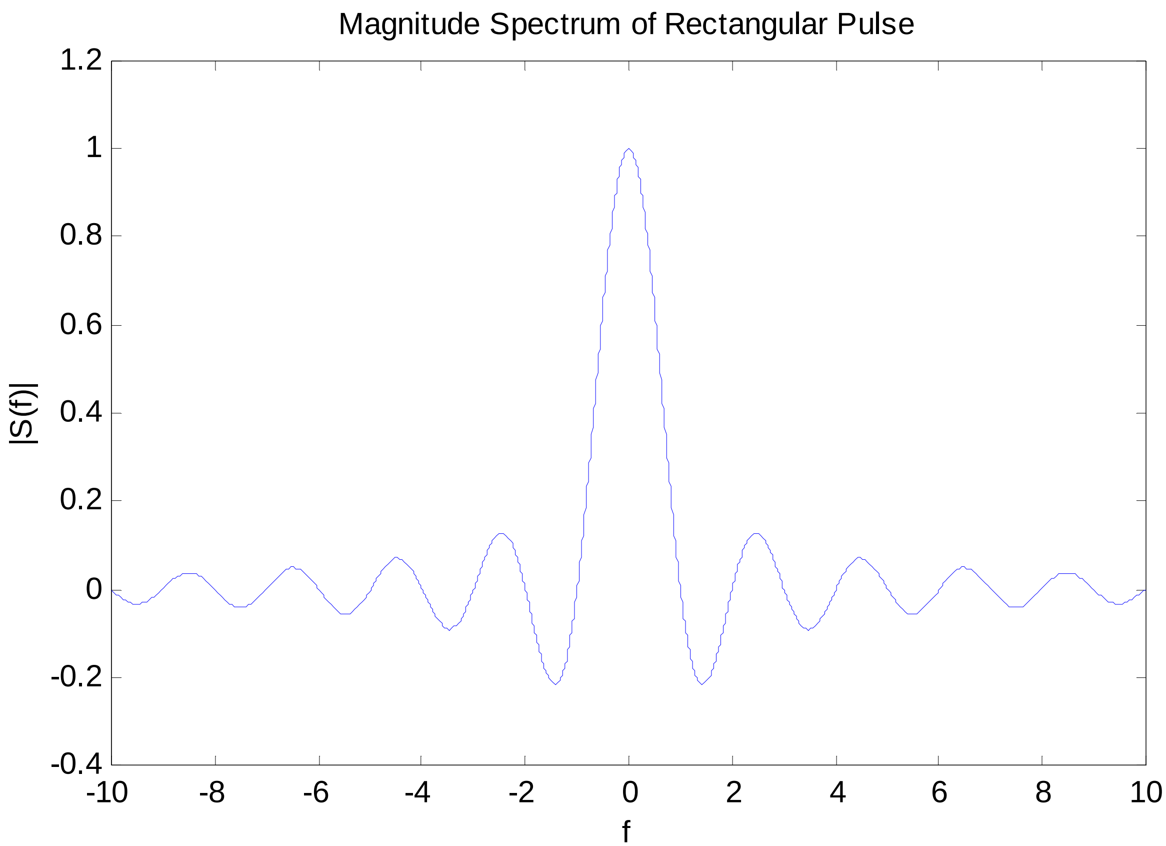
Pulse Function: Time Domain



Derive Fourier transform on the board

$$s(t) = \begin{cases} 1 & -T/2 < t < T/2 \\ 0 & \text{otherwise} \end{cases}$$

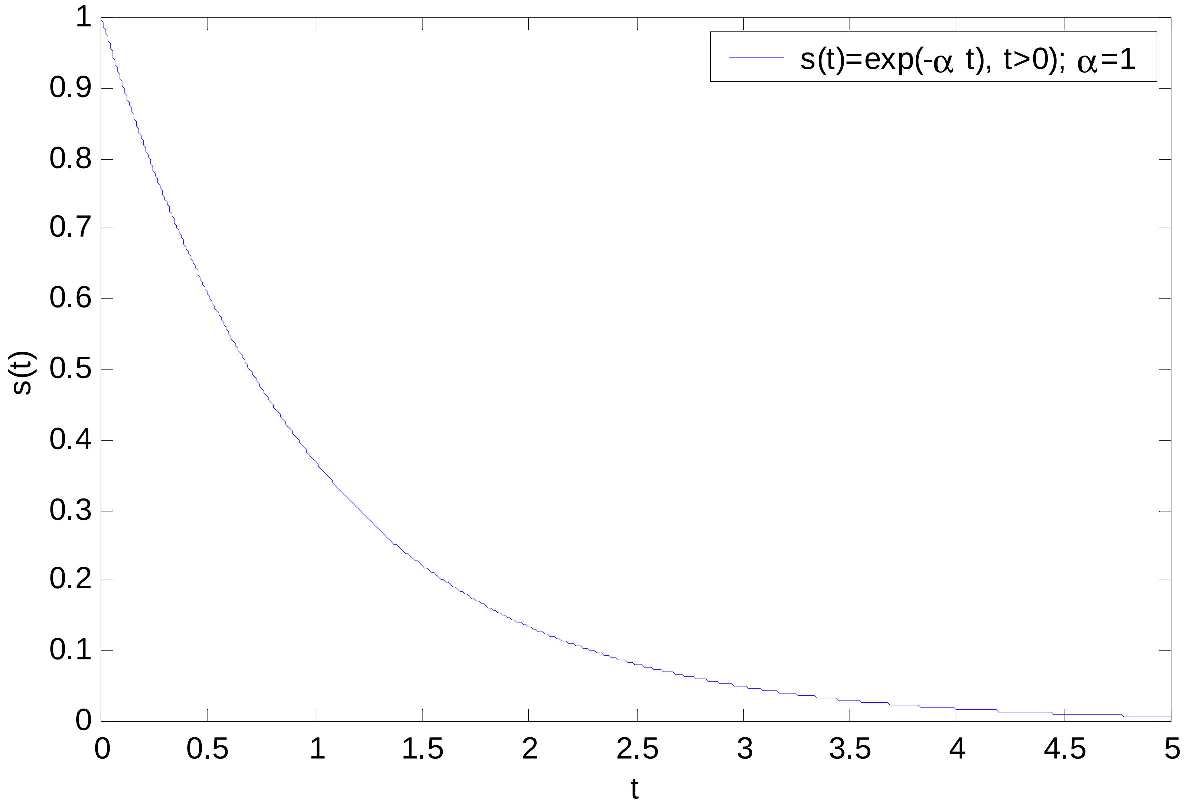
Pulse Function: Spectrum



The peaks of the FT magnitude drops slowly. This is because the pulse function has sharp transition, which contributes to very high frequency in the signal.

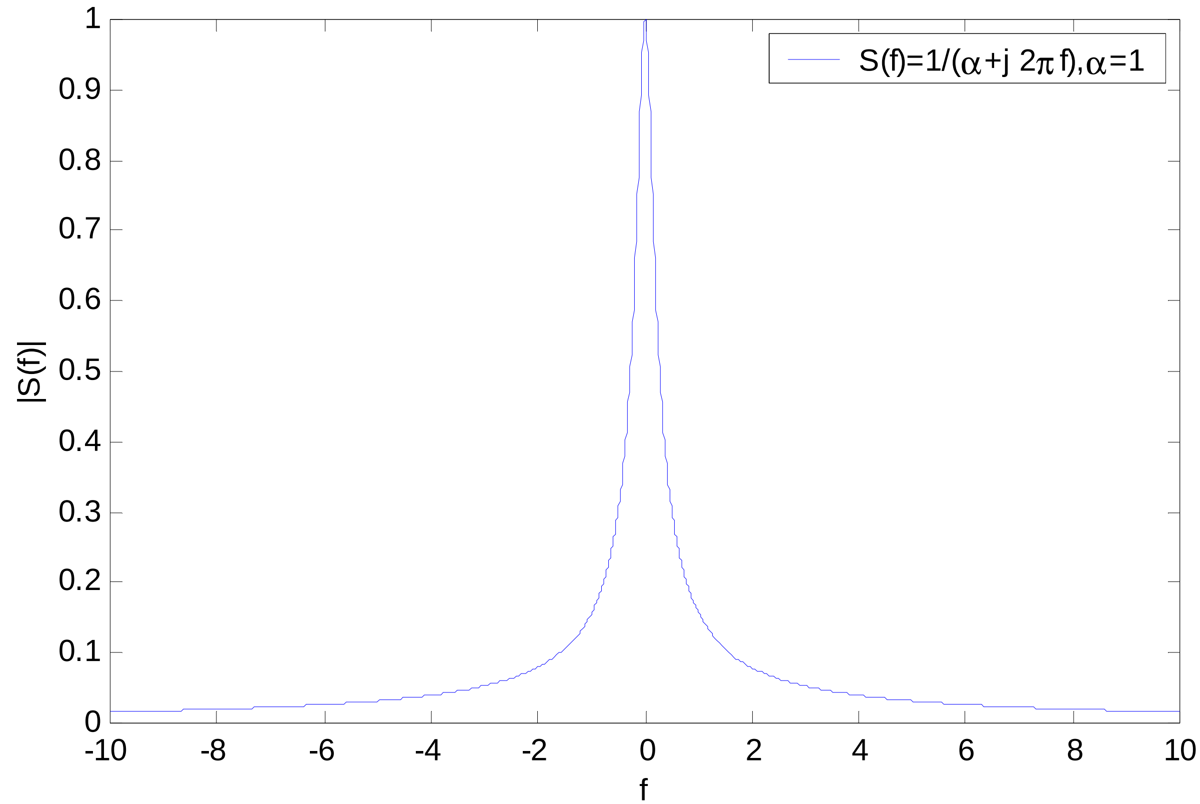
$$s(t)=\begin{cases}1 & -T/2<t<T/2\\ 0 & \text{otherwise}\end{cases} \Leftrightarrow S(f)=T\frac{\sin(\pi Tf)}{\pi Tf}=T\,\mathrm{sinc}(Tf)$$

Exponential Decay: Time Domain



$$s(t)=\begin{cases}\exp(-\alpha t) & t>0\\ 0 & \text{otherwise}\end{cases} \Leftrightarrow S(f)=\frac{1}{\alpha+j2\pi f};\left|S(f)\right|=\frac{1}{\sqrt{\alpha^2+4\pi^2 f^2}}$$

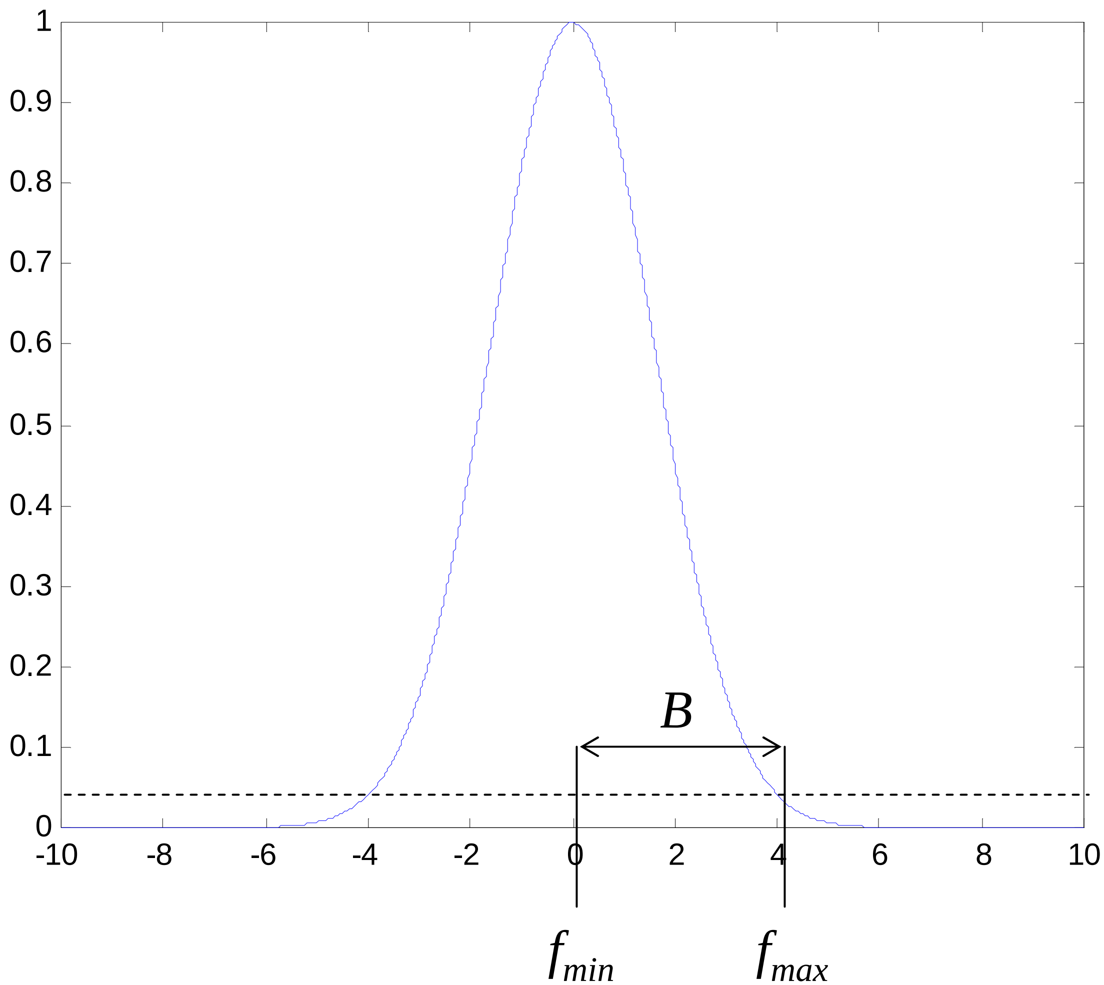
Exponential Decay: Spectrum



The FT magnitude drops much faster than for the pulse function. This is because the exponential decay function does not has sharp transition.

$$s(t)=\begin{cases}\exp(-\alpha t) & t>0\\ 0 & \text{otherwise}\end{cases} \Leftrightarrow S(f)=\frac{1}{\alpha+j2\pi f};|S(f)|=\frac{1}{\sqrt{\alpha^2+4\pi^2 f^2}}$$

(Effective) Bandwidth



- f_{min} (f_{ma}): lowest (highest) frequency where the FT magnitude is above a threshold
- Bandwidth:

  $B=f_{max}-f_{min}$

- The threshold is often chosen with respect to the peak magnitude, expressed in dB
- dB=10 log10(ratio)
- 10 dB below peak = 1/10 of the peak value
- 3 dB below=1/2 of the peak

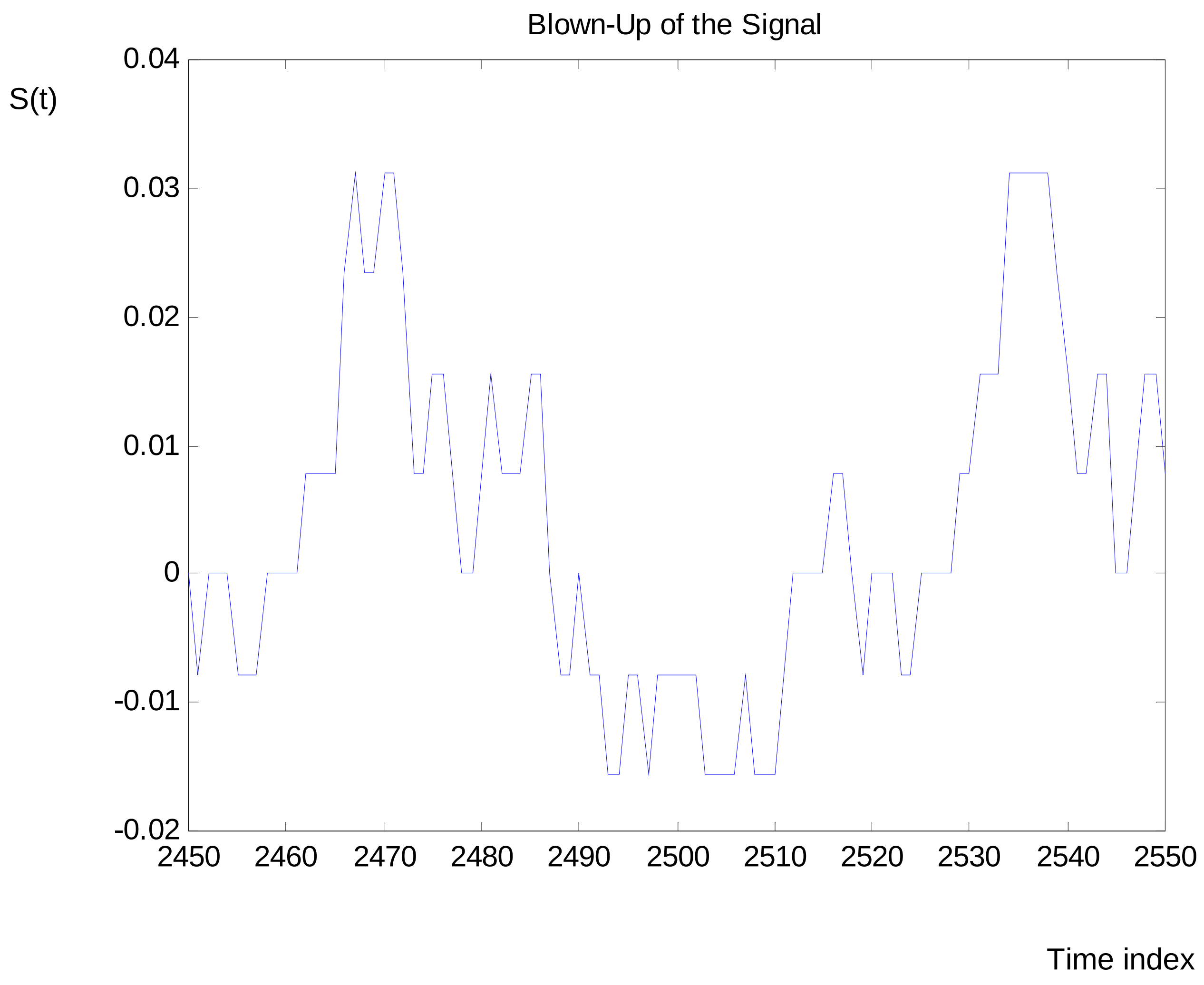
More on Bandwidth

- Bandwidth of a signal is a critical feature when dealing with the transmission of this signal
- A communication channel usually operates only at certain frequency range (called channel bandwidth)
  - The signal will be severely attenuated if it contains frequencies outside the range of the channel bandwidth
  - To carry a signal in a channel, the signal needed to be modulated from its baseband to the channel bandwidth
  - Multiple narrowband signals may be multiplexed to use a single wideband channel

How to Observe Frequency Content from Waveforms?

- A constant -> only zero frequency component (DC compoent)
- A sinusoid -> Contain only a single frequency component
- Periodic signals -> Contain the fundamental frequency and harmonics -> Line spectrum
- Slowly varying -> contain low frequency only
- Fast varying -> contain very high frequency
- Sharp transition -> contain from low to high frequency
- Music: contain both slowly varying and fast varying components, wide bandwidth
- Highest frequency estimation?
  - Find the shortest interval between peak and valleys
- Go through examples on the board

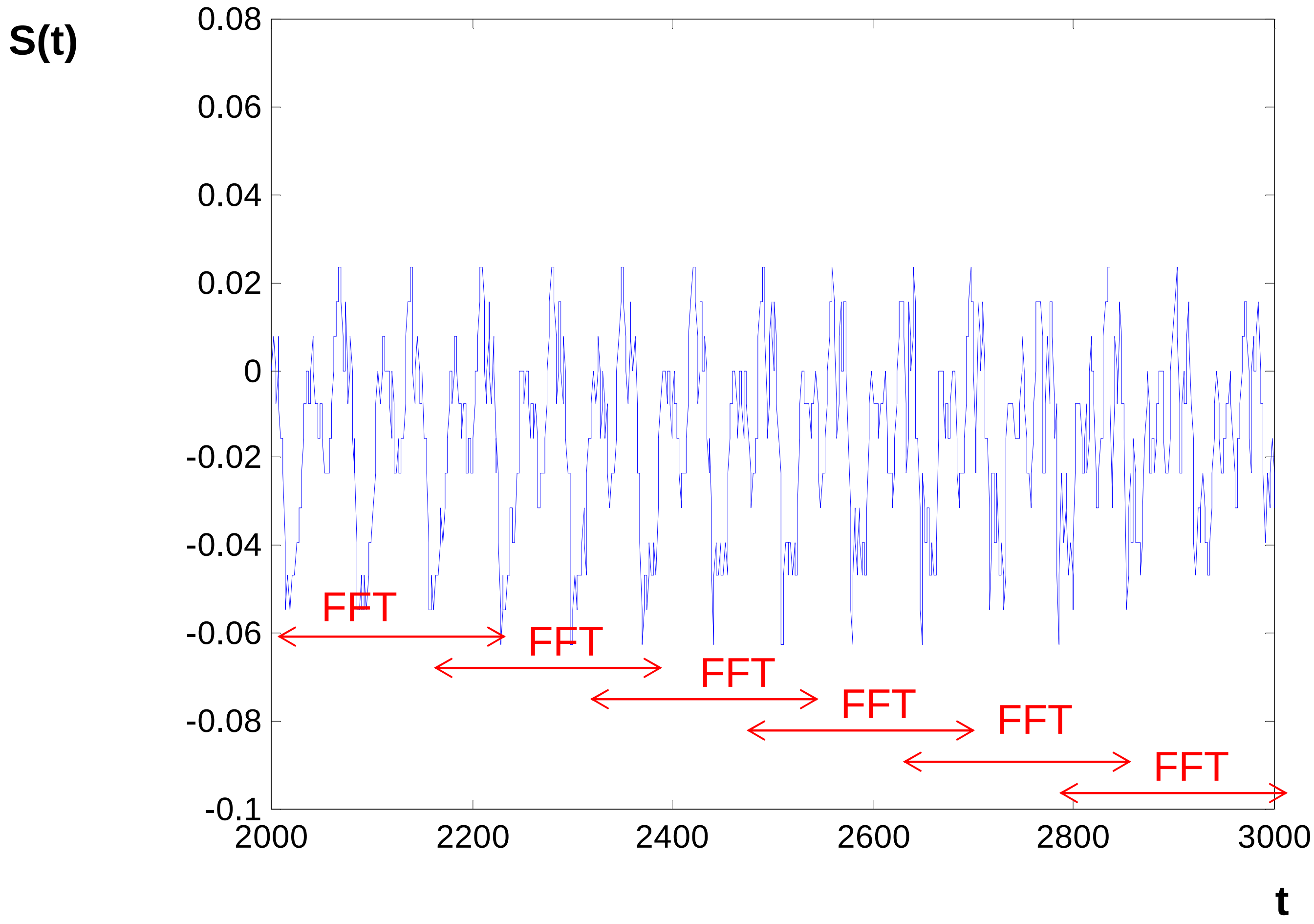
Estimation of Maximum Frequency



Numerical Calculation of FT

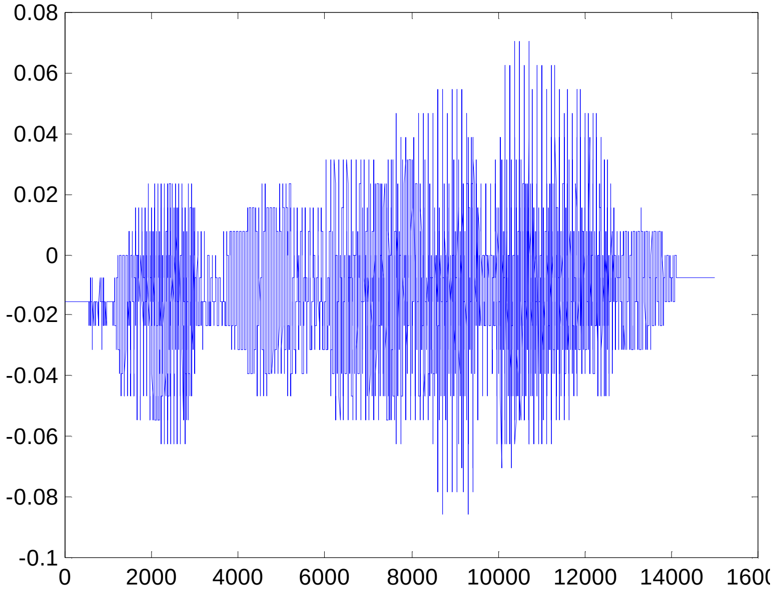
- The original signal is digitized, and then a Fast Fourier Transform (FFT) algorithm is applied, which yields samples of the FT at equally spaced intervals.
- For a signal that is very long, e.g. a speech signal or a music piece, spectrogram is used.
  – Fourier transforms over successive overlapping short intervals

Spectrogram

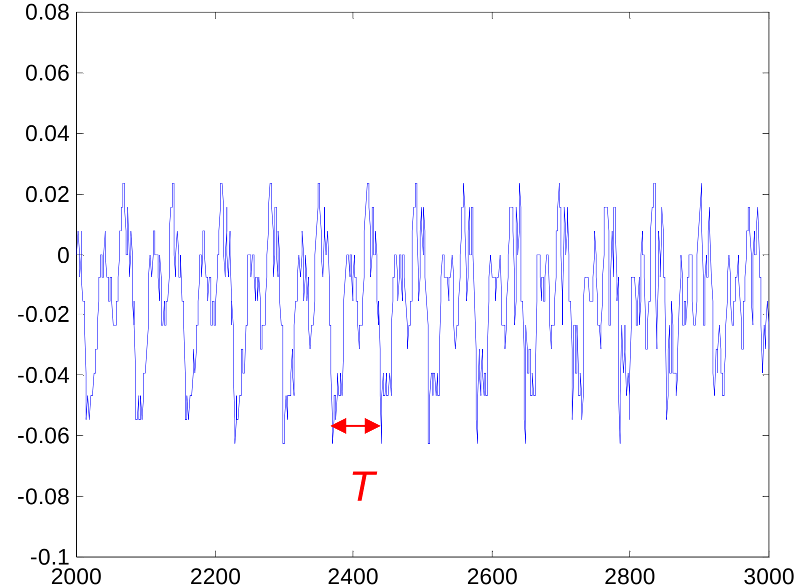


Sample Speech Waveform

(click to hear the sound)



Entire waveform

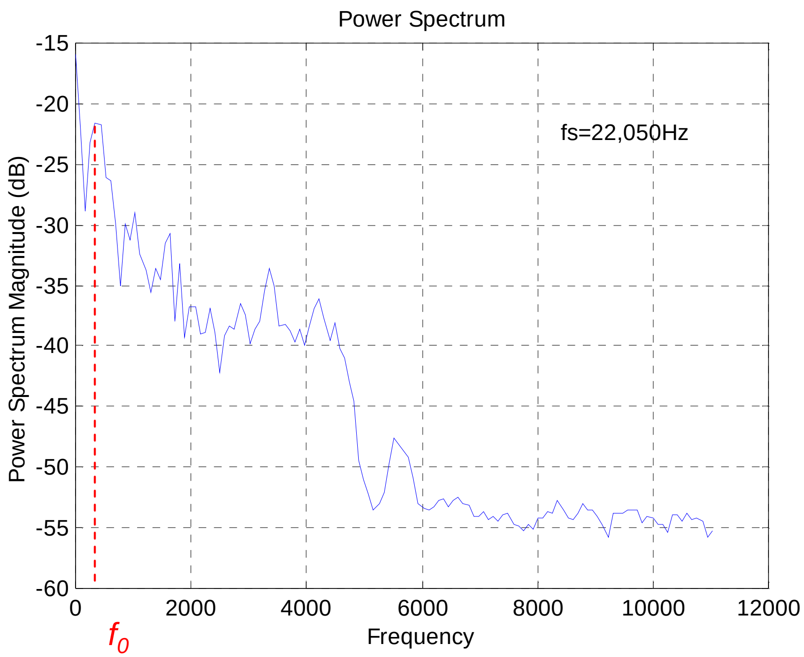


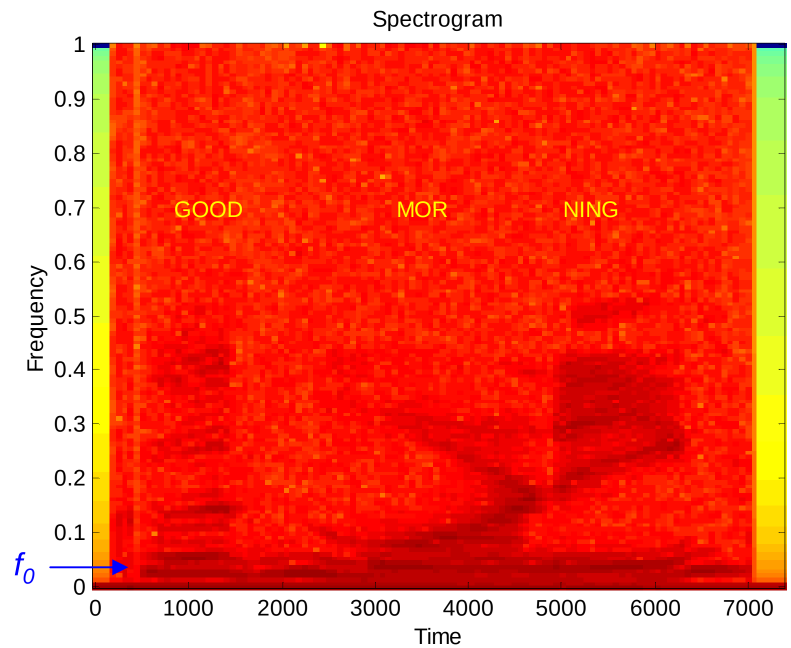
Blown-up of a section.

Signal within each short time interval is periodic. The period *T* is called "pitch".
The pitch depends on the vowel being spoken, changes in time. T~70 samples in this ex.

$f_0=1/T$ is the fundamental frequency (also known as formant frequency). $f_0=1/70fs=315$ *Hz.*
$k*f_0$ (*k*=integers) are the harmonic frequencies.

Sample Speech Spectrogram





» figure;
» psd(x,256,fs);

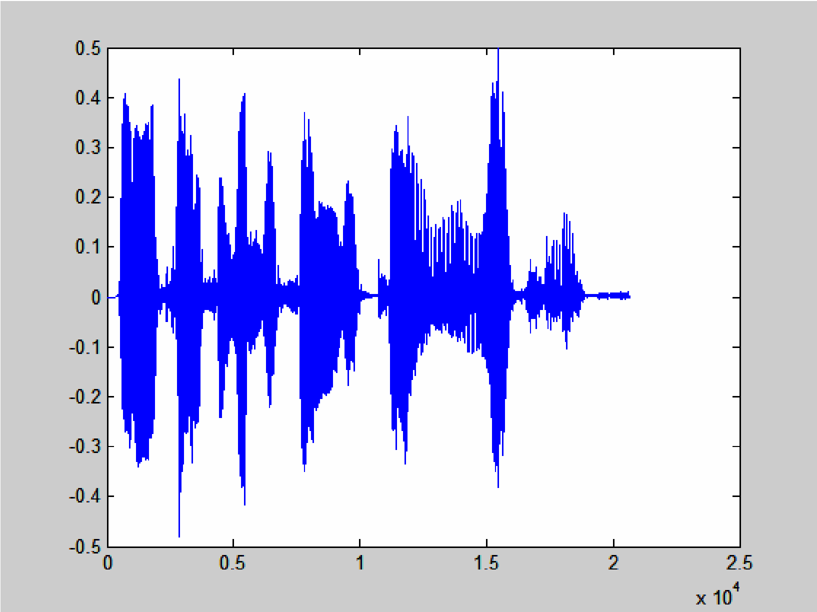
Signal power drops sharply at about 4KHz

» figure;
» specgram(x,256,fs);

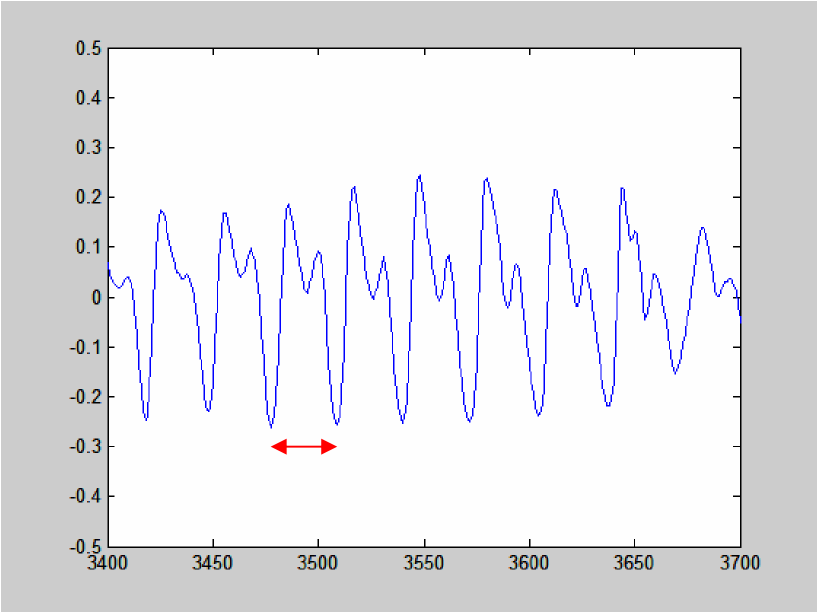
Line spectra at multiple of f0, maximum frequency about 4 KHz

What determines the maximum freq?

Another Sample Speech Waveform



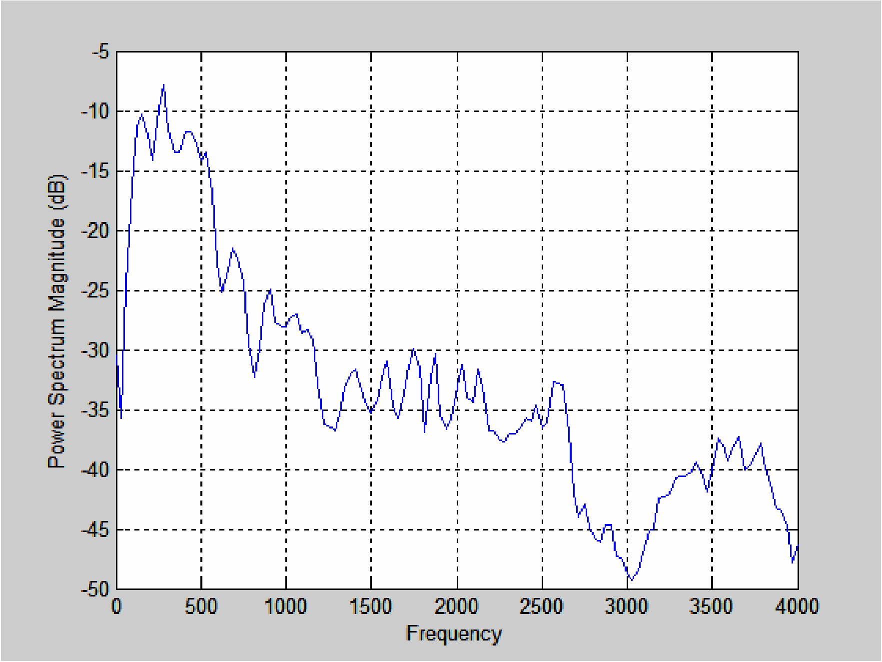
Entire waveform



Blown-up of a section.

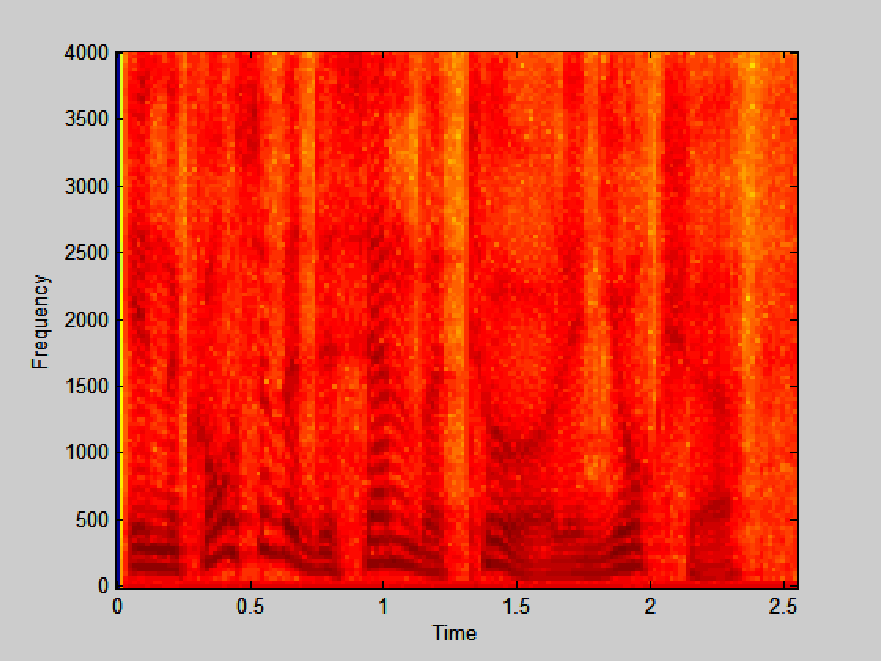
"In the course of a December tour in Yorkshire"

Speech Spectrogram



» figure;
» psd(x,256,fs);

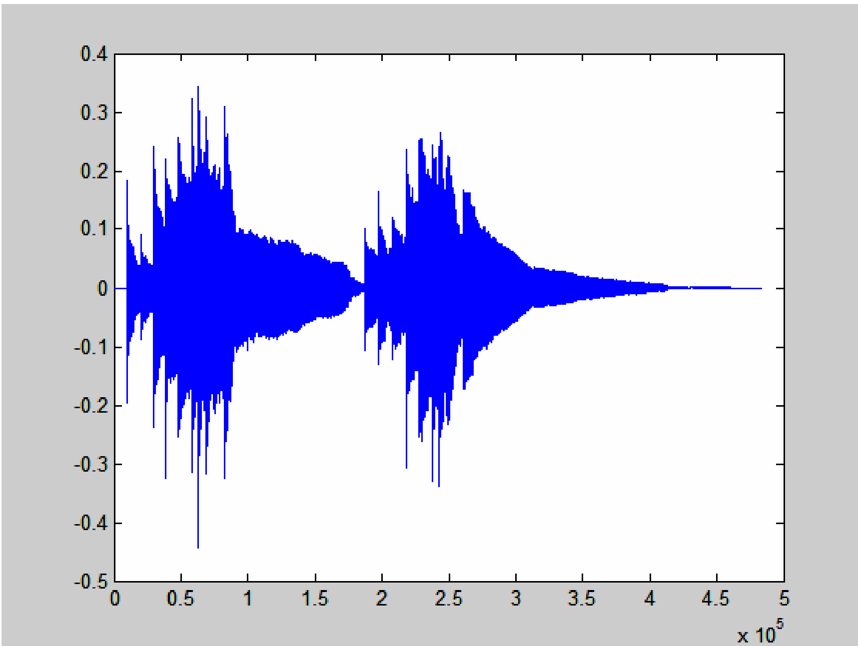
Signal power drops sharply at about 4KHz



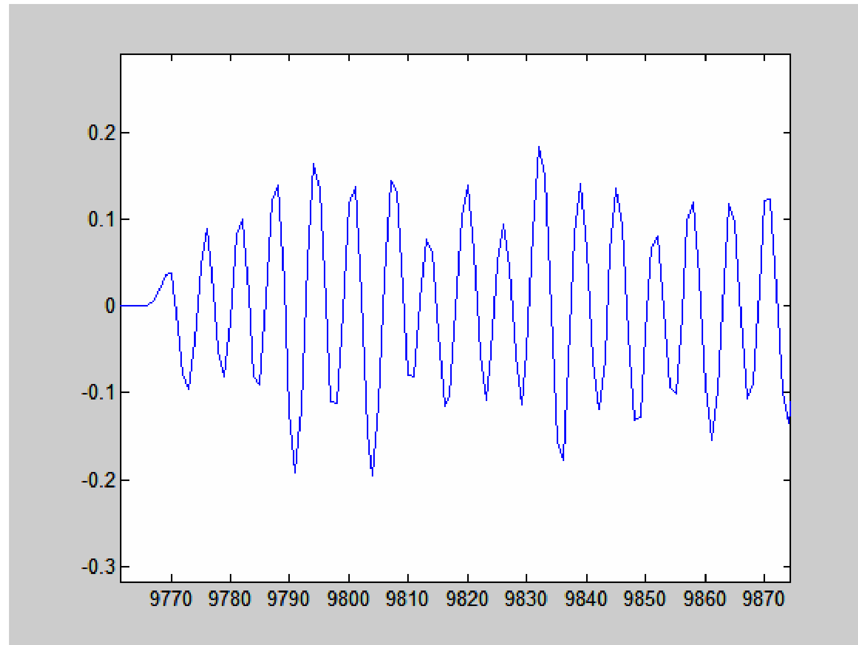
» figure;
» specgram(x,256,fs);

Line spectra at multiple of f0,
maximum frequency about 4 KHz

Sample Music Waveform



Entire waveform



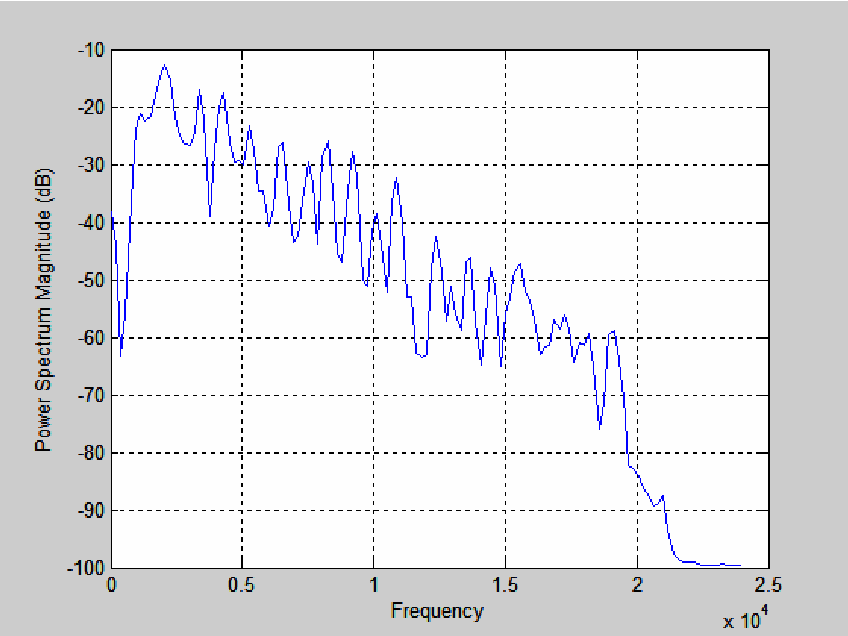
Blown-up of a section

```
» [y,fs]=wavread('sc01_L.wav');
» sound(y,fs);
» figure; plot(y);
```

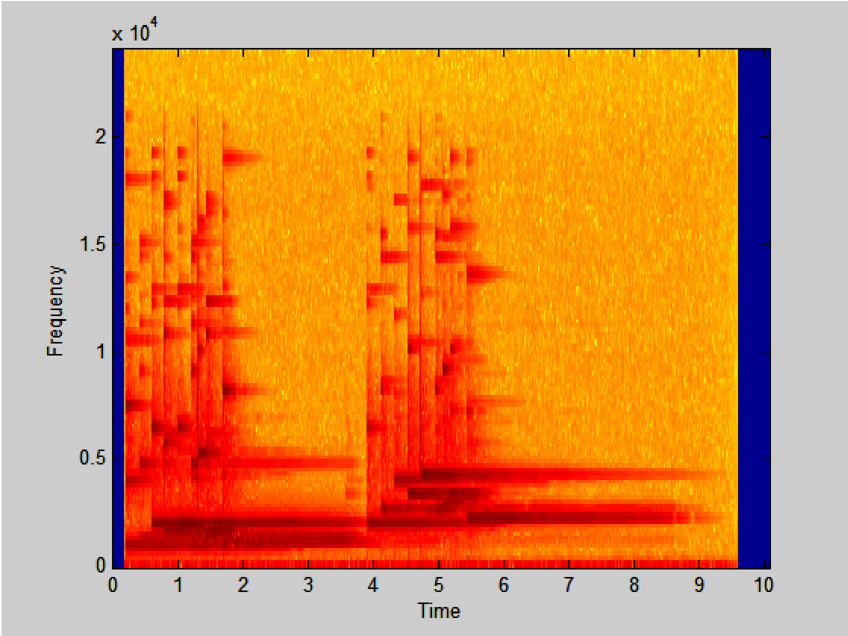
```
» v=axis;
» axis([1.1e4,1.2e4,-.2,.2])
```

Music typically has more periodic structure than speech
Structure depends on the note being played

Sample Music Spectrogram



» figure; » psd(y,256,fs);



» figure; » specgram(y,256,fs);

Signal power drops gradually in the entire frequency range

Line spectra are more stationary, Frequencies above 4 KHz, more than 20KHz in this ex.

Summary of Characteristics of Speech & Music

- Typical speech and music waveforms are semi-periodic
  - The fundamental period is called pitch period
  - The inverse of the pitch period is the fundamental frequency (f0)
- Spectral content
  - Within each short segment, a speech or music signal can be decomposed into a pure sinusoidal component with frequency f0, and additional harmonic components with frequencies that are multiples of f0.
  - The maximum frequency is usually several multiples of the fundamental frequency
  - Speech has a frequency span up to 4 KHz
  - Audio has a much wider spectrum, up to 22KHz

Demo

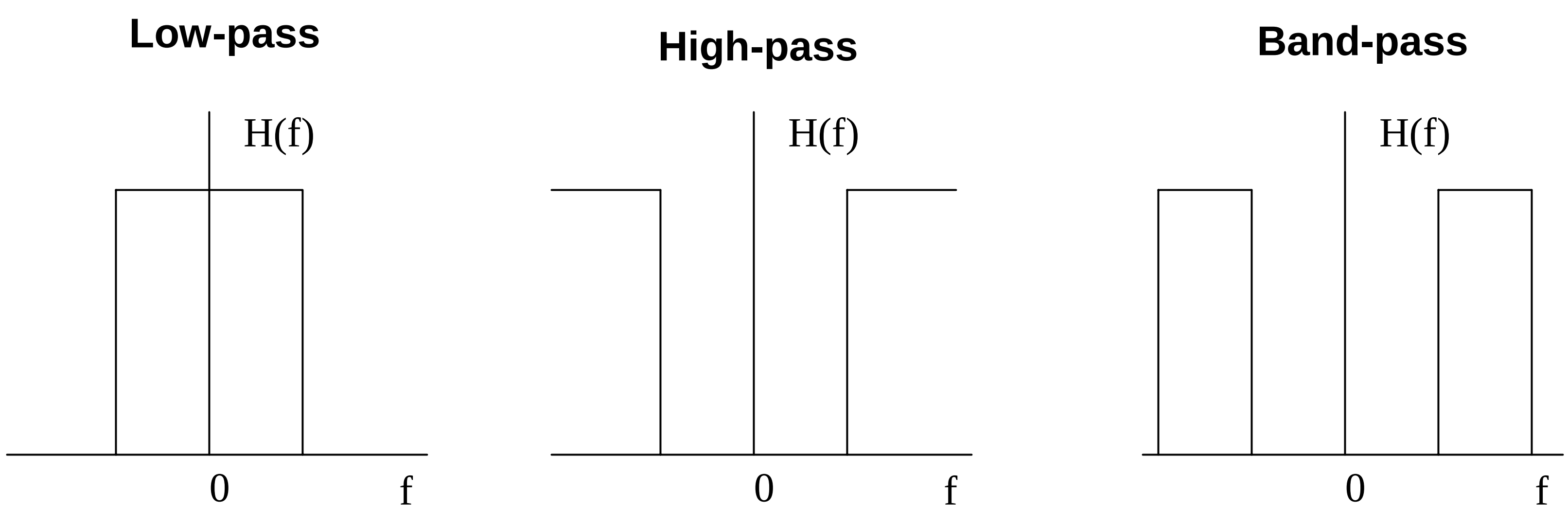
- Demo in DSP First, Chapter 3, Sounds and Spectrograms
  - Look at the waveform and spectrogram of sample signals, while listening to the actual sound
  - Simple sounds
  - Real sounds

Advantage of Frequency Domain Representation

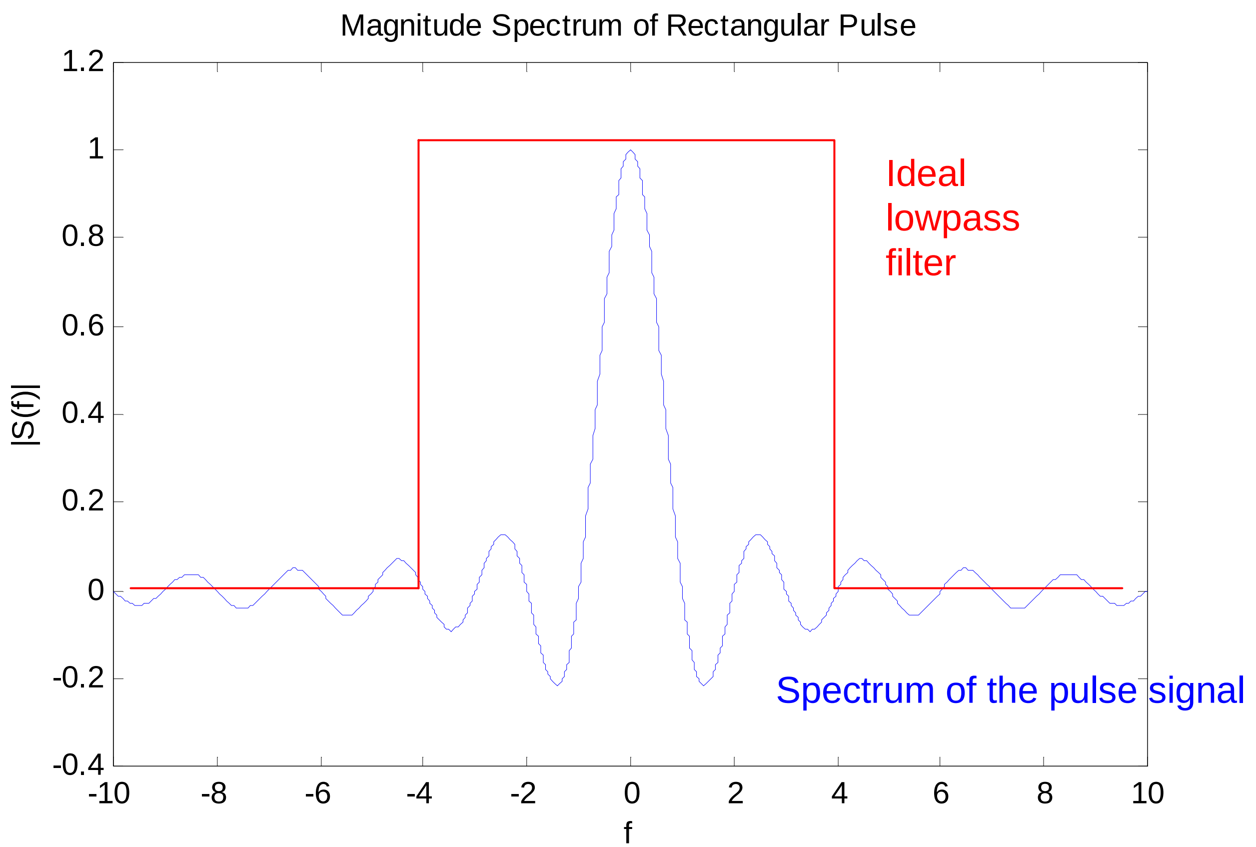
- Clearly shows the frequency composition of the signal
- One can change the magnitude of any frequency component arbitrarily by a filtering operation
  - Lowpass -> smoothing, noise removal
  - Highpass -> edge/transition detection
  - High emphasis -> edge enhancement
- One can also shift the central frequency by modulation
  - A core technique for communication, which uses modulation to multiplex many signals into a single composite signal, to be carried over the same physical medium.

Typical Filters

- Lowpass -> smoothing, noise removal
- Highpass -> edge/transition detection
- Bandpass -> Retain only a certain frequency range



Low Pass Filtering
(Remove high freq, make signal smoother)

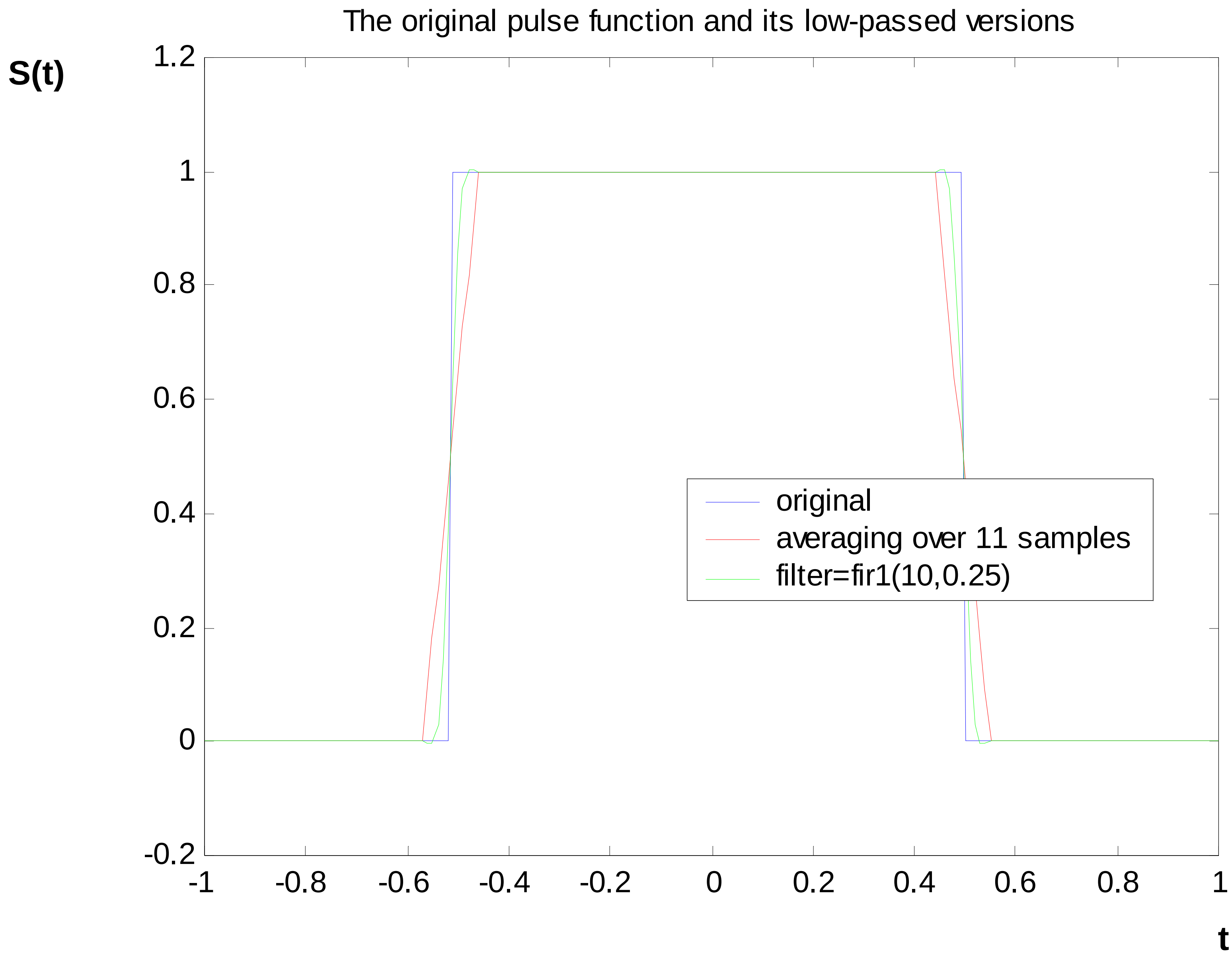


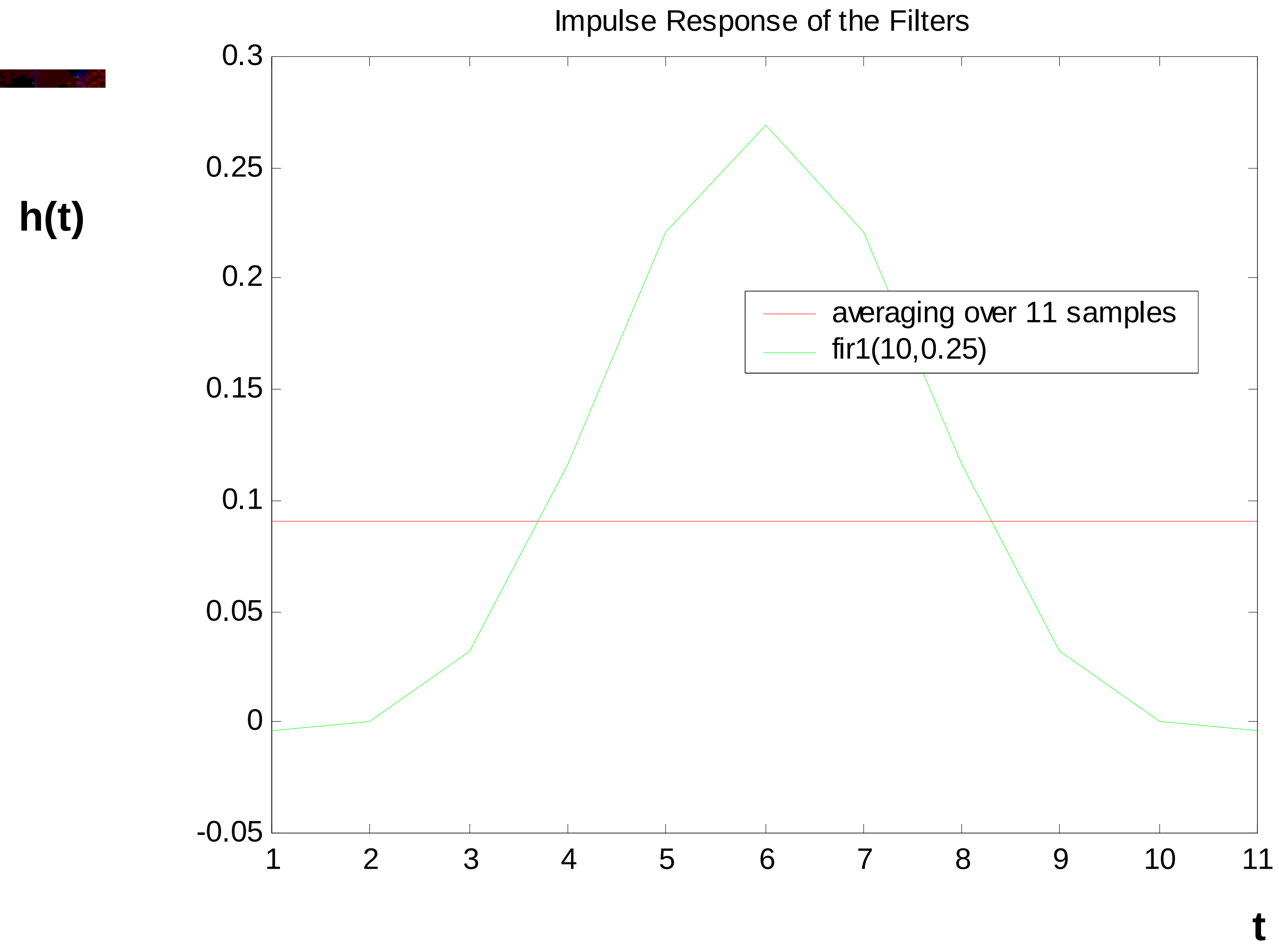
Filtering is done by a simple multiplication:

Y(f)= X(f) H(f)

H(f) is designed to magnify or reduce the magnitude (and possibly change phase) of the original signal at different frequencies.

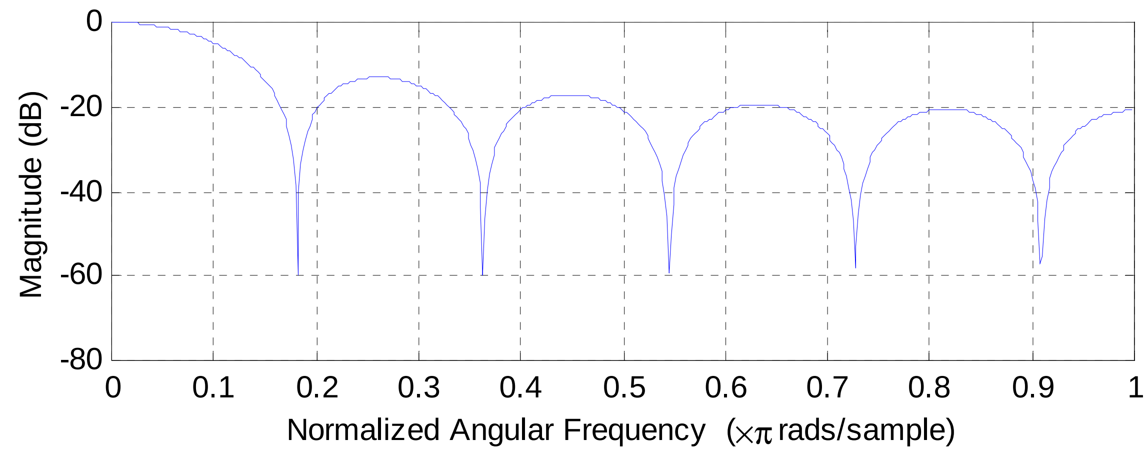
A pulse signal after low pass filtering (left) will have rounded corners.


The original pulse function and its low-passed versions
S(t)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
t
original
averaging over 11 samples
filter=fir1(10,0.25)

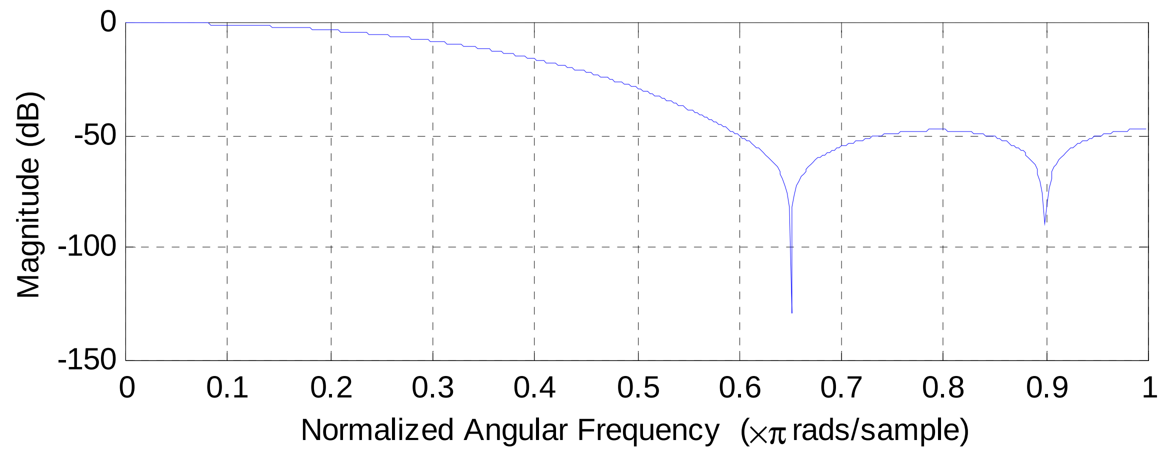

Impulse Response of the Filters
h(t)
t
averaging over 11 samples
fir1(10,0.25)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
1
2
3
4
5
6
7
8
9
10
11

Frequency Response of the Filters

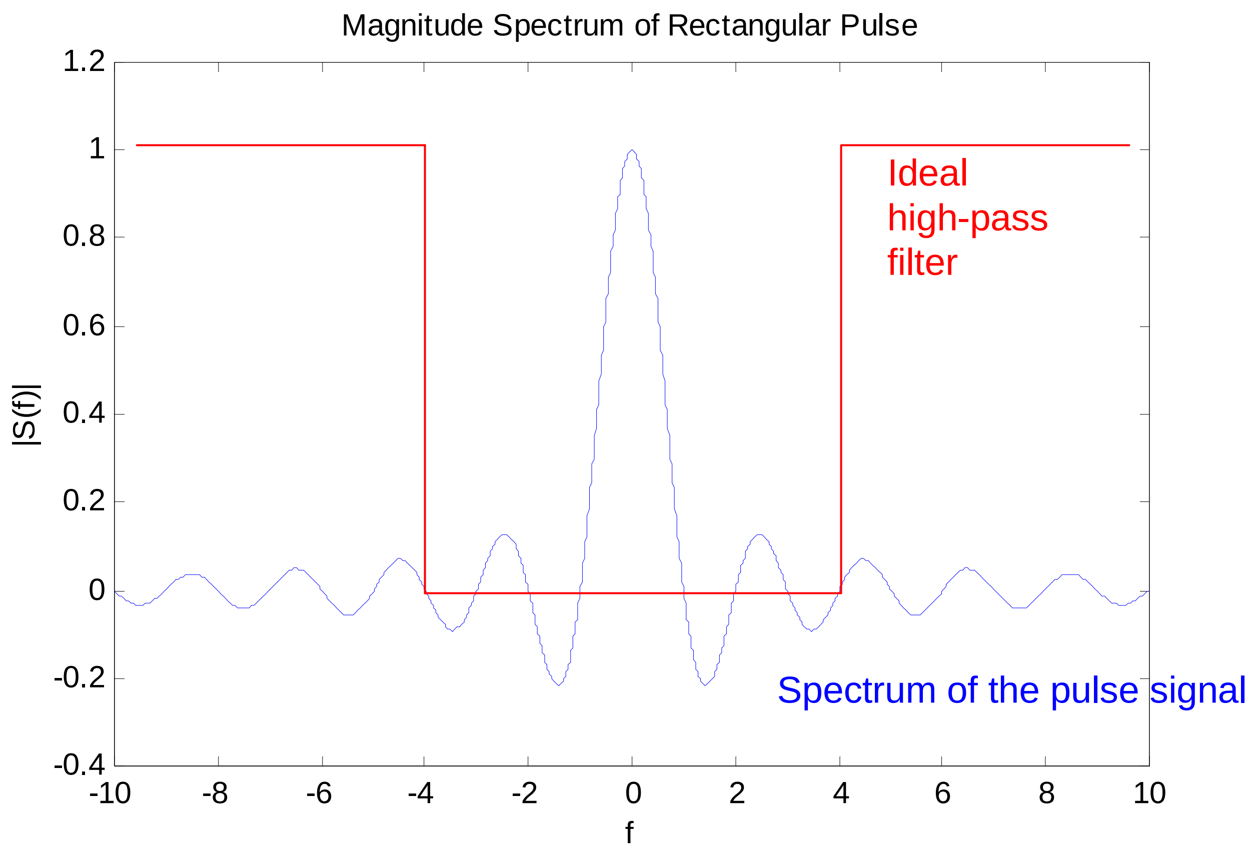
Averaging

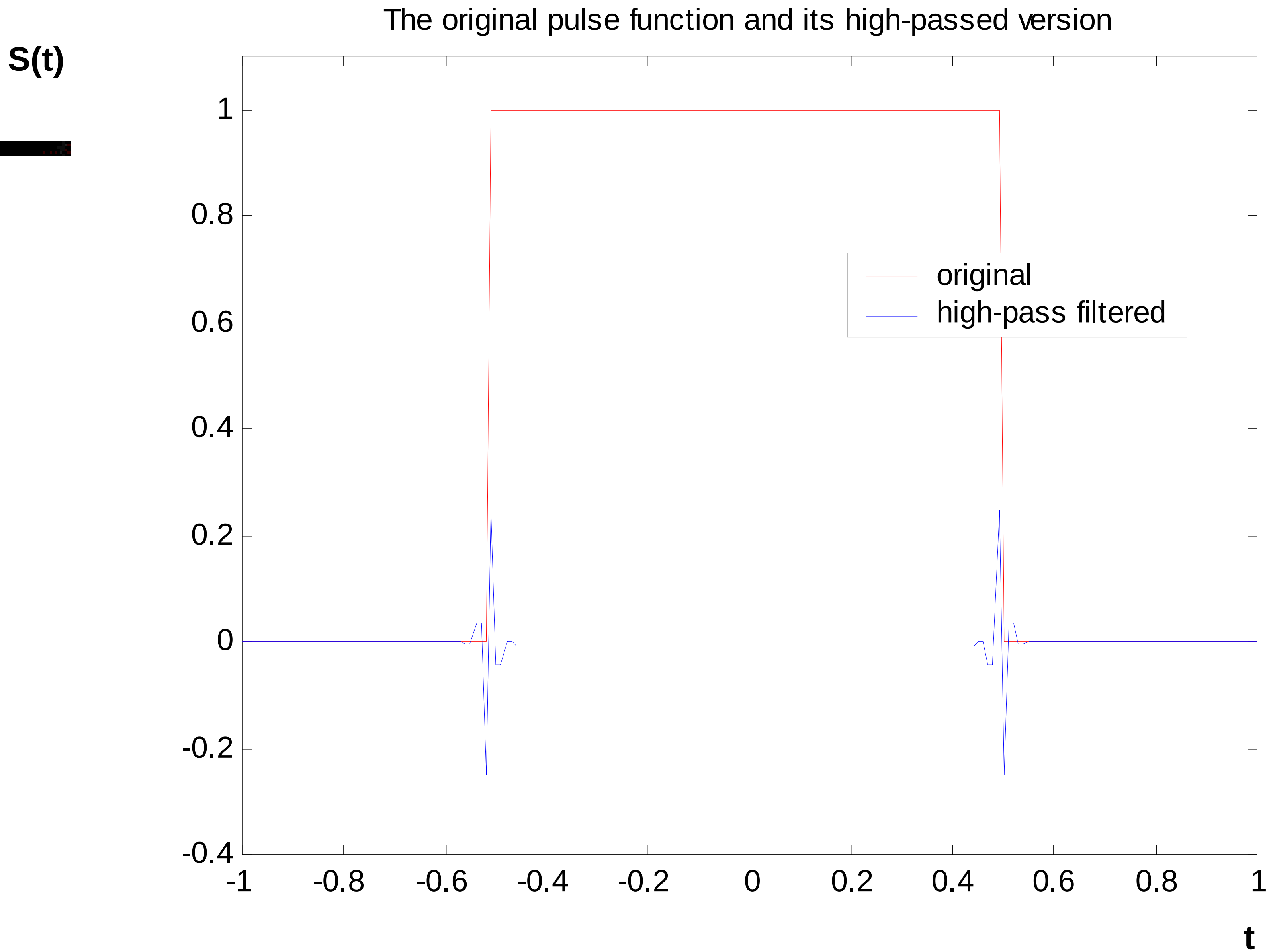


fir11(10,0.25)



High Pass Filtering (remove low freq, detect edges)



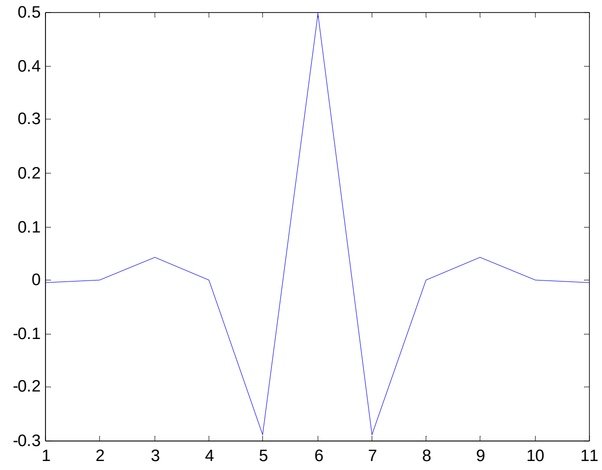

The original pulse function and its high-passed version
S(t)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
original
high-pass filtered
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
t

The High Pass Filter

fir1(10,0.5,'high');

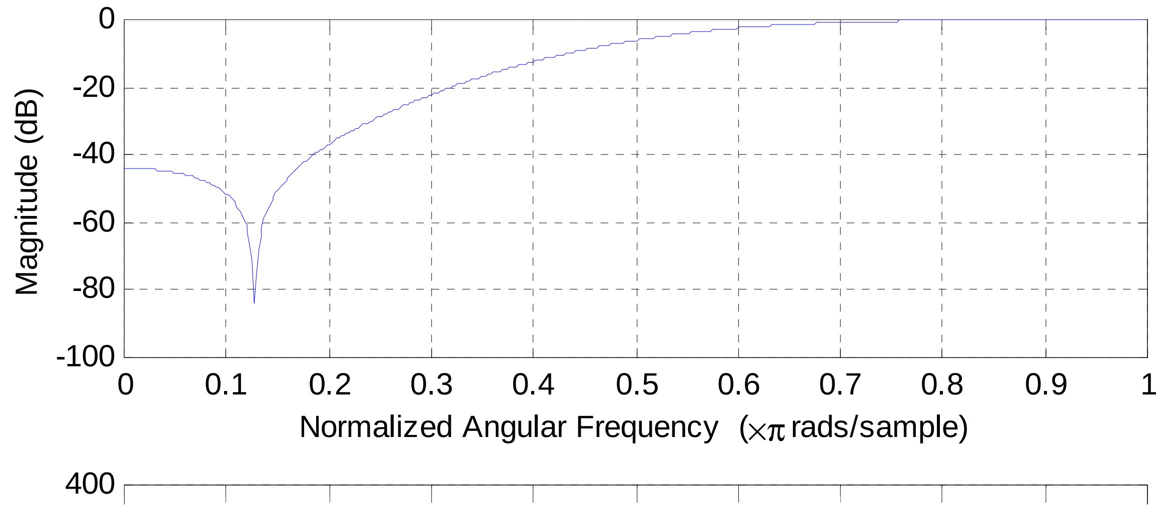
Impulse response:
Current sample –
neighboring samples

h(t)



t

Frequency response



Filtering in Temporal Domain (Convolution)

- Convolution theorem

$$X(f)H(f) \Leftrightarrow x(t) * h(t)$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

- Interpretation of convolution operation
  - replacing each pixel by a weighted sum of its neighbors
  - Low-pass: the weights sum = weighted average
  - High-pass: the weighted sum = left neighbors –right neighbors

Implementation of Filtering

- Frequency Domain
  – FT -> Filtering by multiplication with H(f) -> Inverse FT
- Time Domain
  – Convolution using a filter h(t) (inverse FT of H(f))

- You should understand how to perform filtering in frequency domain, given a filter specified in frequency domain
- Should know the function of the filter given H(f)
- Computation of convolution is not required for this lecture
- Filter design is not required.

What Should You Know (I)

- Sinusoid signals:
  - Can determine the period, frequency, magnitude and phase of a sinusoid signal from a given formula or plot
- Fourier series for periodic signals
  - Understand the meaning of Fourier series representation
  - Can calculate the Fourier series coefficients for simple signals (only require double sided)
  - Can sketch the line spectrum from the Fourier series coefficients
- Fourier transform for non-periodic signals
  - Understand the meaning of the inverse Fourier transform
  - Can calculate the Fourier transform for simple signals
  - Can sketch the spectrum
  - Can determine the bandwidth of the signal from its spectrum
  - Know how to interpret a spectrogram plot

What Should You Know (II)

- Speech and music signals
  - Typical bandwidth for both
  - Different patterns in the spectrogram
  - Understand the connection between music notes and sinusoidal signals
- Filtering concept
  - Know how to apply filtering in the frequency domain
  - Can interpret the function of a filter based on its frequency response
    - Lowpass -> smoothing, noise removal
    - Highpass -> edge detection, differentiator
    - Bandpass -> retain certain frequency band, useful for demodulation

References

- Oppenheim and Wilsky, Signals and Systems, Sec. 4.2-4.3 (Fourier series and Fourier transform)
- McClellan, Schafer and Yoder, DSP First, Sec. 2.2,2.3,2.5 (review of sinusoidal signals, complex number, complex exponentials)