EE3054
Signals and Systems

Amplitude Modulation

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LECTURE OBJECTIVES

- Review of FT properties
  - Convolution $\leftrightarrow$ multiplication
  - Frequency shifting

- Sinewave Amplitude Modulation
  - AM radio
  - Frequency-division multiplexing
  - FDM

- Reading: Chapter 12, Section 12-2
Table of Easy FT Properties

**Linearity Property**
\[ ax_1(t) + bx_2(t) \iff aX_1(j\omega) + bX_2(j\omega) \]

**Delay Property**
\[ x(t - t_d) \iff e^{-j\omega t_d} X(j\omega) \]

**Frequency Shifting**
\[ x(t)e^{j\omega_0 t} \iff X(j(\omega - \omega_0)) \]

**Scaling**
\[ x(at) \iff \frac{1}{|a|} X(j\frac{\omega}{a}) \]
Table of FT Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convolution</td>
<td>$x(t) * h(t) \iff H(j\omega)X(j\omega)$</td>
</tr>
<tr>
<td>Time Scaling</td>
<td>$x(t)p(t) \iff \frac{1}{2\pi} X(j\omega) * P(j\omega)$</td>
</tr>
<tr>
<td>Frequency Shifting</td>
<td>$x(t)e^{j\omega_0 t} \iff X(j(\omega - \omega_0))$</td>
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Differentiation Property

<table>
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</tr>
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<tbody>
<tr>
<td>Differentiation</td>
<td>$\frac{dx(t)}{dt} \iff (j\omega)X(j\omega)$</td>
</tr>
</tbody>
</table>
Convolution Property

- Convolution in the time-domain

\[ y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau \]

corresponds to **MULTIPLICATION** in the frequency-domain

\[ Y(j\omega) = H(j\omega)X(j\omega) \]
Cosine Input to LTI System

\[ Y(j\omega) = H(j\omega)X(j\omega) \]

\[ = H(j\omega)[\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)] \]

\[ = H(j\omega_0)\pi\delta(\omega - \omega_0) + H(-j\omega_0)\pi\delta(\omega + \omega_0) \]

\[ y(t) = H(j\omega_0)\frac{1}{2} e^{j\omega_0 t} + H(-j\omega_0)\frac{1}{2} e^{-j\omega_0 t} \]

\[ = H(j\omega_0)\frac{1}{2} e^{j\omega_0 t} + H^*(j\omega_0)\frac{1}{2} e^{-j\omega_0 t} \]

\[ = \left| H(j\omega_0) \right| \cos(\omega_0 t + \angle H(j\omega_0)) \]
Ideal Lowpass Filter

\( H_{lp}(j\omega) \)

\[ x(j\omega) = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0) \]

\[ y(t) = x(t) \quad \text{if} \quad \omega_0 < \omega_{co} \]

\[ y(t) = 0 \quad \text{if} \quad \omega_0 > \omega_{co} \]
Ideal LPF: Fourier Series

\[ H(j\omega) = \begin{cases} 
1 & |\omega| < \omega_{co} \\
0 & |\omega| > \omega_{co} 
\end{cases} \]

\[ f_{co} \quad \text{"cutoff freq."} \]

\[ y(t) = \frac{4}{\pi}\sin(50\pi t) + \frac{4}{3\pi}\sin(150\pi t) \]
Frequency Shifting Property

\[ x(t)e^{j\omega_0 t} \iff X(j(\omega - \omega_0)) \]

\[ x(t)\cos(\omega_c t) \iff Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c)) \]
The way communication systems work

How do we share bandwidth?
\textbf{Amplitude Modulator}

- \( x(t) \) modulates the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of \( X(j\omega) \).

\[
\begin{align*}
Y(j\omega) &= \frac{1}{2} X(j(\omega - \omega_c)) \\
&\quad + \frac{1}{2} X(j(\omega + \omega_c))
\end{align*}
\]
\[ y(t) = x(t) \cos(\omega_c t) \iff \]
\[ Y(j \omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c)) \]

\[ x(t) = \begin{cases} 
1 & |t| < T \\
0 & |t| > T 
\end{cases} \iff X(j \omega) = 2 \frac{\sin(\omega T)}{\omega} \]

\[ y(t) = x(t) \cos(\omega_c t) \iff \]
\[ Y(j \omega) = \frac{\sin((\omega - \omega_c)T)}{(\omega - \omega_c)} + \frac{\sin((\omega + \omega_c)T)}{(\omega + \omega_c)} \]
\[ y(t) = x(t) \cos(\omega_c t) \iff \]

\[ Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c)) \]
DSBAM Modulator

\[ x(t) \]
\[ X(j\omega) \]
\[ \cos(\omega_c t) \]
\[ y(t) = x(t)\cos(\omega_c t) \]

\[ Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) \]
\[ + \frac{1}{2} X(j(\omega + \omega_c)) \]

- If \( X(j\omega)=0 \) for \( |\omega| > \omega_b \) and \( \omega_c > \omega_b \), the result in the frequency-domain is two shifted and scaled exact copies of \( X(j\omega) \).
DSBAM Waveform

- In the time-domain, the “envelope” of sine-wave peaks follows $|x(t)|$
Double Sideband AM (DSBAM)

"Typical" bandlimited input signal

Frequency-shifted copies

Upper sideband

Lower sideband

$0.5X(j(\omega + \omega_c))$

$0.5X(j(\omega - \omega_c))$

$X(j\omega)$

$Y(j\omega)$
DSBAM DEModulator

\[ w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos(2\omega_c t) \]

\[ W(j\omega) = \frac{1}{2} X(j\omega) + \frac{1}{4} X(j(\omega - 2\omega_c)) + \frac{1}{4} X(j(\omega + 2\omega_c)) \]

\[ V(j\omega) = H(j\omega) W(j\omega) \]
DSBAM Demodulation

\[ V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \quad \text{if} \quad \omega_b < \omega_{co} < 2\omega_c - \omega_b \]
Frequency-Division Multiplexing (FDM)

- Shifting spectrum of signal to higher frequency:
  - Permits transmission of low-frequency signals with high-frequency EM waves
  - By allocating a frequency band to each signal multiple bandlimited signals can share the same channel
  - AM radio: 530-1620 kHz (10 kHz bands)
  - FM radio: 88.1-107.9 MHz (200 kHz bands)
FDM Block Diagram (Xmitter)

Spectrum of inputs must be bandlimited
Need $|\omega_{c2} - \omega_{c1}| > 2\omega_b$
Frequency-Division De-Mux

\[ Y(j\omega) \]

\[-\omega_{c2} \quad -\omega_{c1} \quad 0 \quad \omega_{c1} \quad \omega_{c2} \]

\[ \cos(\omega_{c1}t) \]

\[ \cos(\omega_{c2}t) \]

Demultiplexer

Demodulators
Bandpass Filters for De-Mux

(a) $H_{bp1}(j\omega)$

- $(\omega_1 - \omega_b)$
- $(\omega_1 + \omega_b)$
- $(\omega_2 - \omega_b)$
- $(\omega_2 + \omega_b)$

(b) $H_{bp2}(j\omega)$

- $(\omega_1 - \omega_b)$
- $(\omega_1 + \omega_b)$
- $(\omega_2 - \omega_b)$
- $(\omega_2 + \omega_b)$
QAM

- Modulate two signals using the same carrier freq but orthogonal in phase
- Go through in class: modulator and demodulator
- Application in color TV transmission
Quadrature Amplitude Modulation (QAM)

- A method to modulate two signals onto the same carrier frequency, but with 90° phase shift
QAM in more detail

Proof (in time and freq. domain) the demodulator can separate the signal on board!
Multiplexing of Luminance and Chrominance

\[
\sum \sim A \cos(2\pi f_c t)
\]

\[\text{Gate} \quad \text{Color burst signal}\]

\[\text{BPF 2-4.2MHz} \quad \sum \quad \text{Composite video}\]

\[\text{LPF 0-0.5MHz} \quad \text{LPF 0-1.5MHz} \quad \text{LPF 0-4.2MHz} \]

\[Y(t) \quad I(t) \quad Q(t)\]
DeMultiplexing of Luminance and Chrominance

Composite video → Comb Filter 0-4.2MHz → ∑ → + → LPF 0-1.5MHz → Y(t)

Composite video → Gate → 2Acos(2πf₀t) → ~ → -π/2 → LPF 0-0.5MHz → Q(t)

Horizontal sync signal

Gate
Phase comparator
Voltage controlled oscillator
Multiplexing of luminance, chrominance and audio

- Luminance
- I
- I and Q
- Audio

- Picture carrier
- Color subcarrier
- Audio subcarrier

- $f_p$
- $f_c$
- $f_a$

- 6.0 MHz
- 4.5 MHz
- 4.2 MHz
- 3.58 MHz
- 1.25 MHz
Luminance/Chrominance Separation

- In low-end TV receivers, a low pass filter with cut-off frequency at 3MHz is typically used to separate the luminance and chrominance signal.
  - The high frequency part of the I component (2 to 3 Mhz) is still retained in the luminance signal.
  - The extracted chrominance components can contain significant luminance signal in a scene with very high frequency (luminance energy is not negligible near $f_c$)
  - These can lead to color bleeding artifacts
- For better quality, a comb filter can be used, which will filter out harmonic peaks correspond to chrominance signals.
- Show example of comb filter on board
What will a Monochrome TV see?

- The monochrome TV receiver uses a LPT with cut-off at 4.2 MHz, and thus will get the composite video (baseband luminance plus the I and Q signal modulated to $f_c = 3.58$ MHz)
  - Because the modulated chrominance signal is at very high frequency (227.5 cycles per line), the eye smoothes it out mostly, but there can be artifacts
  - The LPF in Practical TV receivers have wide transition bands, and the response is already quite low at $f_c$. 
Composite Video Viewed as a Monochrome Image w/o filtering

Original Y

Composite Signal as Y

On the right is what a B/W receiver will see if no filtering is applied to the baseband video signal
Recovered Y with Filtering

Original Y

Recovered Y

On the right is what a B/W receiver will see if a lowpass filter with cutoff frequency at about 0.75 MHz is applied to the baseband video signal. This is also the recovered Y component by a color receiver if the same filter is used to separate Y and QAM signal.
Color TV Broadcasting and Receiving

RGB ---+ YC1C2

Luminance, Chrominance, Audio Multiplexing

Modulation

De-Multiplexing

De-Modulation

RGB --> YC1C2

---+ Luminance, Chrominance, Audio Multiplexing

Modulation

De-Multiplexing

De-Modulation
Transmitter in More Details

RGB to YIQ conversion:
- R(t)
- G(t)
- B(t)

Y(t) and I(t) are processed through LPFs:
- LPF 0-4.2MHz
- LPF 0-1.5MHz
- LPF 0-0.5MHz

Audio signal is processed through an FM modulator at 4.5MHz.

Color burst signal:
- $A\cos(2\pi f_c t)$

Gate with $-\pi/2$ phase shift.

BPF 2-4.2MHz.

Summation for VSB signal.

To transmit antenna.
Receiver in More Details

- **Composite video**
- **Comb Filter 0-4.2MHz**
- **LPF 0-1.5MHz**
- **LPF 0-0.5MHz**
- **Gate**
- **2Acos(2πct) ~**
- **Phase comparator**
- **Voltage controlled oscillator**
- **Y(t)**
- **I(t)**
- **Q(t)**
- **YIQ to RGB conversion**
- **BPF, 4.4-4.6MHz**
- **BPF, 0.4-2 MHz**
- **VSB Demodulator**
- **From antenna**
- **Horizontal sync signal**
- **To CRT**
- **To speaker**
- **Audio**

**FM demodulator**

**From antenna**

**BPF, 4.4-4.6MHz**

**BPF, 0.4-2 MHz**

**VSB Demodulator**

**Gate**

**2Acos(2πct) ~**

**Phase comparator**

**Voltage controlled oscillator**

**Y(t)**

**I(t)**

**Q(t)**

**YIQ to RGB conversion**

**To CRT**

**To speaker**

**Audio**
Readings

- Sec. 12.2