

# ADAPTIVE MODULATION FOR CODED COOPERATIVE SYSTEMS

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## ABSTRACT

Throughput plays a key role in data communications over a wireless link. In this paper, we use adaptive modulation for a cooperative relaying system with the goal of maximizing the data throughput. We compare the throughput performance of three different schemes with adaptive modulation: Direct transmission, multihop and coded cooperation. We find that cooperation with adaptive modulation provides large throughput improvement over multihop and direct transmissions. We also observe that to maximize the data throughput in direct transmission and multihop, the modulation mode for each hop only depends on the channel quality from the transmitter to the receiver. However, in coded cooperation a system wide optimization is necessary.

## 1. INTRODUCTION

Cooperative communication enables a relaying node to process signals overheard from a source and to retransmit to the destination. The destination combines signals from the source and the relay, thus creating a form of spatial diversity. It has been shown in [1–5] that cooperation provides full diversity and improves the system performance in terms of outage probability or frame error rate (FER). In most of the past research, the cooperating nodes use a fixed modulation mode that is same across nodes.

In wireless services, high data rate is one of main design considerations. Nodes in 802.11 systems can transmit their data at multiple rates [6]. Furthermore, all the nodes are allowed to adapt their data rates to match the channel conditions such that the throughput is maximized [7]. Motivated by this fact, we consider adaptive modulation for coded cooperative systems. The goal is to maximize the data throughput. As multihop also uses relays, but only to mitigate path loss, we consider multihop with adaptive modulation as well in this paper. We analyze the throughput of coded cooperation with adaptive modulation and multihop with adaptive modulation, and compare the two schemes

with direct transmission. We investigate how average channel qualities affect the source and the relay node in their selection of the modulation rates. In multihop and direct transmissions, the modulation mode selection only depends on a node's average received signal to noise ratio (SNR) at its receiver. However, in cooperative systems, all three links, namely source to relay, source to destination and relay to destination, affect the choice of modulation. We find that in low and medium SNR, multihop with adaptive modulation increases the data throughput compared with direct transmission. However, when SNR is high enough for direct transmission to use the highest order modulation, direct transmission outperforms multihop in terms of throughput. Coded cooperation with adaptive modulation has much better throughput performance than the previous two schemes, providing as much as a factor of two throughput gain over direct transmission.

The paper is organized as follows. In the next section, we present the system model. Section 3 provides analytical formulation for the throughput of cooperative, multihop and direct cases. In Section 4, we give the numerical results on the throughput performance of the three transmission schemes and illustrate how the throughput depends on the users' channel qualities. Section 5 concludes our work.

## 2. SYSTEM MODEL

We assume that all nodes are implemented with one transmitter and receiver antenna. We let  $S$  be the source,  $R$  be the relay and  $D$  be destination as in Fig. 1. We denote  $\gamma_1$  and  $\gamma_2$  as the average received SNR at destination from  $S$  and  $R$  respectively, and  $\gamma_{12}$  as the average received SNR for the between  $S$  and  $R$ . Fig. 1 also shows corresponding distances,  $D_1$ ,  $D_2$  and  $D_{12}$ . We consider three different schemes: Direct transmission, multihop and coded cooperation. We assume each channel block (or packet) is used to send  $B$  data bits. When channel coding is applied, this results in  $N$  coded bits per packet. We consider a system in which the average transmitted energy per symbol is upper bounded by  $E_s$ .

In direct transmission,  $S$  transmits  $N$  coded bits to des-

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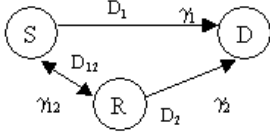


Fig. 1. Example of a relaying system.

tion directly with the transmit energy  $E_s$  per symbol. In multihop, the packet is first transmitted from  $S$  to  $R$ . Upon successful reception, the relay forwards the packet to the destination. In multihop, all the packets are transmitted through two hops to mitigate path loss. For fair comparison, we assume that  $S$  and  $R$  use half transmit energy individually, that is  $\frac{E_s}{2}$  per symbol. In coded cooperation, we use the framework of [4]:  $S$  transmits half of coded bits which are overhead by the relay,  $R$ , and the destination. If  $R$  decodes the information bit correctly (which can be checked by using Cyclic Redundancy Check (CRC)), it encodes the information and sends the remaining bits. The destination combines signals from  $S$  and  $R$ , thus creating spatial diversity. If the inter-user channel is not reliable,  $S$  continues transmission by sending the remaining coded bits. Note that in coded cooperative transmission, if cooperation takes place,  $S$  and  $R$  send half of each packet to destination separately. Hence, the transmit energy per symbol in this scheme is also assumed to be  $E_s$ .

Each link exhibits path loss and independent flat Rayleigh fading. The average received SNR,  $\gamma_i$  is proportional to  $D_i^{-\alpha}$ , where  $\alpha$  is the path loss exponent. We assume a slowly fading channel that stays constant during a single transmission of a packet, but re-transmissions of the same packet and consecutive packets observe independent fading. This can be guaranteed by interleaving. Hence, for direct and multihop transmission, each transmission observes a quasi-static channel, whereas cooperative coding results in a block fading environment.

For adaptive modulation, we have three modulation modes: BPSK, QPSK and 16-QAM. Our results can simply be extended to include 64-QAM in IEEE 802.11 [6]. We let  $M_1$  and  $M_2$  denote the number of bits per symbol sent by  $S$  and  $R$  respectively when cooperation protocol is performed, and  $N_1$  is the number of bits per symbol transmitted by  $S$  for direct transmission. We assume that  $K_1$  and  $K_2$  are the number of bits per symbol when  $S$  communicates  $R$  and  $R$  communicates destination respectively in multihop. Hence,  $M_1, M_2, N_1, K_1, K_2 \in \{1, 2, 4\}$ . The three transmission schemes are illustrated in Fig. 2. Note that because each scheme may use a different mode, transmission of each packet can take a different amount of time. For example, in multihop since each hop spans a shorter distance, a higher order modulation can be used, leading to a shorter hop time

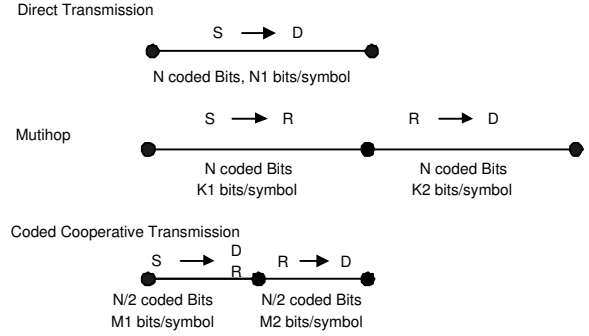


Fig. 2. Three transmission schemes than direct.

### 3. THROUGHPUT ANALYSIS

In this section, we derive the analytical formulas for the throughput of the three transmission schemes. Then we compare these throughput performances and investigate how channel qualities affect the modulation mode selection in each scheme.

Our analysis includes the following assumptions:

- The receivers use CRC to detect all errors in each packet and the probability of undetected transmission error is negligible.
- When the receiver detects an error in a packet, the system uses ARQ to resend the packet until the packet is successfully delivered to the receiver.
- There is no error in the transmission of acknowledgments from the receiver to the transmitter and this transmission is instantaneous.
- The symbol transmission rate for each user is fixed as  $R_s$  symbols/sec and each user uses the same convolutional code with rate  $R_c$ .
- Each packet contains  $B$  data bits. Here we ignore the overhead bits, i.e., CRC bits, as this number is very small compared with the number of data bits. Therefore, each frame/packet consists of  $N$  coded bits, where  $N = \frac{B}{R_c}$ .
- The throughput is defined as the number of payload bits per second received correctly [8]:  $\mathcal{T} = \frac{B}{E(T)}$ , where  $E(T)$  is the total expected time for each packet to be received at the destination successfully.

**Direct Transmission:**  $S$  transmits each packet directly to the destination. The packet is resent from  $S$  until the destination receives it correctly. Assume that the average FER for the quasi-static channel from  $S$  to the destination in direct transmission is  $P_{d,1}^{QS}$ . Using the fact that each retransmission observes independent fading, it is easy to see that on the average it takes  $\frac{1}{(1-P_{d,1}^{QS})}$  number of re-transmissions for the packet to correctly reach the destination. The total

time for each transmission is  $\frac{B}{N_1 R_s R_c}$  seconds. Then for direct transmission  $E(T) = \frac{1}{(1-P_{d,1}^{QS})} \frac{B}{N_1 R_s R_c}$ . Therefore, the data throughput of  $S$  for direct transmission can be written as:

$$\mathcal{T}_{direct} = \frac{B}{N_1 R_s R_c \frac{1}{(1-P_{d,1}^{QS})}} = N_1 R_s R_c (1 - P_{d,1}^{QS}) \quad (1)$$

Note that  $P_{d,1}^{QS}$  is a function of  $N_1$  and  $\gamma_1$ . For higher order modulation,  $P_{d,1}^{QS}$  increases, but so does the multiplicative factor in  $\mathcal{T}_{direct}$ . Hence, there exists an optimal modulation scheme  $N_1$  which depends on the average received SNR from  $S$  at the destination.

**Multihop:**  $S$  re-transmits the coded packet to  $R$  until the packet is successfully received by  $R$ . Then  $R$  relays the packet to the destination. If there is an error in the received packet at the destination, only  $R$  re-transmits. We let  $P_{m,12}^{QS}$  and  $P_{m,2}^{QS}$  denote the average FER of the channel code for the quasi-static channel from  $S$ -to- $R$  and from  $R$ -to-destination in multihop respectively. For the multihop scheme, it takes on the average  $\frac{1}{(1-P_{m,12}^{QS})}$  transmissions in the first hop (from  $S$  to  $R$ ) and  $\frac{1}{(1-P_{m,2}^{QS})}$  transmissions in the second hop (from  $R$  to the destination) to get one packet through. Hence, on the average, the first hop transmission takes  $\frac{B}{(1-P_{m,12}^{QS})K_1 R_s R_c}$  seconds and the second hop takes  $\frac{B}{(1-P_{m,2}^{QS})K_2 R_s R_c}$  seconds. Summing these up from  $S$  to  $D$ , we get  $E(T)$  and the data throughput of  $S$  in multihop is:

$$\mathcal{T}_{mhop} = \frac{R_s R_c}{\left( \frac{1}{(1-P_{m,12}^{QS})K_1} + \frac{1}{(1-P_{m,2}^{QS})K_2} \right)} \quad (2)$$

We find from the above equation that as  $P_{m,12}^{QS}$  and  $P_{m,2}^{QS}$  depend on the channel quality from  $S$  to  $R$  and from  $R$  to the destination separately,  $S$  and  $R$  adapt their modulation rates  $K_1$  and  $K_2$  based on channel qualities of  $S$ - $R$  and  $R$ - $D$  independently.

**Coded Cooperative Transmission:** The flowchart for cooperative scheme is shown in Fig. 3. If  $R$  can decode the information sent by  $S$  in the first transmission of the packet,  $S$  and  $R$  cooperatively transmit to the destination until the destination gets the packet correctly, which is the cooperative mode. Even though  $R$  may have a higher average SNR than  $S$  towards the destination, cooperative transmission coming from both  $S$  and  $R$  results in a lower FER than a retransmission from  $R$  only due to increased spatial diversity. If  $R$  cannot decode  $S$ 's information in the first transmission of each packet,  $S$  continues by transmitting the remaining coded bits and all re-transmissions will come directly from  $S$ . We call this the non-cooperative mode. Note that we do not allow for  $R$  to listen to subsequent retransmission of  $S$  and start the cooperative mode later on.

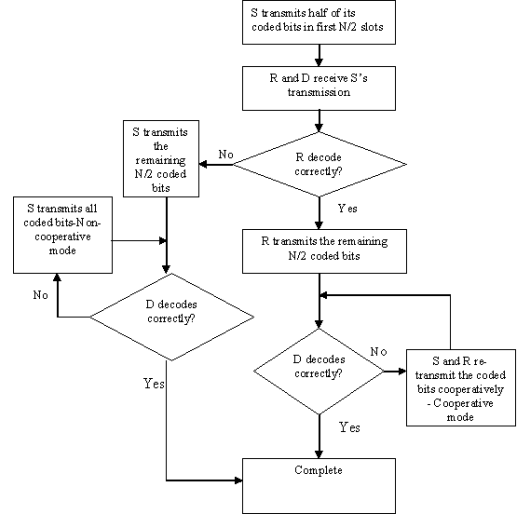


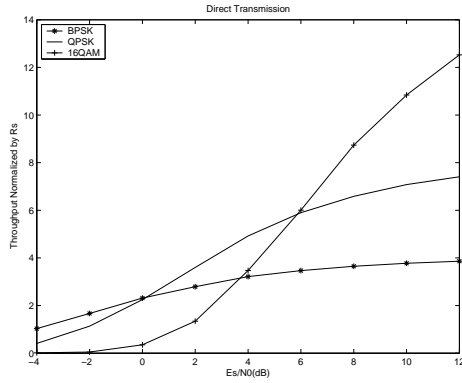
Fig. 3. Flowchart of Coded Cooperative Transmission.

While this may limit the performance, it allows us to free up the relay resources quickly. We will observe in Section 4 that we still enjoy the benefits of cooperation.

In order to calculate the throughput, we let  $P_{c,12}^{QS}$  denote the average FER of the first half channel code for the quasi-static channel of  $S$ -to- $R$ . Similarly,  $P_{c,1}^{QS}$  denotes the average FER for the quasi-static  $S$ -to- $D$  channel when  $S$  transmits the whole packet by itself and  $P_c^{BFF}$  denotes the average FER for the cooperative block fading channel when destination receives half of the packet from  $S$  and the remaining half from  $R$ . When  $S$  and  $R$  transmit cooperatively, which happens with probability  $(1 - P_{c,12}^{QS})$ , we need an average of  $\frac{1}{(1-P_c^{BFF})}$  retransmissions, with each transmission of a packet taking  $\frac{B}{2M_1 R_s R_c} + \frac{B}{2M_2 R_s R_c}$  seconds. When  $S$  transmits by itself, which happens with probability  $P_{c,12}^{QS}$ , we need an average of  $\frac{1}{(1-P_{c,1}^{QS})}$  retransmissions, with each transmission taking  $\frac{B}{M_1 R_s R_c}$  seconds. Therefore, for coded cooperation  $E(T) = (1 - P_{c,12}^{QS}) \frac{1}{(1 - P_c^{BFF})} \left( \frac{B}{2M_1 R_s R_c} + \frac{B}{2M_2 R_s R_c} \right) + P_{c,12}^{QS} \frac{1}{(1 - P_{c,1}^{QS})} \frac{B}{M_1 R_s R_c}$ . Then we get the throughput of the cooperative scheme as:

$$\mathcal{T}_{coop} = \frac{R_s R_c}{\frac{(1 - P_{c,12}^{QS})}{(1 - P_c^{BFF})} \left( \frac{1}{2M_1} + \frac{1}{2M_2} \right) + \frac{P_{c,12}^{QS}}{1 - P_{c,1}^{QS}} \frac{1}{M_1}} \quad (3)$$

Comparing the direct transmission and cooperative transmission schemes,  $P_{c,1}^{QS}$  is not necessarily equal to  $P_{d,1}^{QS}$  as  $S$  may have a different modulation scheme for direct transmission and cooperative scheme. Note that in multihop,  $S$  transmits all the coded bits to  $R$  with transmit energy  $\frac{E_s}{2}$  per symbol, but in coded cooperation,  $S$  sends only half of the coded bits to  $R$  with transmit energy  $E_s$  per symbol. Hence,



**Fig. 4.** Throughput of direct transmission.

$P_{m,12}^{QS}$  is different from  $P_{c,12}^{QS}$ . We observe from Eqn. (3) that the throughput of  $S$  in coded cooperative system depends on  $P_c^{BF}$ ,  $P_{c,12}^{QS}$  and  $P_{c,1}^{QS}$ , with the FER's depending on all three link SNRs,  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_{12}$ . Therefore, to optimize  $\mathcal{T}_{coop}$ ,  $S$  and  $R$  should base their modulation modes not only on their own channel quality to the destination but on all these links. We illustrate this in the next section.

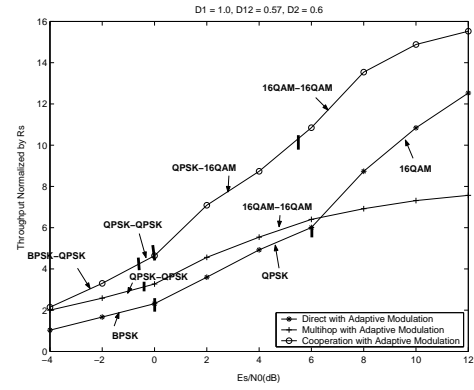
#### 4. ADAPTIVE MODULATION EXAMPLES

In this section, we present numerical calculations of the throughput formulas of Section 3.

We use rate 1/4 [5, 7, 5, 7] convolutional code as the channel code in all transmission schemes. The path loss exponent,  $\alpha$  is 4. We first assume the normalized distances  $D_1 = 1.0$ ,  $D_2 = 0.6$  and  $D_{12} = 0.57$  and SNR per symbol is  $\frac{E_s}{N_0}$ . Hence, for the direct transmission and cooperation, the received SNR at the destination from  $S$  is  $\gamma_1 = \frac{E_s}{N_0} + 10 \log_{10}(D_1^{-\alpha}) = \frac{E_s}{N_0}$  and the received SNR from  $R$  at the destination is  $\gamma_2 = \frac{E_s}{N_0} + 10 \log_{10}(D_2^{-\alpha}) = \frac{E_s}{N_0} + 8.87(\text{dB})$ . For multihop, the transmitter uses only half transmitted energy, the received SNR at  $R$  from  $S$  is  $\gamma_{m,12} = \frac{E_s}{N_0} + 10 \log_{10}(0.5) + 10 \log_{10}(D_{12}^{-\alpha}) = \frac{E_s}{N_0} + 6.87(\text{dB})$  and the received SNR at the destination from  $R$  is  $\gamma_{m,2} = \frac{E_s}{N_0} + 10 \log_{10}(0.5) + 10 \log_{10}(D_2^{-\alpha}) = \frac{E_s}{N_0} + 5.86(\text{dB})$ .

Fig. 4 illustrates how the throughput of direct transmission changes as a function of SNR per symbol and modulation mode. Vertical axis denotes  $\frac{\mathcal{T}}{R_s}$ , throughput normalized by symbol rate  $R_s$ . We can find from Fig. 4 that among BPSK, QPSK and 16QAM, when the received SNR is below 0 dB, direct transmission with BPSK has the highest throughput; when the received SNR is 0dB - 6dB, QPSK is preferred and when the received SNR is higher than 6 dB, 16QAM modulation is the best modulation.

Fig. 5 shows the normalized throughput of direct transmission, multihop and cooperative scheme utilizing the best modulation modes for each scheme. We find from Fig. 5



**Fig. 5.** Throughput of direct transmission, multihop and coded cooperation with adaptive modulation for  $D_1 = 1.0$ ,  $D_2 = 0.6$  and  $D_{12} = 0.57$ .

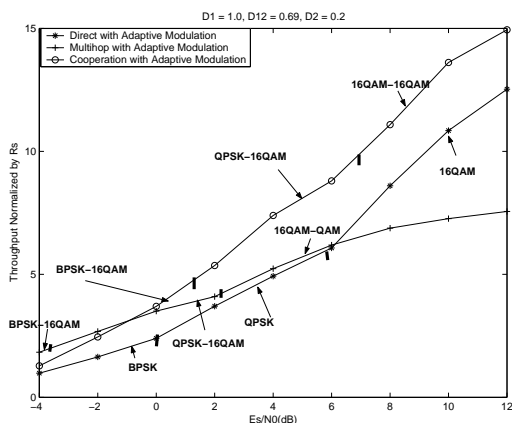
that coded cooperation with adaptive modulation leads to much higher throughput than multihop transmission with adaptive modulation and direct transmission with adaptive modulation. We find in this example that cooperation brings at least 20% throughput gain over direct transmission. When SNR is below 4dB, the gain is as much as 100%. The gains associated with cooperation are many-fold: Gain due to diversity at the receiver, gain due to channel coding cooperatively, gain due to multihop.

The gain purely due to multihop is also illustrated in Fig. 5. We find that multihop transmission with adaptive modulation, while inferior to cooperation, improves the throughput over direct transmission in low and medium SNR. But when SNR is high enough, the throughput of direct transmission is higher than that of multihop transmission. There are mainly two reasons for this behavior. The first is the fact that we only have a finite set of modulation modes. For our example, even when the received SNR in every hop is very high, the largest modulation mode is 16QAM with limited throughput. Secondly, when SNR increases, all the FER's decrease. This results in smaller difference of FER between the direct transmission and each hop of multihop. In other words, the path loss does not have a significant effect in FER performance when SNR is high enough. Therefore, multihop transmission does not have any advantage over direct transmission in terms of throughput for high SNR.

From Fig. 5, we find that as illustrated in Section 3, in multihop, the optimum of  $K_1$  and  $K_2$  are determined by the channel qualities of  $S$ -to- $R$  and  $R$ -to- $D$  separately. When  $\gamma_{m,12}$  is from  $-4 + 6.86 = 2.86\text{dB}$  to  $-2 + 6.86 = 4.86\text{dB}$  and  $\gamma_{m,2}$  is from  $-4 + 5.86 = 1.86\text{dB}$  to  $-2 + 5.86 = 3.86\text{dB}$ ,  $S$  and  $R$  both select QPSK to maximize the throughput. As SNR increases, i.e for  $\frac{E_s}{N_0} > 0\text{dB}$ , 16QAM becomes the best modulation. Unlike multihop, in coded cooperation, the throughput is maximized when  $S$  and  $R$  jointly adapt their modulation modes. For example, when  $\frac{E_s}{N_0} = 0\text{dB}$  which results in  $\gamma_1 = 0\text{dB}$  and  $\gamma_2 = 8.87\text{dB}$ , if

$S$  and  $R$  just base their modulation on their respective channel qualities to the destination, they would choose BPSK and QPSK. Note that in cooperative mode of coded cooperation,  $S$  and  $R$  transmit half of the coded bits to the destination. Then their best modulation mode for direct transmission is based on the channel code with rate 1/2, whose throughput performance is not shown in here due to space limitations. However, we find from Fig. 5 that QPSK-QPSK used by  $S$  and  $R$  respectively can give the system its highest throughput. When  $\frac{E_s}{N_0} = 2\text{dB}$ , or  $\gamma_1 = 2\text{dB}$  and  $\gamma_2 = 10.87\text{dB}$ , BPSK and 16QAM at  $S$  and  $R$  would be the best modulation modes separately. But we find that QPSK-16QAM is the best  $S$ - $R$  modulation mode combination.

Results for  $D_1 = 1.0$ ,  $D_2 = 0.2$  and  $D_{12} = 0.69$  illustrated in Fig. 6 show a similar trend. We observe that in low SNR, since the distance between  $S$  and  $R$  is large, the inter-user channel is not very reliable and the cooperative mode in coded cooperation occurs less often. Therefore, the throughput of cooperative transmission is less than that of multihop in low SNR. We expect cooperation to perform better with a more sophisticated cooperative protocol, in which we allow the relay to listen to retransmissions of the source and use the cooperative mode once it decodes correctly. When SNR increases, cooperative mode occurs more frequently and the diversity gain is dominant. For  $\frac{E_s}{N_0} \geq 0\text{dB}$ , the throughput of cooperative transmission is higher than that of multihop. Also since now the relay is close to the destination, it always uses the highest order modulation, 16QAM, both in cooperative and multihop modes.



**Fig. 6.** Throughput of direct transmission, multihop and coded cooperation with adaptive modulation for  $D_1 = 1.0$ ,  $D_2 = 0.2$  and  $D_{12} = 0.69$ .

## 5. CONCLUSION

In this paper, we propose cooperative schemes incorporating adaptive modulation at both the source and the relay

and investigate its effect on the net throughput of the link. We derive analytical throughput expression for direct transmission, multihop and coded cooperative transmission. We investigate how channel qualities affect the throughput performance of the three schemes and find that joint adaptation of modulation modes is essential to maximize the throughput of the cooperative system. From the simulation results, we find that adaptive modulation coupled with diversity and multihop gains provided by cooperation improves the throughput well over direct transmission and multihop. Future work includes a study of variable code rates and a more adaptive cooperative protocol in which the relay updates its estimate of the source signal at each retransmission.

## 6. ACKNOWLEDGMENT

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