

Cooperative Coding for Wireless Networks

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Abstract—User-cooperation represents an effective way of introducing diversity in wireless networks. Spatial diversity gains are obtained through the cooperation of mobile users and the use of the partner’s antenna. In this paper, we design channel codes that are capable of achieving the full diversity provided by user-cooperation, with the constraint that they also provide the best possible performance in the non-cooperative case. The codes continue to perform well, even when the inter-user channel is noisy, still offering significant improvements with respect to the non-cooperative case.

I. INTRODUCTION

Information transfer through wireless networks involves simultaneous communication among multiple source-destination pairs. In cellular systems, coordination of these multiple communications is done via the base station. The base station processes all the signals transmitted from the sources (uplink) and forwards them to their respective destinations (downlink). Ad-hoc networks on the other hand do not have a fixed infrastructure and utilize other mobiles as relays to transfer information from a source to its destination. Motivated by the diversity effects and power efficiency of communicating via relaying, recent research efforts have also focused on the use of multi-hops/ad-hoc mode in the cellular architecture [3], [4], [5], [6].

In the context of network information theory, the use of relays was first considered by Van der Meulen [1]. Capacity for the degraded relay channel was found by Cover and El Gamal [2]. The classical relay model consists of a source who wishes to communicate with a destination through the use of a relay. The relay receives the signal transmitted by the source through a noisy channel, processes it and forwards to the destination. The destination observes a superposition of the source and the relay transmissions. The relay does not have any information to send, hence the goal is to maximize the total rate of information flow from the source to the destination.

In wireless networks, unlike the classical relay model, the relay is another mobile who has his own information to transfer. One can then reformulate the problem as two or more mobiles, with each having its own data, cooperating through noisy fading links to achieve larger data rates [4]. Sendonaris *et. al.* [3], [4], [5] showed that cooperating mobiles in a wireless network provide not only higher data rates, but also decreased sensitivity to channel variations. One can obtain spatial diversity through the use of this “virtual” antenna array even if the mobiles are connected via noisy links. Laneman *et. al.* [6] showed that even when the relaying mobile cannot receive and transmit at the same time, cooperation is still beneficial in reducing the outage

probability. They considered different protocols to achieve diversity gains such as amplify and forward or decode and forward. However, from a coding perspective these protocols resemble repetition coding, and there are more effective ways of designing channel codes. Hunter and Nosratinia [7] proposed the use of rate compatible punctured convolutional codes for the partnering mobiles.

In this paper, we argue that the block fading channel model is appropriate for cooperative coding. We show that channel codes designed for block fading channels can fully exploit the potentials of user cooperation. We incorporate the practical constraint that the relay needs to receive and transmit at different channels. Hence, we adopt user-cooperation through orthogonal channels via time division multiplexing as in [6]. Our channel allocation scheme is illustrated in Figure 1, for the case of two users. We note that our results would also hold for other orthogonal signaling schemes such as frequency or code division.

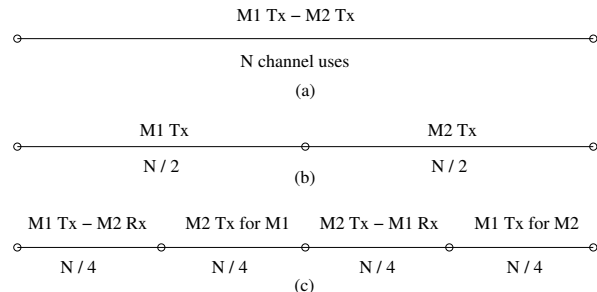


Fig. 1. Time-division channel allocations: (a) simultaneous direct transmission, (b) orthogonal direct transmission and (c) orthogonal cooperative diversity transmission.

The paper is organized as follows. In the next section we present the system model and establish notation. We also review the pairwise error probability for block fading channels and the design criteria for channel codes over block fading channels. The analysis of the cooperative coding approach is given in section III. Simulation results demonstrating the excellent performance and robustness of these codes are given in section IV. Finally, we conclude with section V.

II. THE SYSTEM MODEL

We consider mobile users that operate in a wireless cellular network, LAN, or an ad-hoc network. Each mobile user employs one antenna at the transmitter and there is also one antenna at the receiver. The information bits are encoded by a channel encoder. The coded bits are multiplexed in order to be allocated to the cooperating users so that the maximum possible diversity is achieved. The multiplexed bits are passed through a serial to parallel converter, and are mapped to a particular signal constellation.

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At each time slot t , the output of the modulator is a signal $c(t)$. The block diagram of the transmitter is given in Figure 2.

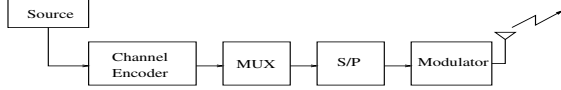


Fig. 2. The block diagram of the transmitter.

At time t the received signal, denoted by $r(t)$, is given by

$$r(t) = \sqrt{E_s}\alpha(t)c(t) + \eta(t)$$

where the noise samples $\eta(t)$ are modeled as independent realizations of a zero mean complex Gaussian random variable with variance $N_0/2$ per dimension and E_s is the average received energy per symbol. The coefficient $\alpha(t)$ is the path gain from the transmit antenna to the receive antenna. We assume flat Rayleigh fading, hence the path gains $\alpha(t)$ are modeled as samples of independent zero mean complex Gaussian random variable with variance 0.5 per dimension. We consider quasi-static fading, i.e., the path gains are constant during the transmission of any given user, but are independent from user to user. We will concentrate on time division form of channel allocation, as illustrated in Figure 1. Note that the above model is applicable to the mobile-destination link, as well as to the inter-user link. Under the assumption of user cooperation via time division channel allocation, each user experiences slow quasi-static fading and the fading is independent among users. Hence, among users, a block fading channel model is appropriate [8].

Coding for Block Fading Channels

Coding for block fading channels has been studied independently by Knopp and Humblet [9] and Malkamaki and Leib [10], [11]. We consider codewords that span a frame of length $N = FL$. The pairwise error probability that codeword e was decoded when codeword c was actually transmitted, over a block Rayleigh fading channel, may be upper bounded by [9]

$$P(c \rightarrow e) \leq \frac{1}{2} \left(\frac{1}{\Xi^2(c, e)E_s/4N_0} \right)^{d_H^F}$$

where

$$\Xi^2(c, e) = \left(\prod_{i=1}^{d_H^F} d_f^2(c, e) \right)^{1/d_H^F}.$$

The distance term $d_f^2(c, e) = \sum_{l=0}^{L-1} |c_{f,l} - e_{f,l}|^2$ denotes the squared Euclidean distance among the parts of the two codewords that experience the same fade during the fading block f . The number of nonzero Euclidean distances $d_f^2(c, e)$ is denoted by d_H^F . Note that d_H^F is the exponent of the signal-to-noise ratio E_s/N_0 and it represents the diversity the code achieves when used over a block fading channel with F independently faded blocks.

Hence, the diversity and product distance criteria for channel codes over block fading channels are [9]

- *Diversity criterion:* Maximize the diversity advantage

$$d_H^F = \sum_{f=1}^F \mathcal{I}_{d_f^2}(c, e)$$

over all pairs of distinct codewords $c, e \in \mathcal{C}$, where $\mathcal{I}_{\mathcal{X}} = 1$ if $\mathcal{X} > 0$ and $\mathcal{I}_{\mathcal{X}} = 0$ if $\mathcal{X} = 0$.

- *Product distance criterion:* Maximize the coding advantage

$$\Xi^2(c, e) = \left(\prod_{i=1}^{d_H^F} d_f^2(c, e) \right)^{1/d_H^F}$$

over all pairs of distinct codewords $c, e \in \mathcal{C}$.

Using the above design guidelines, Knopp and Humblet designed channel codes that obtain the maximum diversity and coding gain over the block fading channel [9]. This framework can be utilized to find channel codes that are suitable for user cooperation and that can obtain the diversity provided by the cooperation of users. Note that user cooperation codes need to perform well over the first slowly faded block as well, so that the information can be transferred to the partner. Also, in contrast to the usual block fading model, the inter-user channel quality in the cooperative case may vary, hence it is of interest to determine whether the codes would still provide improvements over the non-cooperative case when the inter-user channel is noisy.

III. COOPERATIVE CODING PERFORMANCE ANALYSIS

In this section, we provide the performance analysis of the cooperative coding approach for wireless networks. Our objective is to demonstrate that cooperative coding can indeed achieve the full diversity provided by user-cooperation. Let P_f^C denote the frame error probability of the channel code when user-cooperation is employed and we code across differently faded users (blocks). We assume that the fixed decode-and-forward protocol is used [12]. Then P_f^C can be written as

$$P_f^C = (1 - P_f^{in})P_f^{BF} + P_f^{in}P_f^{QS}$$

where P_f^{in} denotes the frame error probability of the inter-user channel, P_f^{BF} denotes the frame error probability over the block fading channel when cooperation takes place, and P_f^{QS} denotes the frame error probability over the quasi-static fading channel in the non-cooperative case. We obtain

$$\begin{aligned} P_f^C &= (1 - P_f^{in})P_f^{BF} + P_f^{in}P_f^{QS} \\ &= P_f^{BF} - P_f^{in}P_f^{BF} + P_f^{in}P_f^{QS} \\ &\leq P_f^{BF} + P_f^{in}P_f^{QS}. \end{aligned}$$

Let E_{s_1}/N_0 denote the received signal-to-noise ratio at the destination corresponding to the transmission from user

M_1 . Similarly, let E_{s_2}/N_0 denote the received signal-to-noise ratio at the destination corresponding to the transmission from user M_2 and $E_{s_{in}}/N_0$ denote the received signal-to-noise ratio at user M_2 corresponding to the transmission from user M_1 .

Note that in the block fading model resulting from user cooperation, each block has a different received signal-to-noise ratio. However, the pairwise error probability can be derived in a similar form. Hence, utilizing the pairwise error probability expression for the block Rayleigh fading channel and the union upper bound on the frame error probability, we have

$$P_f^{BF} \leq \sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Xi^2(\mathbf{c}, \mathbf{e}) (E_{s_1}/4N_0)^{\mathcal{I}_{d_1^2}} (E_{s_2}/4N_0)^{\mathcal{I}_{d_2^2}}}$$

where $\Xi^2(\mathbf{c}, \mathbf{e})$ denotes the product of the non-zero squared Euclidean distances $d_1^2(\mathbf{c}, \mathbf{e})$ and $d_2^2(\mathbf{c}, \mathbf{e})$.

For the upper bound in the non-cooperative quasi-static fading channel case, $d_H^F = 1$, hence

$$P_f^{QS} \leq \sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Psi^2(\mathbf{c}, \mathbf{e}) E_{s_1}/4N_0}$$

where $\Psi^2(\mathbf{c}, \mathbf{e}) = d_1^2(\mathbf{c}, \mathbf{e}) + d_2^2(\mathbf{c}, \mathbf{e})$ denotes the squared Euclidean distance between the two entire codewords. Note that the inter-user channel is also quasi-static, hence $d_H^F = 1$ and we have

$$P_f^{in} \leq \sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Phi^2(\mathbf{c}, \mathbf{e}) E_{s_{in}}/4N_0}$$

where $\Phi^2(\mathbf{c}, \mathbf{e}) = d_1^2(\mathbf{c}, \mathbf{e})$ denotes the squared Euclidean distance between the parts of the two codewords that are used in the inter-user channel.

Therefore when mobile M_1 transmits in cooperation with mobile M_2 , the upper bound on the frame error probability, P_f^C , for M_1 is

$$P_f^C \leq \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Xi^2(\mathbf{c}, \mathbf{e}) \left(\frac{E_{s_1}}{4N_0}\right)^{\mathcal{I}_{d_1^2}} \left(\frac{E_{s_2}}{4N_0}\right)^{\mathcal{I}_{d_2^2}}} \right) + \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Phi^2(\mathbf{c}, \mathbf{e}) \frac{E_{s_{in}}}{4N_0}} \right) \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Psi^2(\mathbf{c}, \mathbf{e}) \frac{E_{s_1}}{4N_0}} \right).$$

At this point, we focus on two extreme scenarios. In the first case we assume that we have a good inter-user channel, and we study the performance at high signal-to-noise ratios for all channels. In the second case we will consider the situation when the inter-user channel is of poor quality. Our goal is to determine the achievable diversity in both situations.

Case I: Good Inter-User Channel

First, we consider the performance when the inter-user channel is good. If the inter-user channel has very high

signal-to-noise ratio, then P_f^{in} is small and we simply have $P_f^C \approx P_f^{BF}$. Therefore, full diversity is obtained.

Next we consider the case when $E_{s_1} \approx E_{s_2} \approx E_{s_{in}} = E_s$. This assumption simplifies the diversity analysis and is quite reasonable at high signal-to-noise ratios in all channels. In this case P_f^C can be approximately upper bounded by

$$P_f^C \leq \left(\frac{1}{E_s/4N_0}\right)^2 \left\{ \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Xi^2(\mathbf{c}, \mathbf{e})} \right) + \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Phi^2(\mathbf{c}, \mathbf{e})} \right) \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{\Psi^2(\mathbf{c}, \mathbf{e})} \right) \right\}.$$

For simplicity, at high signal-to-noise ratios, the above bracketted expression may be approximated with the most dominant term in the summation, which may be denoted by, say K , yielding the following approximation at high signal-to-noise ratios

$$P_f^C \approx K \left(\frac{1}{E_s/4N_0}\right)^2.$$

This demonstrates that cooperative coding can indeed achieve the full diversity possible with user cooperation, as clearly indicated by the exponent of the signal-to-noise ratio.

Case II: Poor Inter-User Channel

In this case, we assume that the inter-user channel has poor quality. Then the upper bound on P_f^C is dominated by the term $P_f^{in} P_f^{QS}$. Hence, we obtain

$$P_f^C \leq \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{E_{s_{in}}/4N_0} \frac{1}{\Phi^2(\mathbf{c}, \mathbf{e})} \right) \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{E_{s_1}/4N_0} \frac{1}{\Psi^2(\mathbf{c}, \mathbf{e})} \right).$$

As the inter-user channel quality is low, we can assume that the inter-user channel signal-to-noise ratio is approximately constant for all transmit powers of interest, i.e., $\frac{E_{s_{in}}}{N_0} \approx C_{in}$. We have

$$P_f^C \leq \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{C_{in}} \frac{1}{\Phi^2(\mathbf{c}, \mathbf{e})} \right) \left(\sum_{\mathbf{c}} \sum_{\mathbf{e} \neq \mathbf{c}} \frac{1}{E_{s_1}/4N_0} \frac{1}{\Psi^2(\mathbf{c}, \mathbf{e})} \right).$$

Hence, at high signal-to-noise ratios in the user-destination channel, we obtain the following approximation for P_f^C ,

$$P_f^C \approx \frac{1}{C_{in}} \left(\frac{1}{E_{s_1}/4N_0}\right) \frac{1}{\min_{\mathbf{c}, \mathbf{e}} \{\Phi^2(\mathbf{c}, \mathbf{e}) \Psi^2(\mathbf{c}, \mathbf{e})\}}$$

where $\min_{\mathbf{c}, \mathbf{e}} \{\Phi^2(\mathbf{c}, \mathbf{e}) \Psi^2(\mathbf{c}, \mathbf{e})\}$ denotes the minimum product distance of the code which dominates the performance at high signal-to-noise ratios.

Therefore, for poor inter-user channel quality, the diversity is basically limited to the diversity of the non-cooperative quasi-static fading channel. However, this only holds when the inter-user channel is very noisy and severely faded. Despite the limited diversity, there is still some coding gain with respect to the non-cooperative coding case, as indicated by the squared Euclidean distance product. In the following section it will be demonstrated via simulations that even when the inter-user channel has low quality cooperative coding still proves beneficial, provides coding gains and significantly outperforms the non-cooperative coding case.

IV. SIMULATION RESULTS

In this section we present the performance of the proposed cooperative coding scheme to illustrate the potential benefits. We assume a Rayleigh slowly-fading channel. Hence, we use the quasi-static model, where the fading coefficient remains the same for the duration of the entire frame for each user. Note, however that the users observe independently faded channels. Due to the slow fading, we assume perfect channel state information at the receiver. In the simulations we use the constraint length $K = 4$ convolutional code with generator polynomials (13,15,15,17) in octal notation and BPSK modulation [9]. This is an appealing solution due to the widespread use of convolutional codes and the simple maximum likelihood decoding algorithm [13]. The extensions to higher order modulations are also possible [9]. We consider the case when both users communicate to the same destination, which is a fixed access point. Similar results would also be obtained for the case when mobiles have different destinations. As described in section II, we have a system with one antenna at each user and one antenna at the destination (access point). We present results for the fixed decode-and-forward protocol. We consider two scenarios based on the quality of each cooperating user's channel to the destination. This quality will be represented by the received signal-to-noise ratio. We first consider the symmetric case when both user's channels to the destination are of similar quality. Then we focus on the asymmetric case when one of the users has a better channel to the destination than the other. We will demonstrate that cooperation is beneficial in this case as well, as it improves the error performance of the good user as well as the bad one.

In the symmetric scenario, both users have channels of similar quality to the destination. Our goal is to study the performance of the cooperative coding scheme for various qualities of the inter-user channel. We demonstrate the performance in terms of the frame error rate (FER). Similar results could also be obtained in terms of the bit error rate.

From Figure 3, it can be observed that the cooperative coding scheme with perfect inter-user channel provides a performance improvement of about 4 dB at FER of 10^{-1} , 8 dB at FER of 10^{-2} and about 12 dB at FER of 10^{-3} with respect to the non-cooperative coding approach. Note the graceful degradation in performance with the degradation of the inter-user channel quality and the gain with respect

to the non-cooperative system. Investigating the FER's for high signal-to-noise ratios, it is clear that the cooperative coding scheme achieves the full block fading diversity for moderate and low FER in the inter-user channel. When the inter-user channel quality is poor, resulting in a high inter-user FER, the diversity of the system is limited to the non-cooperative case. Nevertheless, we still observe significant performance improvements with respect to non-cooperative transmission.

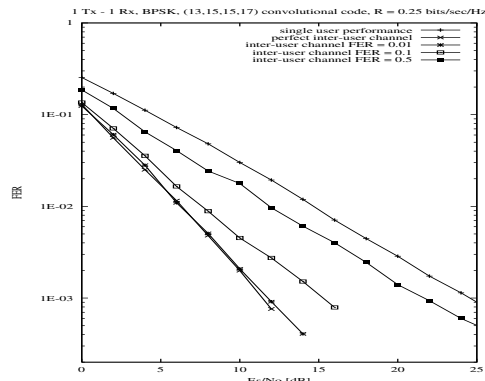


Fig. 3. Single user performance vs. two user cooperation, for different inter-user channel quality. One transmit and one receive antenna.

We next focus on the asymmetric scenario. This happens when one of the users has a better channel to the destination than the other user. We consider this case by fixing one of the users channels to the destination at a relatively high signal to noise ratio. We then vary the quality of the other users channel to the destination and observe the performance of both cooperating users. We assume one transmit antenna and one receive antenna.

We consider the case when we fix user one's channel at 25 dB, which results in a frame error rate of 10^{-3} in the non-cooperative case. The inter-user channel frame error rate is 0.5. We vary the signal-to-noise ratio of user two. We observe from Figure 4 that even in this asymmetric scenario and a bad inter-user channel both users still benefit from cooperation. User one achieves the frame error rate of 10^{-3} when the signal-to-noise ratio of user two is only about 8 dB. At higher signal-to-noise ratios its performance is better than in the non-cooperative case. User two also improves its performance by about 2 dB with respect to the non-cooperative case.

Hence, cooperative coding proves beneficial not only for users with similar channel qualities to the destination, but also in the case when the users have significantly different channel qualities and the inter-user channel has high frame error rate. What is important to note is that both users benefit from cooperation.

Finally, we compare the performance in the symmetric vs. the asymmetric scenario and consider the effect of cooperative coding on packet routing and choice of partners. We envision a scenario depicted in Figure 5. In the symmetric case, user M_1 decides to partner with user M_2 . Hence, both users have similar quality channels to the destination and their inter-user channel is very good. However, both

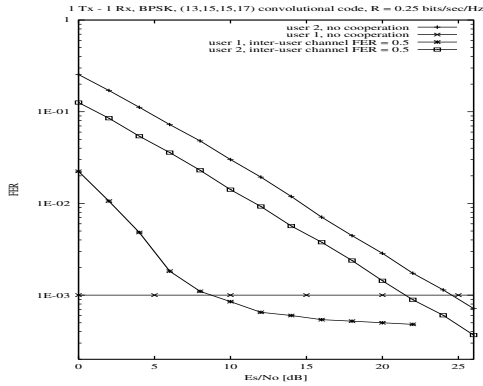


Fig. 4. Single user performance vs. two user cooperation, for two users with different channel qualities. One transmit and one receive antenna.

M_1 and M_2 are far away from the destination, resulting in a somewhat low received SNR. In the asymmetric case, M_1 partners with M_3 who has a better quality channel to the destination, but the inter-user channel is not so good.

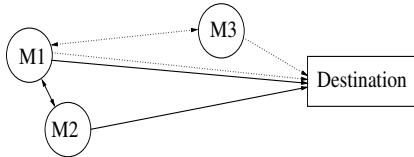


Fig. 5. Various cooperation scenarios.

Figure 6, compares the performance in the asymmetric and the symmetric scenarios in terms of the frame error rate. We assume that we have one transmit and one receive antenna. We focus on the performance of user M_1 . The channel of user M_3 is fixed at 25 dB, resulting in a FER of 10^{-3} in the non-cooperative case. The channel quality of both users M_1 and M_2 varies with the SNR. The inter-user channel between M_1 and M_3 has a FER of 10^{-1} . The inter-user channel between M_1 and M_2 has a FER of 10^{-2} . We observe that the asymmetric case is beneficial when M_1 experiences very low signal-to-noise ratio. However, as the channel quality to the destination of user M_1 (and M_2) improves, at a signal-to-noise ratio of approximately 8 dB, we find that it is better to have a good inter-user channel and take advantage of the diversity benefits. Hence, we do not necessarily need to rely only on a user with a very good signal-to-noise ratio. This is in contrast with routing protocols that only consider path loss and do not exploit diversity gains [14].

V. CONCLUSIONS

In this paper we introduced the cooperative coding concept. This approach is useful due to the size limitations of the mobile units which prevent the use of transmit antenna diversity. We exploited the diversity that is available in the channel by coding across the cooperating mobile users. We used the block fading model to provide a framework for the design of codes suitable for user cooperation. We demonstrated the usefulness of cooperative coding in providing diversity and coding gains both via analysis and sim-

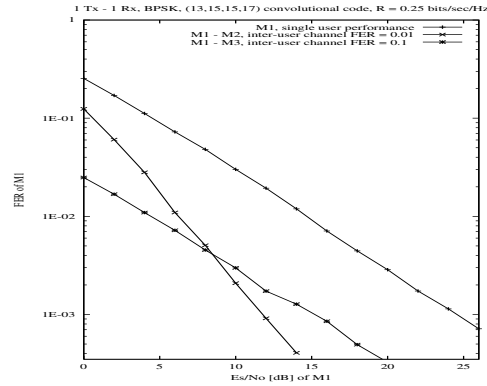


Fig. 6. Performance comparison for two user cooperation for symmetric and asymmetric source-destination channels. One transmit and one receive antenna.

ulation results. Cooperation proves beneficial when both users have similar, as well as different channel qualities to the destination. We also demonstrated that the cooperative coding approach is robust and still provides gains with respect to the non-cooperative case, even when there is a severe degradation in the inter-user channel quality. Our initial results suggest that diversity obtained through cooperative coding can affect the way packets are routed in a wireless network. An interesting future direction is to design effective routing algorithms for cooperating networks.

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