ITW2003, Paris, France, March 31 – April 4, 2003

Cooperative Space–Time Coding for Wireless Networks

Andrej Stefanov and Elza Erkip
Polytechnic University, Brooklyn, NY
{stefanov,elza}@poly.edu

Abstract — **We consider a cooperative system in which the partnering mobiles are equipped with multiple antennas. We present space–time codes that have the capability of providing full cooperation diversity, while achieving maximum possible diversity and best performance in the inter–user channel. Our codes also perform well when cooperation does not take place. We illustrate that cooperative space–time coding offers significant performance improvement over direct transmission even when the inter–user channel is noisy. We also consider code design for users with different number of antennas.**

I. INTRODUCTION

User–cooperation [1, 2] provides spatial diversity by enabling two or more active nodes to jointly transmit their messages towards their destinations. Each wireless transmission can be overheard by neighboring nodes (which we call "partners") which then process this signal and retransmit to provide additional reliability. Hence the partnering nodes can be thought of as an antenna array. However, inter–user channels are noisy and the cooperation schemes need to take into account that the information received by the partner may not be accurate.

Information theoretic studies, mostly confined to two cooperating partners [1, 2, 3], have shown that despite this noisy inter–user channel, cooperation leads to higher data rates and robustness to channel variations. In fact, two–user cooperative systems can achieve full two level diversity. Laneman and Wornell [4] recently extended their two–user results to incorporate multiple partners. They also suggested that space–time block codes based on orthogonal designs could be used across these multiple partners to provide cooperative diversity. However, this requires perfect synchronization among partnering nodes. The space–time cooperative coding addressed in this paper does not refer to this multiple partner scenario, we consider two cooperating nodes that have multiple antennas.

Most of the work on designing channel codes for cooperation has focused on nodes having only one antenna [5, 6]. We consider the case when the partners can have more than one antenna, with possibly different number of antennas on different users. We argue that cooperative diversity can be exploited on top of the already existing spatial diversity in the mobiles. When the users experience quasi–static fading, cooperation through orthogonal channels via time division multiplexing [3] leads to an overall block fading channel model. Hence the literature on space–time coding for block fading channels can be used for cooperative space–time coding. However, we show that the performance of the space–time code in the inter–user channel as well as the non–cooperative case is also crucial in achieving cooperation gains. Also, if a mobile with one antenna cooperates with a mobile with two antennas, the single antenna code should be easily upgraded for a two–antenna space–time code by the partner. In this paper, we present space–time cooperative code design that takes into account the above criteria and achieves full cooperation diversity. We present simulation results to illustrate the performance of the designed codes, analytical bounds on performance can be found in [9].

The paper is organized as follows. In the next section we present the system model and establish notation. The design of space–time codes suitable for user–cooperation is presented in section III. Simulation results illustrating the performance of the cooperative space–time coding approach are presented in section IV. Conclusions are provided in section V.

II. SYSTEM MODEL

We consider a mobile communication system that employs L_t antennas at the transmitter and L_r antennas at the receiver. The information bits are encoded by a channel encoder. The coded bits are multiplexed to achieve the maximum possible diversity in the case of user cooperation. The coded and multiplexed bits are passed through a serial to parallel converter, and are mapped to a particular signal constellation. At each time slot t, the output of the modulator is a signal $c_{t,i}$ that is transmitted using transmit antenna i, for $1 \leq i \leq L_t$. All signals are transmitted simultaneously, each from a different transmit antenna, and all signals have the same transmission period T. The block diagram of the transmitter is given in Figure 1.

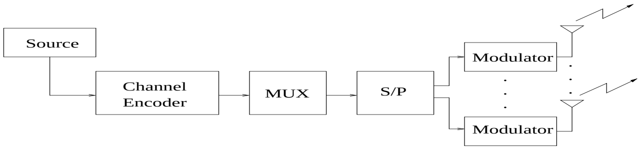

Figure 1: *The block diagram of the transmitter.*

A space–time codeword J is an $L_t \times n$ matrix with elements drawn from a finite alphabet, where n denotes the transmission length in symbols. The code symbol matrix $\boldsymbol{c}$, is $\boldsymbol{c} = f(J)$, where $f(J)$ is an element wise constellation mapping from the finite alphabet to points of the constellation.

At time t the received signal by antenna j, denoted by $r_{t,j}$ is given by

$$r_{t,j} = \sum_{i=1}^{L_t} \alpha_{i,j} c_{t,i} + \eta_{t,j}$$

where the noise samples $\eta_{t,j}$ are modeled as independent samples of a zero mean complex Gaussian random variable with variance $N_0/2$ per dimension. The coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i, $1 \leq i \leq L_t$,to receive antenna j, $1 \leq j \leq L_r$. We assume frequency non–selective quasi–static fading, i.e., the path gains are constant during the transmission of any given user, but are independent from user to user. Hence when users cooperate, the overall channel model becomes block fading [7].

III. CODE DESIGN

We assume that the mobiles cooperate through time division [3] and utilize the cooperation protocol of [6] which is similar to the protocol in [5]. The partner receives the signal transmitted by the mobile and decodes it. Then it performs a CRC check [8] to determine whether the received sequence matches the actual transmitted information sequence. If so, the partner re–encodes the data to obtain the best possible code (in combination with the original mobiles code) which achieves maximum possible diversity. If the transmission is not received successfully, as indicated by the CRC check, then the partner notifies the mobile about it, and the mobile transmits the rest of the coded bits (symbols) itself, instead of relying on cooperation from its partner.

In a recent work [9], we demonstrated that the achieved diversity order in the case of user–cooperation depends on the inter–user channel quality. When the inter–user channel quality is good, full diversity can be obtained with suitable code design. When the inter–user channel has poor quality, the diversity order is limited to the non–cooperative case, nevertheless performance improvement is still possible due to the increased coding gain. Our focus in this paper is on the design of space–time codes that achieve the full user–cooperation diversity, with the constraint that they also provide excellent performance in the inter–user channel and in the non–cooperative case. Hence our codes are suited for a wide range of inter–user channel qualities.

For our designs, we utilize the algebraic space–time coding framework of Hammons and El Gamal [10, 11]. This framework utilizes the ranks of the code generator matrices over the binary field, F, and significantly simplifies the design procedure for BPSK and QPSK space–time codes.

In a communication system with L_t transmit antennas, and B fading blocks per codeword, the maximum achievable transmit diversity is L_tB. The space–time code $\mathcal{C}$ consists of the codewords J, described by

$$J = \begin{bmatrix} xG_{1,1} & xG_{1,2} & \cdots & \cdots & xG_{1,B} \\ xG_{2,1} & xG_{2,2} & \cdots & \cdots & xG_{2,B} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ xG_{L_t,1} & xG_{L_t,2} & \cdots & \cdots & xG_{L_t,B} \end{bmatrix}$$

where x is a $1 \times k$ information vector over F and $G_{i,j}$ are the generator matrices for the i–th antenna in the j–th fading block, $G_{i,j} \in \mathsf{F}^{\mathsf{k} \times \mathsf{n}/\mathsf{B}}$.

The space–time code $\mathcal{C}$ will achieve full transmit diversity L_tB if for every fading block b, $1 \leq b \leq B$, the matrix

$$G = a_1 G_{1,b} \oplus a_2 G_{2,b} \oplus \cdots \oplus a_{L_t} G_{L_t,b}$$

is of full rank k for all $a_1, a_2, \ldots, a_{L_t} \in \mathsf{F}$ unless $a_1 = a_2 = \cdots = a_{L_t} = 0$ [10, 11].

As suggested above, unlike the standard block fading channel, in the case of user–cooperation the space–time codes should provide good performance in the quasi–static inter–user channel, as well as in the non–cooperative case. Therefore, we find codes that provide the best performance in the inter–user channel, and then overlay them into full diversity codes for the case of user–cooperation [12], while also obtaining codes that have excellent performance over the quasi–static channel when cooperation does not take place. Convolutional codes with excellent free distances that satisfy the above criteria are presented in Table I [12]. These generator polynomials can also be used to obtain QPSK space–time trellis codes by utilizing the *lifting* technique from [10, 11]. The resulting Z_4 codes will also achieve the full user–cooperation diversity.

K = ν+1	M_1	M_1 and M_2
3	5,7	5,7,5,7
4	15,17	15,17,13,15
5	23,35	23,35,25,37
6	53,75	53,75,67,71
7	133,171	133,171,117,165

Table 1: *Convolutional codes suitable for cooperative space–time coding.*

IV. SIMULATION RESULTS

In this section we present the performance of the proposed cooperative coding scheme to illustrate the potential benefits. We assume a Rayleigh slowly–fading channel. Hence, we use the quasi–static model, where the fading coefficient remains the same for the duration of the entire frame for each user. Note, however that the users observe independently faded channels. In the simulations we used constraint length, K = 7, convolutional code with generator polynomials (133,171,117,165) in octal notation and BPSK modulation. This is an appealing solution due to the widespread use of the convolutional codes and the simple maximum likelihood decoding algorithm. The extensions to higher order modulations are also possible. We will consider two scenarios based on the quality of each cooperating user's channel to the destination. We first consider the case when both users channels to the destination are of similar quality. Then we focus on the case when one of the users has a better channel to the destination than the other. We will demonstrate that cooperation is beneficial in this case as well.

A. Symmetric Scenario

We first consider the symmetric scenario, when both users have channels of similar quality to the destination. Our goal is to study the performance of the cooperative coding scheme for various qualities of the inter–user channel. We demonstrate the performance in terms of the frame error rate (FER). Similar results could also be demonstrated in terms of the bit error rate.

We consider a system where we have two antennas at each user and two antennas at the destination (access point). From Figure 2, it can be observed that the cooperative coding scheme with perfect inter–user channel provides a performance improvement of about 1 dB at FER of 10^{-1}, 2 dB at FER of 10^{-2} and around 4 dB at FER of 10^{-3}. There is a graceful degradation of performance with the degradation of the inter–user channel quality. Even for a poor inter–user channel of FER=0.5 there is still improvement over direct transmission. If the inter–user channel frame error is less than 10^{-1}, we observe that the performance is virtually the same as perfect inter–user channel. Note that in the case when both users transmit and receive with two antennas, this means near optimal performance even when the inter–user channel signal to noise ratio is rather low.

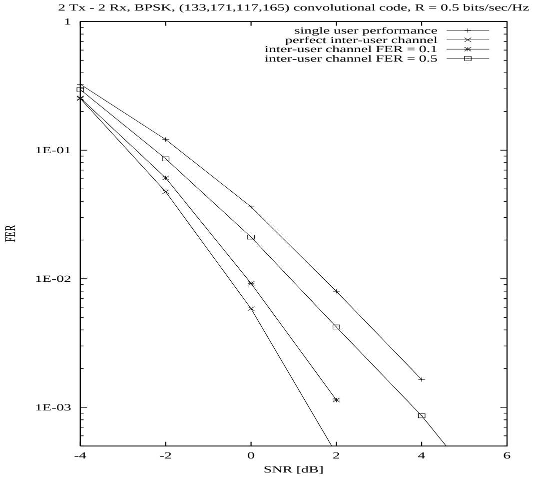

Figure 2: *Two transmit antennas – two receive antennas, symmetric users. Single user performance vs. two user cooperation for different inter-user channel qualities.*

B. Asymmetric Scenario

We next focus on the asymmetric scenario. This happens when one of the users has a better channel to the destination than the other user. We consider this case by fixing one of the users channels to the destination at a relatively high signal to noise ratio. We then vary the quality of the other users channel to the destination and observe the performance of both cooperating users. We illustrate the performance for the case when we have two antennas at each user and two antennas at the destination.

The case when the inter–user channel frame error rate is 0.5, is presented in Figure 3. We fix user one's channel at 5 dB, which in this scenario results in a frame error rate of 10^{-3}. We vary the signal–to–noise ratio of user two. We observe that when we have receive antenna diversity at the destination and a bad inter–user channel both users still benefit from cooperation. User one achieves the frame error rate of 10^{-3} when the signal to noise ratio of user two is about 0 dB. Even with the limited degree of cooperation, due to the bad inter–user channel, user two still improves its performance by about 1 dB with respect to the non–cooperative case.

Hence, cooperative coding proves beneficial not only for users with similar channel qualities to the destination, but also in the case when the users have significantly different channel qualities.

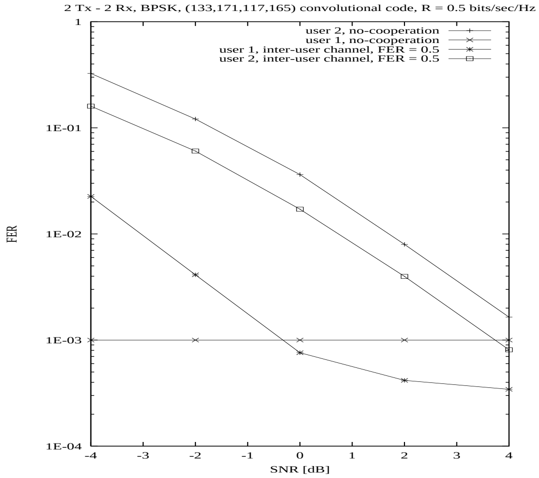

Figure 3: *Two transmit antennas – two receive antennas, asymmetric users. Single user performance vs. two user cooperation for two users with different channel qualities.*

C. Symmetric vs. Asymmetric Scenario

Next, we compare the performance in the symmetric vs. the asymmetric scenario as depicted in Figure 4. We would like to know whether it is best for M_1 to cooperate with M_2 or M_3 for transmitting information to the destination. If M_1 cooperates with M_3 (the asymmetric case), it can enjoy the high signal–to–noise ratio of M_3, but the inter–user channel quality will be low as the distance between M_1 and M_3 is large. If M_1 cooperates with M_2, the inter–user channel will have low FER, but both partners will have the same signal–to–noise ratio towards the destination.

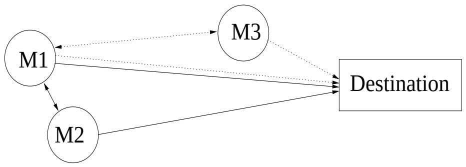

Figure 4: *Various cooperation scenarios.*

Figure 5, compares the performance in the asymmetric and the symmetric scenarios in terms of the frame error rate. We focus on the performance of user M_1. The channel of user M_3 is fixed at 5 dB resulting in a FER of 10^{-3} in the non–cooperative case. The channel quality of both users M_1 and M_2 varies with the SNR. The inter–user channel between M_1 and M_3 has a FER of 10^{-1}. The inter–user channel between M_1 and M_2 has a FER of 10^{-2}. We observe that the asymmetric case is beneficial when M_1 experiences very low signal–to–noise ratio. However, as the channel quality to the destination of user M_1 (and M_2) improves, at a signal–to–noise ratio of approximately 1 dB, we find that it is better to have a good inter–user channel and take advantage of the diversity benefits. Hence, we do not necessarily need to rely only on a user with a very good average signal–to–noise ratio. This is in contrast with routing protocols that only consider path loss and do not exploit diversity gains.

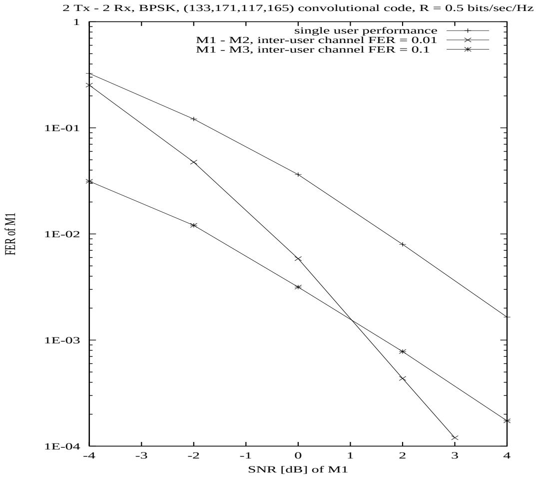

Figure 5: *Two transmit antennas – two receive antennas, performance comparison for two user cooperation for symmetric and asymmetric source–destination channels.*

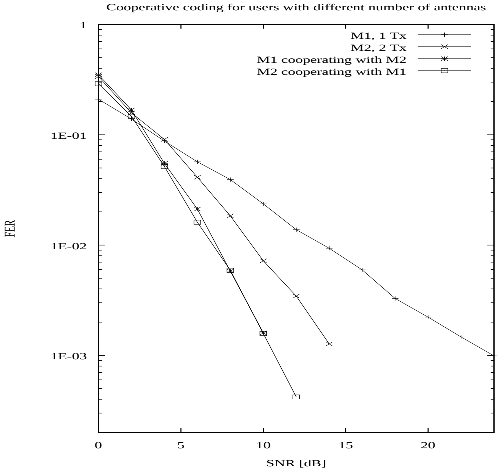

Figure 6: *Cooperative coding between users with different number of antennas. One receive antenna at the destination, perfect inter–user channel.*

D. Cooperative Coding between Mobiles with Different Number of Antennas

Finally, we consider cooperative space–time coding when one of the partners, M_1 has one antenna, and the other partner M_2 has two antennas. We assume the destination has one receive antenna. M_1 utilizes the rate 1/2 constraint length 7 convolutional code, (133,171), during the first half of its frame. If M_2 decodes the transmission from M_1 correctly, it re–encodes it utilizing both of its transmit antennas, using the full diversity code with generator polynomials (117,165,151,137). This results in an underlying rate 1/6, d_{free} convolutional code [12]. The overall spectral efficiency is 0.25 bits/sec/Hz. In case cooperation does not take place M_1 continues its transmission, by choosing the best rate 1/4, d_{free} convolutional code, (133,171,117,165). Similarly, M_2 uses the best space–time code during the first half of its transmission, by using the (133,171) convolutional code, formatted as space–time code. If M_1 recovers the transmission from M_2, it re–encodes it using the generator polynomial 117, obtaining an underlying rate 1/3 convolutional code. The spectral efficiency is 0.5 bits/sec/Hz. In case cooperation does not take place M_2 utilizes the overall (133,171,117,165) convolutional code formatted as space–time code. The performance of this cooperative coding scenario is presented in Figure 6. We assume a perfect inter–user channel. If the inter–user channel is not perfect, user M_1 would have a better inter–user channel than user M_2 by 3 dB, since it has the benefit of receive antenna diversity. In order to make the inter–user channel symmetric, M_1 could transmit at half the power during the first half of its transmission. In case cooperation does not take place, M_1 could then increase the transmission power during the second half of its frame, while maintaining the same overall power consumption. This does not result in a significant loss in performance for user M_1, while ensuring equivalent cooperation for both partners.

V. CONCLUSIONS

In this paper we introduced cooperative space–time coding for cooperating mobiles that have multiple antennas. We used the block fading channel model to provide a framework for the design of space–time codes suitable for user cooperation. We found that cooperative space–time coding is beneficial for both partners when they have similar, as well as different channel qualities to the destination. We also demonstrated that the cooperative space–time coding approach is robust and still provides gains with respect to no cooperation, even when there is a severe degradation in the inter–user channel quality. Finally, we showed how cooperative coding can be utilized when the users have different number of antennas.

REFERENCES

[1] A. Sendonaris, E. Erkip and B. Aazhang, “Increasing Uplink Capacity via User Cooperation Diversity,” in Proc. *IEEE ISIT*, p. 196, Aug. 1998

[2] A. Sendonaris, E. Erkip, and B. Aazhang, “User Cooperation Diversity–Part I: System description,” to appear in *IEEE Trans. on Commun.*

[3] N. Laneman, D. N. C. Tse and G. W. Wornell, “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior,” submitted to *IEEE Trans. on Info. Theory*

[4] N. Laneman and G. W. Wornell, “Distributed Space–Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks,”in Proc. *IEEE GLOBECOM*, Nov. 2002

[5] T. Hunter and A. Nosratinia, “Cooperation Diversity Through Coding,” in Proc. *IEEE ISIT*, Jun. 2002

[6] A. Stefanov and E. Erkip, “Cooperative Coding for Wireless Networks,” in Proc. *IEEE MWCN*, pp. 273-277, Sep. 2002

[7] E. Biglieri, J. Proakis and S. Shamai (Shitz), “Fading Channels: Information–Theoretic and Communications Aspects,” *IEEE Trans. on Info. Theory*, vol. 44, pp. 2619–2692, Oct. 1998

[8] S. Lin and D. J. Costello, “Error Control Coding: Fundamentals and Applications,” *Prentice–Hall*, 1983

[9] A. Stefanov and E. Erkip, “On the Performance Analysis of Cooperative Space–Time Coded Systems,” in Proc. *IEEE WCNC*, Mar. 2003

[10] A. R. Hammons Jr.and H. El Gamal, “On the Theory of Space–Time Codes for PSK Modulation,” *IEEE Trans. on Info. Theory*, vol. 46, pp. 524–542, Mar. 2000

[11] H. El Gamal and A. R. Hammons Jr., “On The Design of Algebraic Space–Time Codes for MIMO Block–Fading Channels,” *IEEE Trans. on Info. Theory*, vol. 49, pp. 151–163, Jan. 2003

[12] H. El Gamal, A. R. Hammons Jr. and A. Stefanov, “Space-Time Overlays for Convolutionally Coded Systems,” to appear in *IEEE Trans. on Commun.*