Diversity-Multiplexing Tradeoff in Half-Duplex Relay Systems

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***Abstract*— We study the multiple antenna half-duplex relay channel from the diversity-multiplexing tradeoff (DMT) perspective. We find performance upper bounds and show that compress-and-forward (CF) protocol achieves the upper bound. We argue that although it is hard to find the exact DMT expressions for decode-and-forward (DF) type protocols, they would be suboptimal in the multiple antenna case. We also study the multiple-access relay channel (MARC), and evaluate how CF works in this system. Our results show that CF is a robust strategy, which performs well in different relay networks and multiple antenna scenarios.**

I. Introduction

In wireless fading channels, cooperation or relaying increases communication reliability and transmission rates [1], [2]. The literature includes numerous papers that compare different cooperation/relaying schemes either in terms of the probability of error or achievable rate. The seminal paper [3] establishes the fundamental tradeoff between reliability and rate, also known as diversity-multiplexing tradeoff (DMT), for multiple antenna systems. DMT is a powerful tool to evaluate the performance of different multiple antenna schemes at high SNR; it is also a useful performance measure for cooperative/relay systems. On one hand it is easy enough to tackle, and on the other hand it is strong enough to show insightful comparisons among different relaying schemes. Furthermore, while the capacity of the relay channel is not known in general, it is possible to find relaying schemes that are optimal from the DMT perspective [4], [5].

The complete DMT of half-duplex relay systems were first studied in [2] and [6]. Amplify-and-forward (AF) and decode-and-forward (DF) are two of the protocols suggested in [2] for a relay system with single antenna nodes. In both protocols, the relay listens to the source during the first half of the frame, and transmits during the second half, while the source remains silent. If the relay does AF, it simply amplifies its received signal and forwards this to the destination. In DF it decodes the source signal and re-encodes it before transmitting. If the relay is unable to decode, it remains silent. To overcome the losses of strict time division between the source and the relay, [2] offers incremental relaying, in which there is a 1-bit feedback from the destination to both the source and the relay, and the relay is used sporadically, i.e. only if the destination cannot decode the source during the first half of the frame. In [7], the authors do not assume feedback, but to improve the AF and DF schemes of [2] they allow the source to transmit simultaneously with the relay. This idea is also used in [6] to study the non-orthogonal amplify-and-forward (NAF) and dynamic decode-and-forward (DDF) protocols in terms of DMT. Both of these new schemes allow the source to transmit for the whole frame. Similar to AF, in NAF the relay listens to the source during the first half of the frame and transmits during the second. The DDF protocol is an incremental redundancy type protocol in which the relay listens to the source until it is able to decode reliably. When this happens, the relay re-encodes the source message and sends it in the remaining portion of the frame. The authors find that DDF is optimal for low multiplexing gains but it is suboptimal when the multiplexing gain is large. This is because at high multiplexing gains, the relay needs to listen to the source longer and does not have enough time left to transmit the high rate source information. This is not an issue when the multiplexing gain is small as the relay usually understands the source message at an earlier time instant and has enough time to transmit.

The multiple-antenna relay channel DMT is studied in [8] for the NAF protocol only. In [8], the authors find a lower bound on the DMT performance and design space-time block codes for the NAF protocol for MIMO relay channel. The lower bound is not tight in general and is valid only if the number of relay antennas is less than or equal to the number of source antennas.

Other works that study the half-duplex relay channel from the DMT perspective are [9] and [10]. In [9], the authors propose space-time coding strategies for a system composed of a single antenna source, single antenna relays and a multiple antenna destination. Bletsas et. al. [10] show that when multiple relays are present, opportunistic relaying, i.e. choosing the best relay, is an efficient way to obtain the DMT gains, and it performs as well as using all the relays together.

In addition to the classical relay channel, another important system utilizing a relay is the multiple-access relay channel (MARC) [11]. In this system, the relay helps multiple sources simultaneously to reach a common destination. In [12], the authors study the MARC and find that DDF is DMT optimal for low multiplexing gains; however, this protocol remains to be suboptimal for high multiplexing gains as in the simple relay channel. In [13], [14], the authors find that the multiple access amplify and forward (MAF) protocol achieves the DMT

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upper bound in the region where DDF is suboptimal in MARC with single antenna nodes.

All the above references have one degree of freedom in the source destination link and study AF and DF type protocols. In [5] we considered the multiple antenna relay channel with a full-duplex relay and compared the performance of the compress-and-forward (CF) protocol with that of DF. In the CF protocol, the relay performs Wyner-Ziv type compression, i.e. it compresses its received signal, taking into account that the signal the destination receives directly from the source will be available as side information. Our results reveal that when each node has a single antenna, CF and DF behave similarly, but this is not true when nodes have multiple antennas. The number of degrees of freedom has a significant effect on the DMT performance. For the multiple antenna full-duplex relay channel the DF protocol is generally suboptimal, whereas the CF protocol always achieves the DMT upper bound.

In this paper, we first study the multiple antenna half-duplex relay channel, find DMT upper bounds and CF type relaying strategies that achieve the upper bound. We also investigate how the CF protocol performs in the half-duplex MARC when each node has multiple antennas and show a significant region on the DMT upper bound is achieved this way.

In Section II we introduce the system model. Section III presents a DMT upper bound for the multiple antenna half-duplex relay channel and shows how to achieve it. In Section IV, we study the MARC DMT. We conclude in Section V.

II. System Model

We study two systems in this paper: The multiple antenna relay channel with one source, one destination and one relay, with m, n and k antennas respectively and the MARC with two sources, one destination and one relay. The two sources have m_1 and m_2 antennas, and the destination and the relay have n and k antennas respectively. The relay is half-duplex. The source(s) transmits for the whole frame length. The relay listens to the source(s) for the first t fraction of the time, and transmits during the remaining $(1-t)$ fraction, where t is a constant real number. When the relay is listening, we say the system is in state q_1 and when the relay is transmitting the system is in state q_2. The models are illustrated in Fig. 1. This type of a transmission, which we call *static*, does not account for protocols, in which the relay transmission is broken into smaller blocks to convey additional information to the destination, while keeping the total transmission time equal to $1-t$. However, in the next section we will argue that it is enough to study *static* protocols.

For the relay channel, at state q_1, the received signals at the relay and the destination are

$$\begin{aligned}\mathbf{Y}_{R,1} &= \mathbf{H}_{SR}\mathbf{X}_{S,1} + \mathbf{Z}_{R,1}\\ \mathbf{Y}_{D,1} &= \mathbf{H}_{SD}\mathbf{X}_{S,1} + \mathbf{Z}_{D,1}\end{aligned}$$

and at state q_2, the received signal at the destination is given as

$$\mathbf{Y}_{D,2} = \mathbf{H}_{SD}\mathbf{X}_{S,2} + \mathbf{H}_{RD}\mathbf{X}_{R,2} + \mathbf{Z}_{D,2}.$$

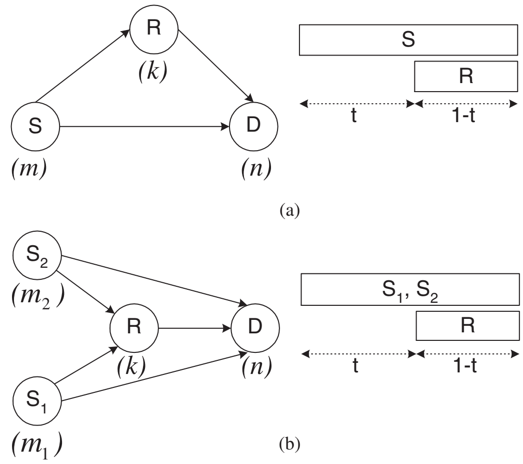

Fig. 1. (a) The relay channel, (b) The multiple-access relay channel, MARC. The relay listens for t fraction of time, transmits for $(1-t)$ fraction. The numbers in parentheses denote the number of antennas each node has.

For the MARC we have

$$\begin{aligned}\mathbf{Y}_{R,1} &= \mathbf{H}_{S_1R}\mathbf{X}_{S_1,1} + \mathbf{H}_{S_2R}\mathbf{X}_{S_2,1} + \mathbf{Z}_{R,1}\\ \mathbf{Y}_{D,1} &= \mathbf{H}_{S_1D}\mathbf{X}_{S_1,1} + \mathbf{H}_{S_2D}\mathbf{X}_{S_2,1} + \mathbf{Z}_{D,1}\end{aligned}$$

at state q_1 and at state q_2, the received signal at the destination is given as

$$\mathbf{Y}_{D,2} = \mathbf{H}_{S_1D}\mathbf{X}_{S_1,2} + \mathbf{H}_{S_2D}\mathbf{X}_{S_2,2} + \mathbf{H}_{RD}\mathbf{X}_{R,2} + \mathbf{Z}_{D,2}.$$

Here $\mathbf{X}_{S,l}$, $\mathbf{X}_{S_1,l}$, $\mathbf{X}_{S_2,l}$ and $\mathbf{X}_{R,l}$ are of size m, m_1, m_2 and k column vectors respectively and denote transmitted signal vectors at node S, S_1, S_2 and R at state q_l. Similarly $\mathbf{Y}_{R,l}$ and $\mathbf{Y}_{D,l}$ are the received signal vectors of size k and n. $\mathbf{H}_{SD}$, $\mathbf{H}_{S_1D}$, $\mathbf{H}_{S_2D}$, $\mathbf{H}_{SR}$, $\mathbf{H}_{S_1R}$, $\mathbf{H}_{S_2R}$, $\mathbf{H}_{RD}$ are the channel gain matrices of size $n\times m$, $n\times m_1$, $n\times m_2$, $k\times m$, $k\times m_1$, $k\times m_2$, $n\times k$ respectively. All inter-user channels are independent and have slow, frequency non-selective Rayleigh fading. Furthermore fading across antennas are assumed to be independent. In other words, $\mathbf{H}_{ij}$ have i.i.d. zero mean complex Gaussian entries with variance 1, and the channel gains remain fixed for one frame length (or an integer multiple of the frame length). Without loss of generality we can assume the source(s) and the relay have the same transmit powers, and we can ignore the path loss in the system. This is because a constant scaling in SNR does not affect the DMT results [3]. Hence, our results would also apply to channels with path loss, where inter-user distances are fixed or when the source transmit power is a constant multiple of the relay transmit power. We denote the average signal to noise ratio at each receive antenna resulting from a single transmit antenna as SNR. The relay and destination noise vectors $\mathbf{Z}_{R,l}$ and $\mathbf{Z}_{D,l}$ are size k and n column vectors with i.i.d. complex Gaussian entries with zero mean and variance 1.

All the receivers have channel state information (CSI) about all the incoming fading levels. Furthermore, the relay has CSI about all the channels in the system. This can happen at a negligible cost, if the destination feeds back its incoming fading levels to the relay. We will explain why we need this information in Section III-B when we explain the CF protocol in detail. The source(s) does not have instantaneous CSI. We also assume the system is delay-limited and requires constant-rate transmission. There is also short-term average power constraint that has to be satisfied for each codeword transmitted both for the source(s) and the relay. For more information about the effect of CSI at the source and variable rate transmission on DMT we refer the reader to [15].

It is assumed that the reader is familiar with the DMT definition [3], and in the rest of the paper $\mathbf{I}_i$ denotes the identity matrix of size $i \times i$, $\dagger$ denotes conjugate transpose, and $|.|$ denotes determinant. For notational simplicity we write $f(\mathrm{SNR}) \doteq \mathrm{SNR}^c$, if $\lim_{\mathrm{SNR}\to\infty} \frac{\log f(\mathrm{SNR})}{\log \mathrm{SNR}} = c$. The inequalities $\dot{\geq}$ and $\dot{\leq}$ are defined similarly.

III. The Relay Channel

A. Upper Bounds

In this subsection we present a DMT upper bound for the half-duplex multiple antenna relay channel of Fig. 1(a). Next subsection discusses how a CF type method achieves the DMT upper bound.

In a communication network we can upper bound the achievable rates with the minimum mutual information over all possible cut-sets. For the half-duplex relay channel we have [16]

$$I_1(t) = tI(\mathbf{X}_S; \mathbf{Y}_R \mathbf{Y}_D | q_1) + (1-t) I(\mathbf{X}_S; \mathbf{Y}_D | \mathbf{X}_R, q_2) \quad (1)$$
$$I_2(t) = tI(\mathbf{X}_S; \mathbf{Y}_D | q_1) + (1-t) I(\mathbf{X}_S \mathbf{X}_R; \mathbf{Y}_D | q_2), \quad (2)$$

where index 1 and 2 refer to the cut-sets around the source and the destination respectively. These upper bounds are only valid for *static* protocols. As we mentioned in Section II, the relay could switch between listen and talk times to convey additional information. Using the results in [17] one can prove that such a transmission scheme can increase the rate at most 1 bit, which means it is enough to study *static* protocols at high SNR.

To find the largest possible upper bound from (1) and (2), we need to choose the input distributions for $\mathbf{X}_S$ and $\mathbf{X}_R$ as complex Gaussian with covariance matrices Q_S and Q_R respectively. We know that for any random channel matrix $\mathbf{H}$ of size $n \times m$ [3],

$$\sup_{Q \geq 0, \mathrm{trace}\{Q\} \leq m\mathrm{SNR}} \log \left| \mathbf{I}_n + \mathbf{H} Q \mathbf{H}^\dagger \right| \leq \log \left| \mathbf{I}_n + m\mathrm{SNR}\mathbf{H}\mathbf{H}^\dagger \right|,$$

which means the performance is upper bounded by the case when the input covariance matrix is diagonal but each antenna has all the total transmit power. We define

$$K_{S,D} \triangleq \left| \mathbf{H}_{SD} \mathbf{H}_{SD}^\dagger m\mathrm{SNR} + \mathbf{I}_n \right| \quad (3)$$
$$K'_{S,RD} \triangleq \left| \mathbf{H}_{S,RD} \mathbf{H}_{S,RD}^\dagger m\mathrm{SNR} + \mathbf{I}_{k+n} \right| \quad (4)$$
$$K_{SR,D} \triangleq \left| \mathbf{H}_{SR,D} \mathbf{H}_{SR,D}^\dagger (m+k)\mathrm{SNR} + \mathbf{I}_n \right|, \quad (5)$$

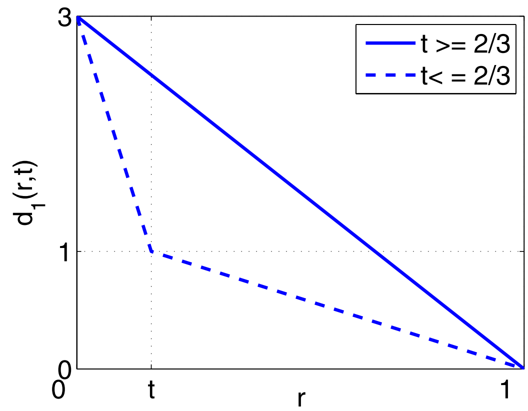

Fig. 2. DMT for the cut-set around the source for $m = 1$, $k = 2$, $n = 1$. Note that as $m = n$, $d_1(r,t) = d_2(r, 1-t)$.

where

$$\mathbf{H}_{S,RD}^\dagger = \left[\mathbf{H}_{SR}^\dagger \; \mathbf{H}_{SD}^\dagger \right], \mathbf{H}_{SR,D} = \left[\mathbf{H}_{SD} \; \mathbf{H}_{RD} \right]. \quad (6)$$

Then we can upper bound $I_1(t)$ and $I_2(t)$ with $C_1(t)$ and $C_2(t)$ as

$$I_1(t) \leq C_1(t) = t \log K'_{S,RD} + (1-t) \log K_{S,D} \quad (7)$$
$$I_2(t) \leq C_2(t) = t \log K_{S,D} + (1-t) \log K_{SR,D}. \quad (8)$$

At high SNR, for a target data rate $R = r \log \mathrm{SNR}$, suppose we have

$$P(C_1(t) < R) \doteq \mathrm{SNR}^{-d_1(r,t)}, \; P(C_2(t) < R) \doteq \mathrm{SNR}^{-d_2(r,t)}. \quad (9)$$

Then using [4], the best achievable diversity for the half-duplex relay channel for fixed t satisfies

$$d(r,t) \leq \min\{d_1(r,t), d_2(r,t)\}. \quad (10)$$

Optimizing over t we find an upper bound on the multiple antenna half-duplex relay channel DMT as

$$d(r) \leq \max_t \min\{d_1(r,t), d_2(r,t)\}. \quad (11)$$

An explicit form for $d_1(r,t)$ is given in the following theorem.

Theorem 1: For $m = 1$, $d_1(r,t)$ is given as

$$d_1(r,t) = \begin{cases} n + k - k\frac{r}{t} & \text{if } \; r \leq t, \text{ and } t \leq \frac{k}{n+k} \\ n\left(\frac{1-r}{1-t}\right) & \text{if } \; r \geq t, \text{ and } t \leq \frac{k}{n+k} \\ (n+k)(1-r) & \text{if } \; t \geq \frac{k}{n+k} \end{cases}.$$

For $n = 1$ and for arbitrary m and k, $d_2(r,t)$ has the same expression as $d_1(r,t)$ if n and t are replaced with m and $(1-t)$ in the above expressions.

Proof: The proof can be found in [18]. ■

It is hard to find $d_1(r,t)$ and $d_2(r,t)$ for general m, n and k as this requires finding the joint eigenvalue distribution of two correlated Hermitian matrices, $\mathbf{H}_{SD}\mathbf{H}_{SD}^\dagger$ and $\mathbf{H}_{S,RD}\mathbf{H}_{S,RD}^\dagger$ or $\mathbf{H}_{SD}\mathbf{H}_{SD}^\dagger$ and $\mathbf{H}_{SR,D}\mathbf{H}_{SR,D}$. However, when $m = 1$ ($n = 1$), both $\mathbf{H}_{SD}\mathbf{H}_{SD}^\dagger$ and $\mathbf{H}_{S,RD}\mathbf{H}_{S,RD}^\dagger$ ($\mathbf{H}_{SR,D}\mathbf{H}_{SR,D}^\dagger$) reduce to vectors and it becomes easier to find $d_1(r,t)$ ($d_2(r,t)$).

Although we do not have an explicit expression for $d_1(r,t)$ or $d_2(r,t)$ in the most general case, we can comment on some special cases. First we observe that the half-duplex upper bound depends on the choice of t and the upper bound of (11) is not always equal to the full-duplex bound. As an example consider $m = n = 1, k = 2$, for which the DMT is shown in Fig. 2. To achieve the full-duplex bound for all r, $d_1(r,t)$ needs to have $t \geq 2/3$, whereas $d_2(r,t)$ needs $t \leq 1/3$. As both cannot be satisfied simultaneously, $\min\{d_1(r,t), d_2(r,t)\}$ will be less than the full-duplex bound.

On the other hand, to maximize the half-duplex DMT it is optimal to choose $t = 1/2$ whenever $m = n$. To see this we compare (7) with (8) and note that both $K'_{S,RD} \geq K_{S,D}$ and $K_{SR,D} \geq K_{S,D}$ for $m = n$. Furthermore, for $m = n$ $d_1(r,t) = d_2(r, 1-t)$ and $d_1(r,t)$ is a non-decreasing function in t. Therefore $\min\{d_1(r,t), d_2(r,t)\}$ must reach its maximum at $t = 1/2$.

B. Achievability

To illustrate how the cut-set upper bound in Section III-A can be achieved, in this subsection we assume the relay performs Wyner-Ziv compression [19], [20], [11]. To do this, the relay utilizes its CSI, namely, both the source-destination and relay-destination channel gains to compute the accurate compression rate. The relay listens to the source for t fraction of time and compresses this received signal, assuming the destination will have side information available, which is the signal received via the direct link from the source at the destination. In addition, the compression rate is such that the compressed signal at the relay can reach the destination error-free in the remaining $(1-t)$ fraction of time, in which the relay transmits. Then, for a fixed t

$$R_{CF} = tI(\mathbf{X}_S; \hat{\mathbf{Y}}_R \mathbf{Y}_D | q_1) + (1-t)I(\mathbf{X}_S; \mathbf{Y}_D | \mathbf{X}_R, q_2) \quad (12)$$

is achievable when

$$tI(\hat{\mathbf{Y}}_R; \mathbf{Y}_R | \mathbf{Y}_D, q_1) \leq (1-t)I(\mathbf{X}_R; \mathbf{Y}_D | q_2), \quad (13)$$

where the source and relay input distributions are independent, $\hat{\mathbf{Y}}_R$ is the auxiliary random vector which denotes the compressed signal at the relay and depends on $\mathbf{Y}_R$ and $\mathbf{X}_R$. The proof is very similar to the case when the relay is full-duplex, thus we refer the reader to [20], [11]. More information can also be found in [21], [22], [23] for the half-duplex case.

Assuming $\mathbf{X}_S$ and $\mathbf{X}_R$ are i.i.d. complex Gaussian with zero mean and covariance matrices $\mathrm{SNR}\mathbf{I}_m$, $\mathrm{SNR}\mathbf{I}_k$, $\hat{\mathbf{Y}}_R = \mathbf{Y}_{R,1} + \hat{\mathbf{Z}}_R$, and $\hat{\mathbf{Z}}_R$ is a vector with i.i.d. complex Gaussian entries with zero mean and variance $\hat{N}_R$ that is independent from all other random variables, we define

$$L_{S,RD} \triangleq \left| \mathbf{H}_{S,RD}\mathbf{H}_{S,RD}^{\dagger}\mathrm{SNR} + \begin{bmatrix} (\hat{N}_R + 1)\mathbf{I}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_n \end{bmatrix} \right|, \quad (14)$$

where $\mathbf{H}_{S,RD}$ is defined in (6). $L_{S,D}$, $L'_{S,RD}$ and $L_{SR,D}$ are defined similarly as in (3), (4) and (5) with both m and $m+k$ are substituted with 1. Then we have

$$I(\hat{\mathbf{Y}}_R; \mathbf{Y}_R | \mathbf{Y}_D, q_1) = \log \frac{L_{S,RD}}{L_{S,D}\hat{N}_R^k},$$
$$I(\mathbf{X}_R; \mathbf{Y}_D | q_2) = \log \frac{L_{SR,D}}{L_{S,D}}.$$

Thus using (13) we can choose the compression noise variance $\hat{N}_R$ to satisfy

$$\hat{N}_R = \sqrt[k]{\frac{L_{S,RD}}{U}}, \text{ with } U = L_{S,D}\left(\frac{L_{SR,D}}{L_{S,D}}\right)^{\left(\frac{1-t}{t}\right)}, \quad (15)$$

and (12) becomes

$$R_{CF} = t \log \frac{L_{S,RD}}{\left(\hat{N}_R + 1\right)^k} + (1-t)\log L_{S,D}. \quad (16)$$

Theorem 2: For the half-duplex relay channel with m antenna source, k antenna relay and n antenna destination, for any t, the CF protocol is optimal in the DMT sense as it achieves the DMT upper bound given in Section III-A.

Proof: We find an upper bound on the probability of outage, which is a sum of two terms that are on the order of $P(C_1(t) < R)$ and $P(C_2(t) < R)$ of (9) respectively. This means the CF DMT $d_{CF}(r,t) \geq \min\{d_1(r,t), d_2(r,t)\}$. Hence the CF protocol achieves the bounds in (10) and (11). The details of the proof can be found in [18]. ■

The optimal behavior of the CF protocol arises as soft information is transmitted to the destination. In CF the destination receives the relay's signal with some distortion, but this additional distortion is not as detrimental as the decoding constraint of DF. Furthermore, the CF protocol provides a balance between the relay's reception and transmission intervals.

When $m = k = n = 1$, the DDF protocol achieves [6]

$$d_{DDF}(r) = \begin{cases} 2(1-r) & \text{if } \quad 0 \leq r \leq \frac{1}{2} \\ (1-r)/r & \text{if } \quad \frac{1}{2} \leq r \leq 1 \end{cases}$$

which is suboptimal for $\frac{1}{2} \leq r \leq 1$. We would like to note that, if the relay had all CSI, the DMT of the DDF protocol would not improve. With this CSI the relay could at best perform beamforming with the source; however, this only brings power gain, which does not improve DMT under short term power constraint. It is also worth mentioning that when only relay CSI is present, incremental DF [2] would not improve the DMT performance either. Unless the source knows if the destination has received its message or not, it will never be able to transmit new information to increase achievable multiplexing gains in incremental relaying .

In general it is hard to extend the single antenna DDF results to the multiple antenna case. This is because the instantaneous mutual information DDF achieves in a multiple antenna relay channel is equal to $I_{DDF} = f \log L_{S,D} + (1-f)\log L_{SR,D}$, where f is the random time instant at which the relay does successful decoding. We see that this mutual information has the same form as the second cut-set in (2), but with a random t. Thus it is even harder to compute the DMT for this case than for the second cut-set. Moreover, we think that the multiple antenna DDF performance will still be suboptimal. In [5], we showed that for a multiple antenna full-duplex relay system, the probability that the relay cannot decode is dominant and the DF protocol becomes suboptimal. Therefore, we do not expect any decoding based protocol to achieve the full DMT upper bound in the multiple antenna half-duplex system either.

IV. The Multiple-Access Relay Channel

In this section we examine the DMT for the MARC as shown in Fig. 1(b). We present our results for the MARC with single antenna nodes to demonstrate the basic idea. The results can be extended to multiple antenna MARC.

A. Upper Bounds

The DMT upper bound for the MARC occurs when both users operate at the same multiplexing gain $r/2$, and is given in [12] as

$$d_{MARC}(r) \leq \begin{cases} 2-r & \text{if } 0 \leq r \leq \frac{1}{2} \\ 3(1-r) & \text{if } \frac{1}{2} \leq r \leq 1 \end{cases}, \quad (17)$$

which follows from cut-set upper bounds on the achievable rate. We see that this upper bound has the single user DMT for $r \leq \frac{1}{2}$, and has the relay channel DMT with a two-antenna source for high multiplexing gains. This is because for low multiplexing gains, the typical outage event occurs when only one of the users is in outage, and at high multiplexing gains, the typical outage event occurs when both users are in outage, similar to multiple antenna multiple-access channels [24]. In the next section we study the achievable DMT with CF in this network.

B. Achievability

For the half-duplex MARC the following rate region

$$\begin{aligned} R_1 &\leq tI(X_{S_1}; \hat{Y}_R Y_D | X_{S_2}, q_1) \\ &\quad + (1-t)I(X_{S_1}; Y_D | X_{S_2} X_R, q_2) \\ R_2 &\leq tI(X_{S_2}; \hat{Y}_R Y_D | X_{S_1}, q_1) \\ &\quad + (1-t)I(X_{S_2}; Y_D | X_{S_1} X_R, q_2) \\ R_1 + R_2 &\leq tI(X_{S_1} X_{S_2}; \hat{Y}_R Y_D | q_1) \\ &\quad + (1-t)I(X_{S_1} X_{S_2}; Y_D | X_R, q_2) \end{aligned}$$

is achievable for independent X_{S_1}, X_{S_2}, and X_R subject to

$$tI(\hat{Y}_R; Y_R | Y_D, q_1) \leq (1-t)I(X_R; Y_D | q_2), \quad (18)$$

where $\hat{Y}_R$ is the auxiliary random variable which denotes the quantized signal at the relay and depends on Y_R and X_R [25].

To calculate this achievable rate, we assume X_1 and X_2 are independent, complex Gaussian with zero mean and variance SNR, and $\hat{Y}_R = Y_{R,1} + \hat{Z}_R$, where $\hat{Z}_R$ is a complex Gaussian random variable with zero mean and variance $\hat{N}_R$ and is independent from all other random variables. We define

$$\begin{aligned} L_{S_1,D} &\triangleq 1 + |h_{S_1 D}|^2 \text{SNR} \\ L_{S_2,D} &\triangleq 1 + |h_{S_2 D}|^2 \text{SNR} \\ L_{S_1 S_2,D} &\triangleq 1 + (|h_{S_2 D}|^2 + |h_{S_2 D}|^2) \text{SNR} \\ L_{S_1 S_2 R,D} &\triangleq 1 + (|h_{S_1 D}|^2 + |h_{S_2 D}|^2 + |h_{RD}|^2) \text{SNR} \\ L_{S_1,RD} &\triangleq 1 + (|h_{S_1 R}|^2 + |h_{S_1 D}|^2) \text{SNR} \\ &\quad + \hat{N}_R (1 + |h_{S_1 D}|^2 \text{SNR}) \\ L_{S_2,RD} &\triangleq 1 + (|h_{S_2 R}|^2 + |h_{S_2 D}|^2) \text{SNR} \\ &\quad + \hat{N}_R (1 + |h_{S_2 D}|^2 \text{SNR}) \\ L_{S_1 S_2,RD} &\triangleq \left| \mathbf{H}_{S_1 S_2,RD} \mathbf{H}^\dagger_{S_1 S_2,RD} \text{SNR} + \begin{bmatrix} \hat{N}_R + 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \end{aligned}$$

where $\mathbf{H}_{S_1 S_2,RD} = \begin{bmatrix} h_{S_1 R} & h_{S_2 R} \\ h_{S_1 D} & h_{S_2 D} \end{bmatrix}$.

Using (18) we can choose the compression noise variance $\hat{N}_R$ to satisfy

$$\hat{N}_R = \frac{L_{S_1 S_2,RD}}{U}, \text{ with } U = L_{S_1 S_2,D} \left(\frac{L_{S_1 S_2 R,D}}{L_{S_1 S_2,D}} \right)^{\left(\frac{1-t}{t}\right)}.$$

Then

$$\begin{aligned} R_1 &\leq t \log \frac{L_{S_1,RD}}{\hat{N}_R + 1} + (1-t) \log L_{S_1,D} && (19) \\ R_2 &\leq t \log \frac{L_{S_2,RD}}{\hat{N}_R + 1} + (1-t) \log L_{S_2,D} && (20) \\ R_1 + R_2 &\leq t \log \frac{L_{S_1 S_2,RD}}{\hat{N}_R + 1} + (1-t) \log L_{S_1 S_2,D}. && (21) \end{aligned}$$

To find a lower bound on DMT, we use the union bound on the probability of outage. For symmetric users with a target sum data rate $R = r \log \text{SNR}$

$$\begin{aligned} P(\text{outage}) &\leq P(R_1 < R/2) + P(R_2 < R/2) \\ &\quad + P(R_1 + R_2 < R). \quad (22) \end{aligned}$$

One can prove that the first and second terms $P(R_1 < R/2)$ and $P(R_2 < R/2)$ behave like $\text{SNR}^{-(1-r/2)}$ at high SNR, for any t. As the relay compresses both sources together, the compression noise is on the order of SNR at high SNR and the contribution of the relay to the individual rates, in (19) and (20), is negligible and we observe the direct link behavior (from S_1 to D or from S_2 to D). Finally, we note that the last term has the upper bound

$$P(R_1 + R_2 < R) \quad \dot{\leq} \quad \text{SNR}^{-\min\{d_2(r,t), d_2(r,1-t)\}},$$

where $d_2(r,t)$ is given by Theorem 1 with $m = m_1 + m_2 = 2$, $k = 1$, $n = 1$. We refer the reader to [18] for the proof. To maximize $\min\{d_2(r,t), d_2(r,1-t)\}$ we need to choose $\frac{1}{3} \leq t \leq \frac{2}{3}$ and thus

$$d_{MARC,CF}(r) \geq \min\left\{1 - \frac{r}{2}, 3(1-r)\right\}.$$

On the other hand, $P(\text{outage}) \geq \max\{P(R_1 < R/2), P(R_2 < R/2)\}$, so $d_{MARC,CF}(r) \leq 1 - r/2$. When we combine this with the upper bound in (17), we have the following theorem.

Theorem 3: For the single antenna, half-duplex MARC the CF strategy achieves the DMT

$$d_{MARC,CF}(r) = \min\left\{1 - \frac{r}{2}, 3(1-r)\right\}.$$

This DMT $d_{MARC,CF}(r)$ becomes equal to the upper bound for $r \geq \frac{4}{5}$.

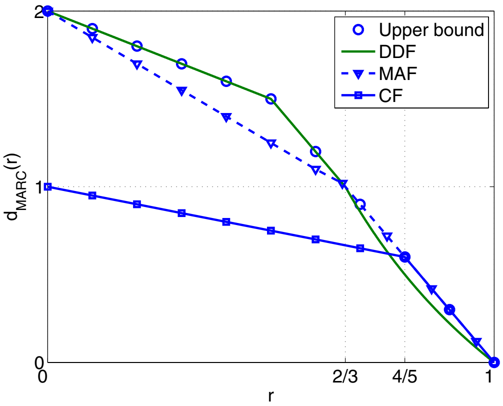

Fig. 3. DMT for MARC. Each node has a single antenna

We observe that the CF relay is not helpful when the single user behavior is dominant, but whenever both users being in outage is the dominant outage event, the system becomes equivalent to the multiple antenna half-duplex relay channel, and CF achieves the DMT upper bound.

We compare our results with the achievable DMT with DDF [12] and MAF [14] for the MARC channel, where

$$d_{MARC,DDF}(r) \geq \begin{cases} 2-r & \text{if } 0 \leq r \leq \frac{1}{2} \\ 3(1-r) & \text{if } \frac{1}{2} \leq r \leq \frac{2}{3} \\ 2\frac{1-r}{r} & \text{if } \frac{2}{3} \leq r \leq 1 \end{cases}$$

$$d_{MARC,MAF}(r) = \begin{cases} 2-3\frac{r}{2} & \text{if } 0 \leq r \leq \frac{2}{3} \\ 3(1-r) & \text{if } \frac{2}{3} \leq r \leq 1 \end{cases}$$

and plot these DMT performances in Fig. 3.

In [5] we observed that for a full-duplex relay channel, when terminals have multiple antennas, DF becomes suboptimal, whereas CF is not. Hence, we conjecture that DDF will not be able to sustain is optimality even in the low multiplexing gain regime when the terminals have multiple antennas. Moreover, it is not easy to extend the MAF protocol for multiple antenna MARC. Even when we have one source, the DMT for the multiple antenna NAF protocol for the relay channel is not known, only a lower bound exists [8]. On the other hand, the CF protocol can easily be extended to multiple antenna MARC. For the multiple antenna case CF will still be optimal whenever decoding all sources together is the dominant error event. However, for some antenna numbers m_1, m_2, and n, single-user behavior will always dominate [24].

V. Conclusion

In this work we show that for the multiple-antenna half-duplex relay channel the CF protocol achieves the DMT upper bound. Although it is hard to find the DMT upper bound explicitly for arbitrary m, k and n, we have solutions for special cases, which show that the half-duplex bound provides a tighter bound than the full-duplex bound in general. We also argue that the DDF protocol or any DF type protocol would be suboptimal in the multiple antenna half-duplex relay channel as they are suboptimal in the full-duplex case. We also study the MARC. In MARC the CF protocol achieves the upper bound for high multiplexing gains, when both users being in outage is the dominant outage event.

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