Cooperative Interference Management: The Role of Cognitive Relaying

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Motivation

• Ad-hoc networks are limited by multiuser interference.
• Information theoretic characterization is challenging
• Exact capacity of interference channel only known in special cases
  • Very strong, strong, noisy interference
• Other approaches:
  • Degrees of freedom
  • Capacity within 1-bit
• **Selfish Transmission**: Interference Channel,
  - [Shannon 61], [Carleial 75], [Han,Kobayashi 81], [Sato 81], [Costa 87],
  [Shang et.al 08], [Annapureddy et.al. 08], [Motahari et.al. 08].
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- **Cognitive Transmission**: Cognitive Relay Channel
  - [Devroye et.al. 06], [Wu et.al. 06], [Maric et.al. 06]
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• **Cooperative Transmission in Selfish Environment**: Interference Relay Channel
Gaussian Interference Relay Channel

- Relay helps both sources simultaneously
  - Time division: Serbetli, Yener
- The sources do not cooperate
- Relay can
  - Decode-forward (Sahin, Erkip 07)
  - Have non-causal information from both $S_1$ and $S_2$; a “cognitive relay”
    - Can model a scenario when relay is very close to sources
- Goal: Uncover effect of relaying on interference
Gaussian Interference Channel

- Best achievable region is due to Han-Kobayashi
  - Rate splitting: $X_1 = X_{1c} + X_{1p}$, $X_2 = X_{2c} + X_{2p}$
    - $X_{1c}$, $X_{2c}$: Common information to be decoded at both destinations
    - $X_{1p}$, $X_{2p}$: Private information to be decoded at $D_1$, $D_2$, respectively
  - Joint decoding at the destinations
  - Time division among the sources. $S_1$-$D_1$ for $t \in [0,t_o]$, $S_2$-$D_2$ for $t \in (t_o,1]$
Capacity of Gaussian Interference Channel

Capacity known under

- **Strong interference**: $a_{12} \geq 1$, $a_{21} \geq 1$
  - Only common information, $X_{1c}, X_{2c}$, is transmitted.
- **Noisy interference**: $0 < a_{12} < a_{12}^*$, $0 < a_{21} < a_{21}^*$.
  - Only private information is rate-sum optimal [Shang et.al 08], [Annapureddy et.al. 08], [Motahari et.al 08]

TDMA gives best rate-sum under

- **Moderate interference**: $a_{12}^* < a_{12} < 1$, $a_{21}^* < a_{21} < 1$
Achievable Region for Interference Channel with a Cognitive Relay

- Rate-splitting at the sources: $X_1 = X_{1c} + X_{1p}$, $X_2 = X_{2c} + X_{2p}$.
- Relay knows both messages non-causally: $X_R = X_{Rc} + X_{R\hat{c}} + X_{Rp}$
  - Beamforming and carbon-copying for common messages
  - Beamforming, interference cancellation and dirty-paper coding (generalized DPC) for private messages
Beamforming for Common Messages

Beamforming: \[ X_{Re} = \sqrt{\frac{P_{R1c}}{P_{1c}}} X_{1c} + \sqrt{\frac{P_{R2c}}{P_{2c}}} X_{2c} \]

Both destinations decode \((X_{1c}, X_{2c})\) by treating all the private information as noise

- More on this later

Rate constraints

\[ R_{ic} \leq \min\{R_{ic}^{D1}, R_{ic}^{D2}\}, i = 1,2 \]

\[ R_{1c} + R_{2c} \leq \min\{R_{tot,c}^{D1}, R_{tot,c}^{D2}\} \]
Carbon Copying for Common Messages

- Destinations first remove \((X_{1c}, X_{2c})\)

- The remaining interference at each destination is different

- Relay treats this interference as state

- Relay uses \(X_{R\hat{c}}\) to carbon copy \((Khisti \ et \ al, \ 07)\) to send additional common information to both destinations at rate \(R_{\hat{c}}\)
Generalized DPC for Private Messages

- From $D_1$’s perspective $X_{2p}$ is interference known at the relay
  - Beamforming/interference cancellation for $(X_{1p}, X_{2p})$, DPC on the remaining interference (similar to Somekh-Baruch et al, 06)

\[ X_{Rp} = X_{1p} \phi_1 \sqrt{\frac{P_{R1p}}{P_{1p}}} X_{1p} + \phi_2 \sqrt{\frac{P_{R2p}}{P_{2p}}} X_{2p} \quad \phi_1 = \pm 1, \phi_2 = \pm 1 \]
Achievable Region

**Theorem:** In a Gaussian interference channel with a cognitive relay, the following gives an achievable rate region,

\[
R_1 = R_{1c} + R_{1\hat{c}} + R_{1p} + R_{1\hat{p}}
\]

\[
R_2 = R_{2c} + R_{2\hat{c}} + R_{2p}
\]

for all source and relay power allocations
Outer Bound-1: Very Strong Interference

\[ R_1 + R_2 \leq \max_{\alpha} \left\{ \log \left( 1 + \frac{(a_{11} \sqrt{P_1} + a_{R1} \sqrt{\alpha P_R})^2}{N_1} \right) + \log \left( 1 + \frac{(a_{22} \sqrt{P_2} + a_{R2} \sqrt{\bar{\alpha} P_R})^2}{N_2} \right) \right\} \]

\( \alpha + \bar{\alpha} = 1 \)

- Destinations remove the interference
S\textsubscript{1} also knows message 2 non-causally

Equivalent to MIMO cognitive channel

\textit{(Sridharan, Vishwanath, 07)}

By symmetry, another bound when S\textsubscript{2} also knows message 1 non-causally
Cognitive Relaying for One-sided Interference

- $D_2$ receives only from its own source $S_2$
- Relay has non-causal information from both sources
- Relay helps $S_1$ by
  - Increasing the signal strength of $X_1$
  - Minimizing the interference due to $X_2$
One-sided Gaussian Interference Channel

- For strong interference, $a_{21} \geq 1$
  - Decoding $X_2$ is optimal, i.e. $X_2 = X_{2c}$.
- For weak interference $a_{21} < 1$
  - Considering $X_2$ as noise is optimal for sum-capacity (*Sason 04*), i.e., $X_2 = X_{2p}$. 
To Treat as Interference or To Decode?

Consider \((S_1, R)\) to \(D_1\) channel only.
Can consider \(S = a_{21}X_2\) as state known at \(R\) only (such as Somekh-Baruch et.al. 06)
However, \(S\) has a structure and may be decoded
Best strategy depends on link gains
Cognitive Relaying for One-sided Interference

- Rate-splitting is not necessary at $S_1$, only performed at $S_2$:
  $X_1 = X_1$, $X_2 = X_{2c} + X_{2p}$

- At relay
  - Beamform with $X_1$ and $X_{2c}$
  - No carbon-copying
  - Remove $X_{2p}$ partly, then DPC

$$X_R = \sqrt{\frac{\rho_1 P_R}{P_1}} X_1 + \sqrt{\frac{\rho_2 P_R}{P_{2c}}} X_{2c} + \phi_2 \sqrt{\frac{\rho_3 P_R}{P_{2p}}} X_{2p} + \sqrt{\rho_3 P_R} X_{1p}, \quad \phi_2 = -1$$
Achievable Rate Region for One-Sided Relay

**Theorem:** An achievable region for one-sided Gaussian interference channel with a cognitive relay is given as

\[ \begin{align*}
R_1^{ach} &= R_1 + R_{1p}, \quad R_2^{ach} = R_{2c} + R_{2p}
\end{align*} \]

where

\[
\begin{align*}
R_1 &\leq C \left( \frac{(a_{11} \sqrt{P_2} + a_{R1} \sqrt{\rho_1 P_R})^2}{N_{eff}} \right) \\
R_{2c} &\leq C \left( \frac{(a_{21} \sqrt{P_{2c}} + a_{R1} \sqrt{\rho_2 P_R})^2}{N_{eff}} \right) \\
R_1 + R_{2c} &\leq C \left( \frac{(a_{11} \sqrt{P_2} + a_{R1} \sqrt{\rho_1 P_R})^2 + (a_{21} \sqrt{P_{2c}} + a_{R1} \sqrt{\rho_2 P_R})^2}{N_{eff}} \right) \\
R_{2c} &\leq C \left( \frac{a_{22}^2 P_{2c}}{N_2} \right), \quad R_{2p} \leq C \left( \frac{a_{22}^2 P_{2p}}{N_2} \right), \quad R_{2c} + R_{2p} \leq C \left( \frac{a_{22}^2 P_2}{N_2} \right) \\
R_{1p} &\leq C \left( \frac{a_{R1}^2 \rho_4 P_R}{N_1} \right) \quad N_{eff} = (a_{21} \sqrt{P_{2p}} - a_{R1} \sqrt{\rho_3 P_R})^2 + a_{R1}^2 \rho_4 P_R + N_1
\end{align*}
\]
Suppose $D_1$ knows $W_2$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{(a_{11} \sqrt{P_1} + a_{R1} \sqrt{P_R})^2}{N_1} \right) + \frac{1}{2} \log \left( 1 + \frac{a_{22}^2 P_2}{N_2} \right)$$

Bound is achievable for very strong relay interference condition

$$a_{21} \geq a_{22} \sqrt{\frac{N_1 + (a_{11} \sqrt{P_1} + a_{R1} \sqrt{P_R})^2}{N_2}}$$
• $S_1$ also knows message 2 non-causally

• $(S_1, R)$ do DPC with respect to $X_2$
  • Same as no interference
  • Two parallel channels, same as Outer Bound-1
Achievable Rate Sum: Channel Parameters

\[ a_{11} = 1, \ a_{22} = 4, \ a_{R1} = 1, \]
\[ P_1 = P_2 = P_R = 10, \ N_1 = N_2 = 1 \]

\[ \alpha_{2c} = \text{Power of } X_{2c} \]
\[ \rho_1 = \text{Beamforming power for } X_1 \]
\[ \rho_2 = \text{Beamforming power for } X_{2c} \]

Very-strong interference cut-off
Achievable Rate Sum: Channel Parameters

\[ \rho_4 = \text{DPC power} \]

\[ -\rho_3 = \text{Interference cancellation for } X_{2p} \]
Achievable Rate-Sum and Outer Bound

Very-strong interference cut-off

\[ R_{D1}^{MARC} \]
Conclusion

- Interference channel with a cognitive channel
  - Achievable region
  - Outer bounds
- One-sided interference with a cognitive relay
- Relays can be used to
  - Increase data rates
  - Manage interference
- Treat as interference or decode?
  - Depends on whether help is available
  - Presence of a relay alters the interference regimes