Simple Derivation of the Exact Pairwise Error Probability for Rayleigh Block Fading Channels

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1 Derivation of the Pairwise Error Probability

We consider a single input single output (SISO) flat Rayleigh block fading channel with L blocks. This model arises when the coherent time of the channel is much longer than one symbol interval (such as GSM, IS54-TDMA) [1], or in cooperative systems where partner mobiles help the original node transmit parts of the codeword [2]. For the block fading channel, the averaged pairwise error probability (PEP) between two arbitrary codewords $\underline{c}$ and $\underline{e}$ can be obtained by calculating the expected value of the conditional PEP with respect to Rayleigh fading r_i [1], $P(\underline{c} \rightarrow \underline{e}) = E_{r_i}\left[Q\left(\sqrt{\sum_{i=1}^{L} r_i^2 a_i}\right)\right]$, where r_i's are i.i.d. Rayleigh with parameter 1. Based on the values of a_i, which is the product of the code Euclidean distance, d_i^2, and the signal to noise ratio, γ_i, in block i, we classify the PEP calculation into two types, one is the symmetric case where all a_i's are equal; the other is the asymmetric case where a_i's are not necessarily identical.

In the symmetric case, we let $r = r_1$ and $z_j = \frac{r_{j+1}}{r}, j = 1, \ldots, L-1$. Then PEP can be evaluated as the expected value with respect to r and z_j. Note that r and z_j are correlated. By computing $f(r, z_1, \ldots, z_{L-1})$ and using the integral property of the Q-function [3, eqn(3-63)], we can easily obtain the PEP.

In the asymmetric case where a_i's are not identical for all L blocks, we evaluate PEP with the aid of the alternate finite integral form of the Q-function [4]. Hence, we can write the PEP as, $P(\underline{c} \rightarrow \underline{e}) = \frac{1}{\prod_{i=1}^{L} a_i} \cdot \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{\prod_{i=1}^{L}\left(\frac{1}{\sin^2\theta} + \frac{1}{a_i}\right)} \mathrm{d}\theta$, where $a_i = \frac{\gamma_i d_i^2}{4}$. Performing partial fraction expansion on the integrand and using our result from the symmetric case, we can easily integrate with respect to θ and get the exact PEP as

$$P(\underline{c} \rightarrow \underline{e}) = \frac{1}{\prod_{i=1}^{L} a_i} \sum_{p=1}^{k} \sum_{m=1}^{p} \sum_{q=1}^{t_p} \frac{A_{q,m}^{(p)} \left(a_q^{(p)}\right)^m}{2^m} \left(1 - \frac{1}{u_q^{(p)}}\right)^m \sum_{l=0}^{m-1} 2^{-l} \binom{m-1+l}{l} \left(1 + \frac{1}{u_q^{(p)}}\right)^l$$

where $a_q^{(p)}$ represents the qth p-repeated pole of the integrand, $u_q^{(p)} = \sqrt{1 + \frac{1}{a_q^{(p)}}}$, $A_{q,m}^{(p)}$ denotes the mth residue associated with $a_q^{(p)}$, t_p is the number of distinct p-repeated poles and k is the largest number of repeated poles.

2 Numerical Examples

In order to validate the general PEP from above, we use BPSK modulation and simulate the PEP between the transmitted all-zero codeword and the shortest error path. In all the examples, we consider [13, 15, 15, 17] convolutional code for 2-block fading channels. The Hamming distance in each block becomes 7 and 6 respectively. Fig. 1 shows the PEP result for the traditional 2-block fading channel, where $\gamma_1 = \gamma_2$. Fig. 2 gives the PEPs for the cases when the SNR in the second block is 0 dB and 4 dB respectively and the SNR in the first block changes. This could be the scenario for a coded cooperative system [2] where the original mobile uses [13, 15] convolutional code and the partner uses parity bits from [15 17] convolutional code to help the original mobile transmit its information bits. We assume the inter-user channel is perfect. The mobile and the partner are assumed to experience independent quasi-static fading levels, hence the block fading channel model is appropriate for this system. From Figures 1 and 2, we can observe that our analytical PEP matches the simulated PEP exactly.

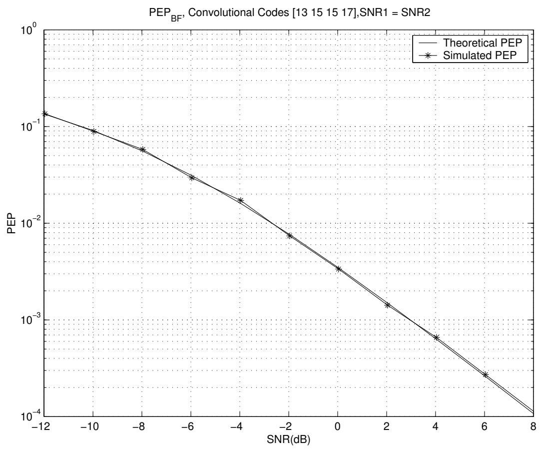

Figure 1: PEP for regular 2-block fading channels, $SNR_1 = SNR_2$

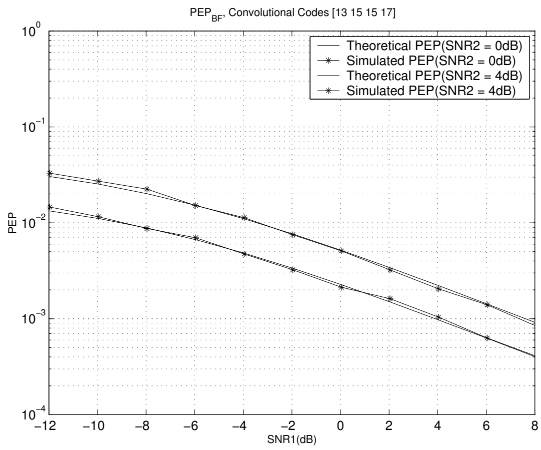

Figure 2: PEP for 2-user coded cooperative systems.

3 Conclusion

In this work, we provide a simple technique to compute the exact PEP for Rayleigh block fading channels. Our technique can be extended to MIMO Rayleigh block fading channels. The exact PEP not only gives us a good guideline in the code design, but also can help us with the analytical study of the cooperation benefits in coded cooperation systems.

References

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