Diversity in Relaying Protocols with Amplify and Forward

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***Abstract*— We examine a network consisting of one source, one destination and two amplifying and forwarding relays and consider a scenario in which destination and relays can have various processing limitations. For all possible diversity combining schemes at the relays and at the destination, we find diversity order results analytically and confirm our findings through numerical calculations of bit error rate (BER) versus signal-to-noise-ratio (SNR) curves. We compare our results with direct transmission, well known transmit diversity methods and traditional multihop transmission and conclude that diversity reception in multihop networks provides the lowest error rate.**

I. INTRODUCTION

In wireless networks path loss is a major problem. Path loss is the attenuation in the amplitude of the signal when it travels from the transmitter to the receiver. To mitigate the effects of path loss and to provide significant power savings, information is routed with multihop transmissions in ad hoc and more recently cellular networks [9].

However, in wireless channels, along with path loss, fading also effects the signal quality. Fading degrades the performance of the system when signal components that are received over different propagation paths add destructively [6]. Temporal, frequency and spatial diversity are the basic diversity techniques to overcome the effects of fading. In addition to these traditional diversity techniques, mobiles can also relay each other's information and the signals coming from the source and the relays can be used to provide additional diversity. The spatial diversity obtained through this virtual array is also referred to as "user cooperation diversity" or "cooperative diversity" [1], [4], [8].

In a wireless network, selection of relays and how to utilize them are two important issues. The former has been investigated in [7] for path loss channels from a minimum energy perspective. How the relays and the destination should process information in the presence of fading is explored in [1], [2], [4]. Laneman *et. al.* examine several relaying protocols for one relay and show that amplifying and forwarding which corresponds to the relay retransmitting its received signal rescaled to its own power level, achieves full diversity. In [1] and [2] it is shown that amplifying and forwarding outperforms both direct transmission and decoded relaying, in which the relays must first decode their received signals. It is also argued that diversity reception, where every node listens to every other node transmitting before itself, has better performance than traditional multihop communication where each node listens to the preceding node only. In addition to these, Gastpar and Vetterli show that amplifying and forwarding will achieve the capacity when number of relays tend to infinity [3]. Motivated by these results, we also assume amplifying and forwarding relays in our problem setup.

As the network size and the number of nodes in the network increase, it will not be feasible for every node to listen to all the preceding nodes due to increasing complexity. Besides, because of large path loss introduced over long distances, diversity gains introduced will be insignificant. This has led us to investigate the effect of different diversity processing schemes at the destination and at the relays.

In this work, we examine the two relay case with relays that amplify and forward their incoming signals. We will compare the performance of all possible diversity combining schemes at the relays and at the destination. This includes an analytical study of the diversity order achieved and numerical BER calculations.

In the next section system model is described. To motivate our analytical diversity order expressions, we present BER graphs for various diversity combining schemes in Section III. Section IV includes a high signal analysis to establish diversity orders of various schemes studied.

II. SYSTEM MODEL

Figure 1 shows our system model where there is only one source-destination pair. There can be up to two relays. We denote source by S, destination by D and the relays as R_1 and R_2. The nodes S, R_1, R_2 and D are labeled as 1, 2, 3 and 4 respectively and each d_{ij} in the figure indicates the distance between node i and j. The various branches illustrate different communication links that are utilized at a given time. For example if destination listens to source, this is illustrated

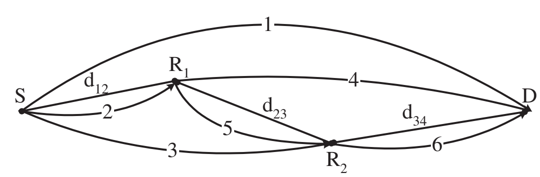

Fig. 1. A two-relay system

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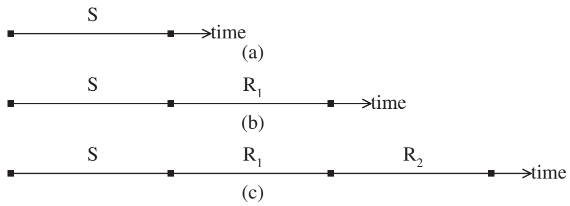


Fig. 2. Communication with relays (a) Direct transmission, (b) One relay, (c) Two relays.

by link 1 in Figure 1. Every link in our system has path loss and flat Rayleigh fading.

We assume time division multiple access between all nodes, although any orthogonal multiple accessing scheme is suitable. For example, if only R_1 is utilized in the system, source transmits in the first time slot and R_1 in the second as shown in Figure 2(b).

All transmitting nodes in the system use BPSK no matter how many hops are taken. The relays amplify and forward each bit separately. This ends up increasing the delay or decreasing the throughput. One can overcome this problem by increasing the size of the constellations as the number of hops increase. Another method is to utilize distributed space-time coded protocols which requires synchronization among the relays [5]. However, the spectral efficiency loss introduced is not a bottleneck in delay tolerant applications. Furthermore some delay is inherent in multihop systems and is probably inevitable for communicating over large distances with limited power.

In the one relay case, source transmits in the first time slot and R_1 listens to source over link 2 in Figure 1. Relay amplifies, that is normalizes the received signal power to its own power value, and forwards the signal to the destination over link 4. Then, destination can either process the signal from R_1, which we denote as $R_1(S)$, or can combine the source signal, which is received in the first time slot through link 1, with the signal from the relay. We denote this second scheme by $S + R_1(S)$. The terms in the parentheses denote the links the relay listens to and the terms added with a plus sign denote the links the destination processes. As another example, $S + R_1(S) + R_2(R_1)$ means destination combines the signals received over the links 1, 4 and 6; R_1 listens to link 2 and R_2 listens to link 5. In the particular case of $R_2(S, R_1)$, R_2 listens to both links 3 and 5 and forms a resultant signal using maximal ratio combining. Then this combined signal is amplified and forwarded to the destination over link 6. We assume all receiving nodes have perfect channel state information to perform maximal ratio combining.

All possible schemes resulting from two relays are listed in Table I as well as their diversity orders. The diversity orders will be discussed in Sections III and IV. In our comparisons, we keep the total network power the same for all schemes and assume it is equally shared between the source and the relays. We will first present BER versus SNR characteristics corresponding to all different processing schemes in Table I.

Scheme	Diversity Order
S	1
$R_1(S)$	1
$R_2(R_1)$	1
$R_2(S, R_1)$	1
$S + R_1(S)$	2
$S + R_2(R_1)$	2
$S + R_2(S, R_1)$	2
$R_1(S) + R_2(S)$	2
$R_1(S) + R_2(R_1)$	1
$R_1(S) + R_2(S, R_1)$	2
$S + R_1(S) + R_2(S)$	3
$S + R_1(S) + R_2(R_1)$	2
$S + R_1(S) + R_2(S, R_1)$	3

TABLE I
LIST OF SCHEMES AND DIVERSITY ORDERS.

The BER is calculated by Monte-Carlo integration for two different cases. In the first case, we fix the relay locations. In the second case, we alter this assumption and assume that R_1 and R_2 are uniformly located on the line joining the source and the destination, the one that is closer to the source will be named R_1 and, the other R_2. We then average BER over all locations. Our graphical results illustrated in the next section will be supported by analytical diversity order expressions in Section IV.

III. Performance Analysis

Before we present analytical diversity expressions in the next section, BER versus SNR plots will be shown here for all schemes, grouped according to the diversity orders provided. Both fixed and average results are plotted with respect to total transmitted power. We also plot well known transmit diversity curves in our graphs to provide a reference [6]. Without loss of generality we assume source to destination distance is 1. To demonstrate the ideas R_1 and R_2 locations are fixed at 0.33 and 0.67 respectively on the source-destination line. Path loss exponent α is 4, that is signal energy decreases as $d^{-\alpha}$ where d is the distance between the transmitter and the receiver.

As can be seen from Figure 3, S, $R_1(S)$, $R_2(R_1)$, $R_2(S, R_1)$ and $R_1(S) + R_2(R_1)$ schemes have one level of diversity. This means if only one branch is processed at the destination, at most one level of diversity can be obtained. We also observe the benefits of multihop transmissions: As expected using two relays ($R_2(R_1)$) provides better performance than direct transmission (S) or using only one relay ($R_1(S)$). However, if the second relay has extra processing capability and can combine source and R_1 ($R_2(S, R_1)$), the destination will gain in performance akin to "coding gain" resulting from repetition coding. This gain over traditional multihop is as much as 4 dB. Interestingly, if the branches combined at the destination are correlated, this will not result in diversity gains but substantial coding gains. This is the case in $R_1(S) + R_2(R_1)$. Note that if link 2 fails, this will affect both $R_1(S)$ and $R_2(R_1)$, causing failure at the destination. This intuition will be confirmed by our analytical study in Section IV. The BER curves get closer to each other and the

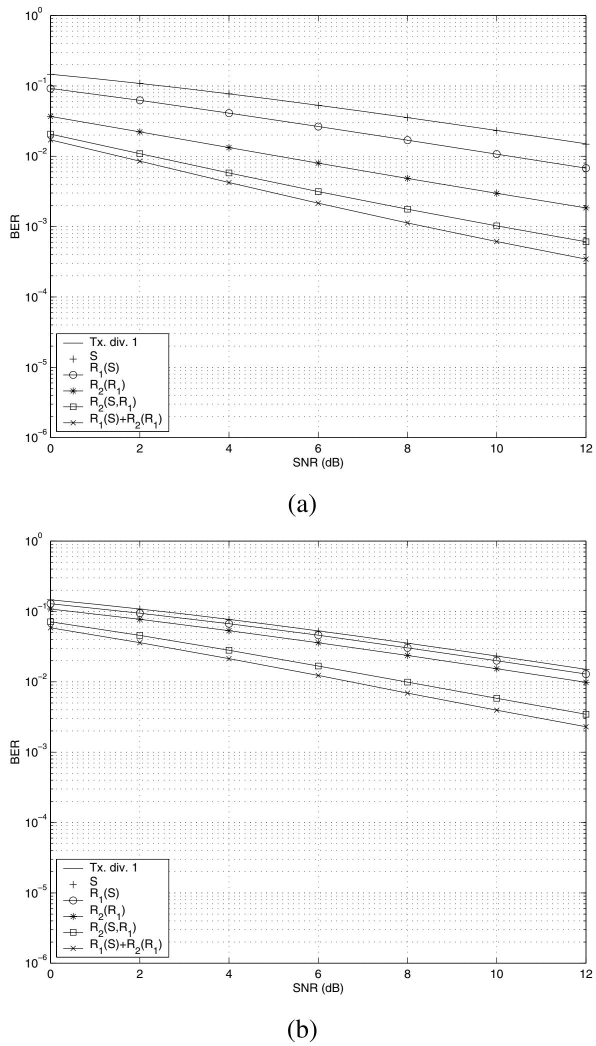


Fig. 3. Diversity order one (a) Fixed distances, (b) Average.

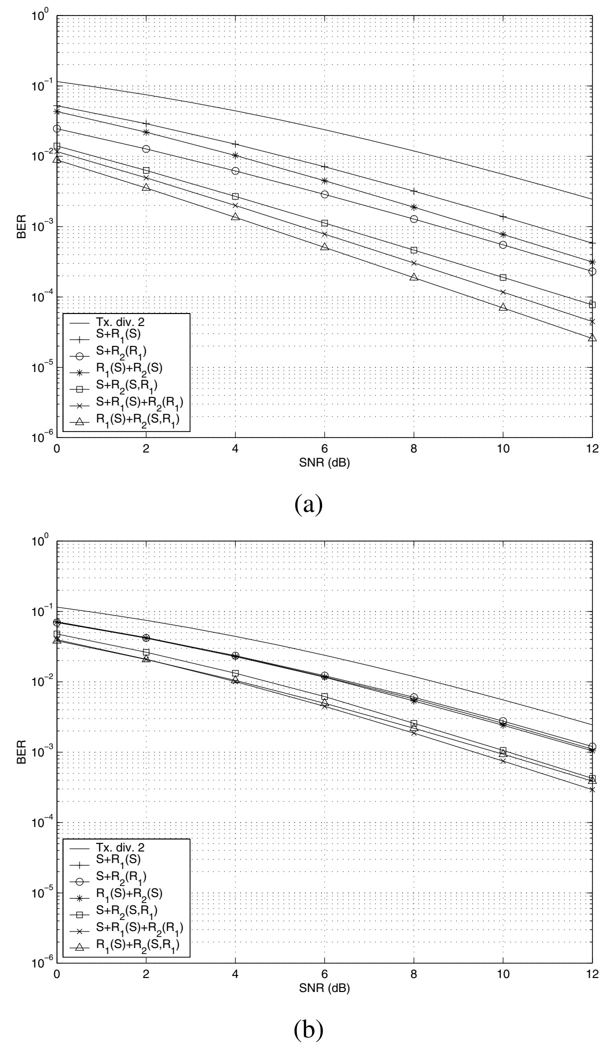


Fig. 4. Diversity order two (a) Fixed distances, (b) Average.

gains over transmit diversity become lower when the BER is averaged over all possible R_1 and R_2 locations.

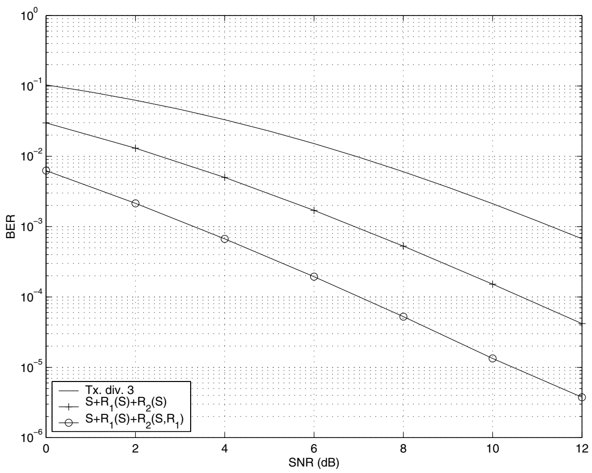
When we compare the BER curves of $S + R_1(S)$, $S + R_2(R_1)$ and $S + R_2(S, R_1)$ in Figure 4, we observe they all have the same slope for high SNR, thus diversity level is 2. Addition of the source branch (link 1 in Figure 1) introduces the second level of diversity. The system favors $S+R_1(S)$ over two level transmit diversity at the source and $R_1(S) + R_2(S)$ over $S + R_1(S)$ since path loss is dominant. This can also be observed for $S + R_2(S, R_1)$ and $R_1(S) + R_2(S, R_1)$. This also means that if certain level of diversity is desired at the destination and if relays are available, then it is not wise to employ transmit diversity for the system investigated. Besides, similar to one diversity case $S + R_1(S) + R_2(R_1)$ has only two levels of diversity because of correlated links. Therefore, if destination has processing limitations, it must choose the branches processed in a smart way because number of branches processed is not always equal to the diversity order.

The schemes $S+R_1(S)+R_2(S)$ and $S+R_1(S)+R_2(S, R_1)$ both have three levels of diversity as shown in Figure 5. The latter is the best among all schemes as it makes full use of all the information present in the system.

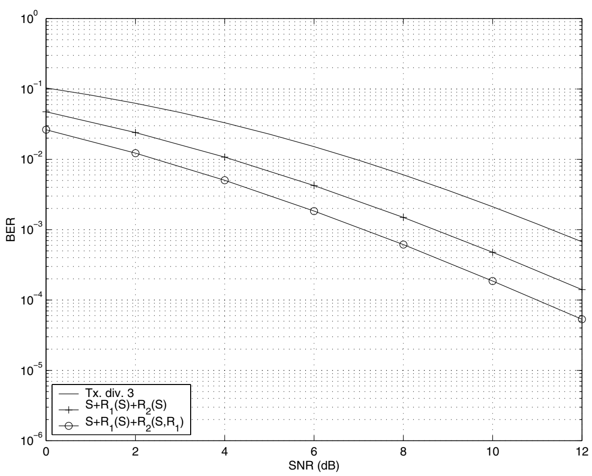
Among all the schemes we investigate, direct transmission (S) is the worst in terms of error probability and full diversity transmission ($S + R_1(S) + R_2(S, R_1)$) is the best. We obtain significant gains if relays are deployed. When destination or relays can combine more than one branch, diversity gains are introduced. If diversity combining is not available due to processing limitations, then the gains multihop transmissions provide will be more emphasized.

IV. Diversity Order Results

In this section we confirm analytically the diversity observations we made in Section III. The diversity orders of S, $R_1(S)$ and $S + R_1(S)$ are proved in [4]. In this paper this analysis is extended to cover two relays. We will analyze the two cases $R_2(R_1)$ and $R_2(S, R_1)$ in detail starting from the transmitted and received signals in the system. Then we will



(a)



(b)

Fig. 5. Diversity order three (a) Fixed distances, (b) Average.

argue that similar expressions for the schemes $R_2(S)+R_2(R_1)$ and $R_1(S)+R_2(S)$ can be derived. Note that relays do not need to be collinear with the source and the destination for the analytical results to be valid.

To prove the diversity order of k, first we argue that as total SNR increases, $P[\gamma_{eq} < c]$, where γ_{eq} is the "equivalent received SNR" of the suggested scheme and c is a constant, is inversely proportional to $\mathrm{SNR}^{\mathrm{k}}$. Then one can easily show that the overall BER, given by $E[Q(\sqrt{2\gamma_{eq}})]$, where expectation is with respect to γ_{eq}, also decays as $\mathrm{SNR}^{-\mathrm{k}}$. Note that arguing large SNR behavior of $P[\gamma_{eq} < c]$ also gives us the asymptotic behavior of the outage probability [4].

A. Diversity of $R_2(R_1)$

Recall that this corresponds to traditional multihop. The transmission scheme follows that of Figure 1(c). In the first time interval source transmits $x_1[n]$ and

$$y_2[n] = a_{12}\sqrt{\mathcal{E}}x_1[n] + z_2[n] \tag{1}$$

will be received by the first relay. In the second time interval R_1 will amplify the signal to its own power level and transmit

$$x_2[n+1] = \beta_2 y_2[n] \tag{2}$$

where

$$\beta_2 = \sqrt{\frac{\mathcal{E}}{|a_{12}|^2\mathcal{E} + \mathcal{N}_o}} \tag{3}$$

is the amplification factor. R_2 receives

$$y_3[n+1] = a_{23}x_2[n+1] + z_3[n+1]. \tag{4}$$

Then

$$\begin{aligned} x_3[n+2] &= \beta_3 y_3[n+1], && (5)\\ \beta_3 &= \sqrt{\frac{\mathcal{E}}{|a_{23}|^2\mathcal{E} + \mathcal{N}_o}} && (6)\end{aligned}$$

is transmitted to the destination, here β_3 is the amplification factor of R_2. Finally destination receives

$$y_4[n+2] = a_{34}x_3[n+2] + z_4[n+2]. \tag{7}$$

Path loss and fading effects are included in a_{ij}, which is a zero mean, complex Gaussian random variable and is independent for different (i,j) pairs. Both real and imaginary parts of a_{ij} have variance σ_{ij}^2 which is proportional to $1/d_{ij}^{\alpha}$ where α is the path loss exponent. Then $|a_{ij}|^2$ has an exponential distribution with parameter λ_{ij} with $\lambda_{ij} = 1/(2\sigma_{ij}^2)$. Also, the source and both relays are assumed to have equal energy per bit $\mathcal{E}$ and equal noise variance $\mathcal{N}_o$.

The signal in equation (7) will result in an equivalent received SNR of $f(f(\gamma_{12},\gamma_{23}),\gamma_{34})$ at the destination where $f(x,y) = \frac{xy}{x+y+1}$ and $\gamma_{ij} = |a_{ij}|^2\mathcal{E}/\mathcal{N}_o$. For this particular γ_{eq}, following similar steps as in [4], one can show that:

$$\liminf_{\mathrm{SNR}\to\infty} P(f(f(\gamma_{12},\gamma_{23}),\gamma_{34}) < c) \geq \frac{c}{\mathrm{SNR}}(\lambda_{12}+\lambda_{23}+\lambda_{34})$$

$$\limsup_{\mathrm{SNR}\to\infty} P(f(f(\gamma_{12},\gamma_{23}),\gamma_{34}) < c) \leq \frac{c}{\mathrm{SNR}}(\lambda_{12}+\lambda_{23}+\lambda_{34})$$

Therefore

$$\lim_{\mathrm{SNR}\to\infty} P(f(f(\gamma_{12},\gamma_{23}),\gamma_{34}) < c) = \frac{c}{\mathrm{SNR}}(\lambda_{12}+\lambda_{23}+\lambda_{34}) \tag{8}$$

This means that $R_2(R_1)$, which corresponds to multihop transmission, has one level of diversity.

B. Diversity of $R_2(S,R_1)$

This scheme is different from the previous one because R_2 combines all signals it receives, namely from the source and R_1. Hence in addition to $y_3[n+1]$, $y_3[n]$ is also processed at R_2. These two signals are combined with maximal ratio combining using the coefficients

$$c_1 = a_{13}^*, \qquad c_2 = \frac{a_{12}^* a_{23}^* \beta_2}{|a_{23}|^2|\beta_2|^2 + 1}. \tag{9}$$

Thus R_2 transmits

$$x_3[n+2] = \beta_3(c_1 y_3[n] + c_2 y_3[n+1]) \tag{10}$$

with

$$\beta_3 = \frac{\mathcal{E}/\mathcal{N}_o}{\sqrt{[\gamma_{13} + f(\gamma_{12},\gamma_{23})][\gamma_{13} + f(\gamma_{12},\gamma_{23}) + 1]}}. \tag{11}$$

The signal received from the destination $y_4[n+2]$ again follows equation (7). Similar to the previous scheme, destination only processes $y_4[n+2]$ and the equivalent SNR, γ_{eq}, becomes $f(\gamma_{13} + f(\gamma_{12},\gamma_{23}),\gamma_{34})$. We can show that

$$\begin{aligned} \liminf_{\mathrm{SNR}\to\infty} P(f(\gamma_{13} + f(\gamma_{12},\gamma_{23}),\gamma_{34}) < c) &\geq \frac{c}{\mathrm{SNR}}\lambda_{34}, \\ \limsup_{\mathrm{SNR}\to\infty} P(f(\gamma_{13} + f(\gamma_{12},\gamma_{23}),\gamma_{34}) < c) &\leq \frac{c}{\mathrm{SNR}}\lambda_{34} \end{aligned}$$

and

$$\lim_{\mathrm{SNR}\to\infty} P(f(\gamma_{13} + f(\gamma_{12},\gamma_{23}),\gamma_{34}) < c) = \frac{c}{\mathrm{SNR}}\lambda_{34}. \tag{12}$$

Although R_2 receives two copies of the source signal over two independent channels, this does not result in an extra diversity level at the destination. This is because the overall performance is limited by link 6 that connects R_2 to the destination. However, this extra processing provides gains resembling network coding. This has its interesting implications on larger networks. That is, even if the destination is bound to process one branch, if the network has some powerful nodes that can listen to as many branches as they can, then destination will obtain extra benefits.

C. Diversity of $R_1(S) + R_2(R_1)$

Although it sounds counter-intuitive, $R_1(S) + R_2(R_1)$ has also one level of diversity. Hence even if destination processes both relays, no extra diversity is gained. This is because when x and y are non-negative, $f(x,y) \leq \min(x,y)$ and for this scenario

$$\begin{aligned} \gamma_{eq} &= f(\gamma_{12},\gamma_{24}) + f(f(\gamma_{12},\gamma_{23}),\gamma_{34}) \\ &\leq 2\gamma_{12}. \end{aligned} \tag{13}$$

Hence

$$P(\gamma_{eq} < c) \geq P(2\gamma_{12} < c) \tag{14}$$

and

$$\lim_{\mathrm{SNR}\to\infty} P(\gamma_{12} < c) = \frac{c}{\mathrm{SNR}}\lambda_{12}. \tag{15}$$

We conclude that $R_1(S) + R_2(R_1)$ has at most one level of diversity. On the other hand, it is obvious that this scheme has to provide at least one level of diversity, because in the worst case, destination can choose not to process one of the branches and we already know that each branch that are combined in this scheme has one level of diversity. This interesting result is due to the fact that these two branches are correlated via the common second branch in Figure 1.

D. Diversity of $R_1(S) + R_2(S)$

This is similar to one relay amplify and forward of [4] but now we have two relays.

$$\begin{aligned} &P(f(\gamma_{12},\gamma_{24}) + f(\gamma_{13},\gamma_{34}) < c) \\ &\quad\leq P(f(\gamma_{12},\gamma_{24}) < c)P(f(\gamma_{13},\gamma_{34}) < c) \\ &\quad= \frac{c^2}{\mathrm{SNR}^2}(\lambda_{12} + \lambda_{24})(\lambda_{13} + \lambda_{34}) \end{aligned} \tag{16}$$

and

$$\begin{aligned} &P(f(\gamma_{12},\gamma_{24}) + f(\gamma_{13},\gamma_{34}) < c) \\ &\quad\geq P(f(\gamma_{12},\gamma_{24}) < \frac{c}{2})P(f(\gamma_{13},\gamma_{34}) < \frac{c}{2}) \\ &\quad= \frac{c^2}{4\mathrm{SNR}^2}(\lambda_{12} + \lambda_{24})(\lambda_{13} + \lambda_{34}). \end{aligned} \tag{17}$$

Hence we obtain two level diversity. Diversity orders of other schemes can be easily derived using similar arguments. The results are listed in Table I. We observe that our analytical results agree with the observations made in Section III.

V. Conclusion

In this paper we investigated the diversity order effects of various processing schemes at the destination and at the relays in a two relay network. Relays employ the amplify and forward strategy. It is shown that if the links combined are independent, then the diversity order is equal to the number of links combined and a network with r relays can provide up to r level diversity. On the other hand, combining correlated links does not provide diversity but extra coding gains due to repetition of information. In addition to these, if the relays can do extra processing, even though the number of branches combined at the destination is kept constant, substantial gains over direct and traditional multihop transmission are obtained.

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