The Matlab Residue Command

The Matlab command `residue` allows one to do partial fraction expansion.

```
>> help residue

 RESIDUE Partial-fraction expansion (residues).
    [R,P,K] = RESIDUE(B,A) finds the residues, poles and direct term of
    a partial fraction expansion of the ratio of two polynomials B(s)/A(s).
    If there are no multiple roots,
       B(s)       R(1)       R(2)             R(n)
       ----  =  -------- + -------- + ... + -------- + K(s)
       A(s)     s - P(1)   s - P(2)         s - P(n)
    Vectors B and A specify the coefficients of the numerator and
    denominator polynomials in descending powers of s.  The residues
    are returned in the column vector R, the pole locations in column
    vector P, and the direct terms in row vector K.  The number of
    poles is n = length(A)-1 = length(R) = length(P). The direct term
    coefficient vector is empty if length(B) < length(A), otherwise
    length(K) = length(B)-length(A)+1.

    If P(j) = ... = P(j+m-1) is a pole of multiplicity m, then the
    expansion includes terms of the form
                 R(j)        R(j+1)                R(j+m-1)
               -------- + ------------   + ... + ------------
               s - P(j)   (s - P(j))^2           (s - P(j))^m

    [B,A] = RESIDUE(R,P,K), with 3 input arguments and 2 output arguments,
    converts the partial fraction expansion back to the polynomials with
    coefficients in B and A.

    See also POLY, ROOTS, DECONV.
```

To perform partial fraction expansion on $T(z)$

$$T(z) = \frac{4z + 1}{z^2 - z - 2}$$

we enter the following Matlab commands.

```
>> num = [4 1];
>> den = [1 -1 -2];
>> [r,p,k] = residue(num,den)

r =
     3
     1


p =
     2
    -1


k =
     []
```

This tells us that,

$$T(z) = \frac{3}{z - 2} + \frac{1}{z + 1}.$$

The `residue` command also works when some poles are complex. For example, to find the partial fraction expansion of $X(z)$

$$X(z) = \frac{4\,z + 3}{2\,z^3 - 3.4\,z^2 + 1.98\,z - 0.406}$$

we can use the following Matlab commands.

```
>> num = [4 3];
>> den = [2 -3.4 1.98 -0.406];
>> [r,p,k] = residue(num,den)


r =
  36.2500
 -18.1250 +13.1250i
 -18.1250 -13.1250i


p =
   0.7000
   0.5000 + 0.2000i
   0.5000 - 0.2000i


k =
     []
```

This tells us that,

$$X(z) = \frac{36.25}{z - 0.7} + \frac{-18.125 + 13.125i}{z - (0.5 + 0.2i)} + \frac{-18.125 - 13.125i}{z - (0.5 - 0.2i)}.$$

Notice that the residues and poles appear in complex-conjugate pairs. In fact, this is always the case when the coefficients in $X(z)$ are real.

Here is an example with a repeated pole. To perform partial fraction expansion on $T(z)$

$$T(z) = \frac{2z+1}{z^3+5z^2+8z+4}$$

we enter the following Matlab commands.

```
>> num = [2 1];
>> den = [1 5 8 4];
>> [r,p,k] = residue(num,den)

r =
    1.0000
    3.0000
   -1.0000


p =
   -2.0000
   -2.0000
   -1.0000


k =
     []
```

This tells us that,

$$T(z) = \frac{1}{z+2} + \frac{3}{(z+2)^2} + \frac{-1}{z+1}$$

The `residue` command also works in the other direction. To write $G(z)$,

$$G(z) = \frac{5}{z-3} + \frac{6}{z+4} - \frac{7}{z+1/5}$$

as the ratio of two polynomials we can use the following commands.

```
>> r = [5 6 -7];
>> p = [3 -4 -1/5];
>> [num,den] = residue(r,p,[])


num =


    4.0000   -2.8000   84.4000


den =


    1.0000    1.2000  -11.8000   -2.4000
```

This tells us that

$$G(z) = \frac{4z^2 - 2.8\,z + 84.4}{z^3 + 1.2z^2 - 11.8z - 2.4}.$$

The argument `k` is needed only when the degree of the numerator is greater than or equal to the degree of the denominator, so it was not needed for these examples.

In this example, the degree of the numerator is greater than the degree of the denominator, so `k` is required.

$$X(z) = \frac{3\,z^4 - 1.1\,z^3 + 0.88\,z^2 - 2.396\,z + 1.348}{z^3 - 0.7\,z^2 - 0.14\,z + 0.048}$$

The partial fraction expansion of $X(z)$ can be found using the following Matlab commands.

```
>> num = [3   -1.1    0.88   -2.396    1.348];
>> den = [1   -0.7   -0.14    0.048];
>> [r,p,k] = residue(num,den)


r =
    1.0000
    4.0000
   -3.0000


p =
    0.8000
   -0.3000
    0.2000


k =
    3.0000    1.0000
```

This tells us that

$$X(z) = 3\,z + 1 + \frac{1}{z-0.8} + \frac{4}{z+0.3} - \frac{3}{z-0.2}$$