A NOTE ON CIRCULAR CONVOLUTION

Suppose \( x(n) \) and \( g(n) \) are finite length signals with the same length:

\[
x(n), \quad n = 0, \ldots, N - 1
\]
\[
g(n), \quad n = 0, \ldots, N - 1.
\]

Conventional (linear) convolution is given by

\[
(x * g)(n) = \sum_k x(k)g(n - k)
\]

\[
= x(0)g(n) + x(1)g(n - 1) + \cdots + x(N - 1)g(n - (N - 1))
\]

So the linear convolution

\[
y = [x(0) \; x(1) \; x(2) \; x(3)] * [g(0) \; g(1) \; g(2) \; g(3)]
\]

can be written out as

\[
y = x(0) [g(0) \; g(1) \; g(2) \; g(3) \; 0 \; 0 \; 0] + x(1) [0 \; g(0) \; g(1) \; g(2) \; g(3) \; 0 \; 0] + x(2) [0 \; 0 \; g(0) \; g(1) \; g(2) \; g(3) \; 0] + x(3) [0 \; 0 \; 0 \; g(0) \; g(1) \; g(2) \; g(3)].
\]

The sequence \( y \) is of length 7.

In circular convolution (or ‘periodic convolution’), the shift \( g(n - k) \) is taken to be a circular shift. So the circular convolution

\[
y = [x(0) \; x(1) \; x(2) \; x(3)] \odot [g(0) \; g(1) \; g(2) \; g(3)]
\]

can be written out as

\[
y = x(0) [g(0) \; g(1) \; g(2) \; g(3)] + x(1) [g(3) \; g(0) \; g(1) \; g(2)] + x(2) [g(2) \; g(3) \; g(0) \; g(1)] + x(3) [g(1) \; g(2) \; g(3) \; g(0)]
\]

The sequence \( y \) is of length 4.