Continuous-Time Signals and Systems — complex poles, frequency response

1. A causal continuous-time LTI system is described by the equation

$$y''(t) + 2\,y'(t) + 2\,y(t) = x(t)$$

where x is the input signal, and y is the output signal.

   (a) Find the transfer function $H(s)$ and its ROC.

   (b) Find the impulse response of the system.

   (c) Accurately sketch the pole-zero diagram.

2. The impulse response of an LTI system is

$$h(t) = \mathrm{e}^{-t}u(t) - 2\,\mathrm{e}^{-t}\sin(3\pi t)u(t)$$

   (a) List the poles of the system

   (b) Find the differential equation describing the system

   (c) What is the dc gain of the system?

   (d) Find the output signal produced by input $x(t) = 2$.

3. It is observed of some continuous-time LTI system that the input signal

$$x(t) = \mathrm{e}^{-2\,t}\,u(t)$$

produces the output signal

$$y(t) = 3\,\mathrm{e}^{-2\,t}\,u(t) + 2\,\mathrm{e}^{-3t}\,\cos(2\pi t)\,u(t).$$

What can be concluded about the pole positions of the LTI system?

4. The frequency response of a continuous-time LTI system is given by,

$$H^f(\omega) = \begin{cases} 2\,\mathrm{e}^{-\mathrm{j}\omega}, & |\omega| \le 2\,\pi \\ 0, & |\omega| > 2\,\pi \end{cases}$$

   (a) Accurately sketch the frequency response magnitude $|H^f(\omega)|$.

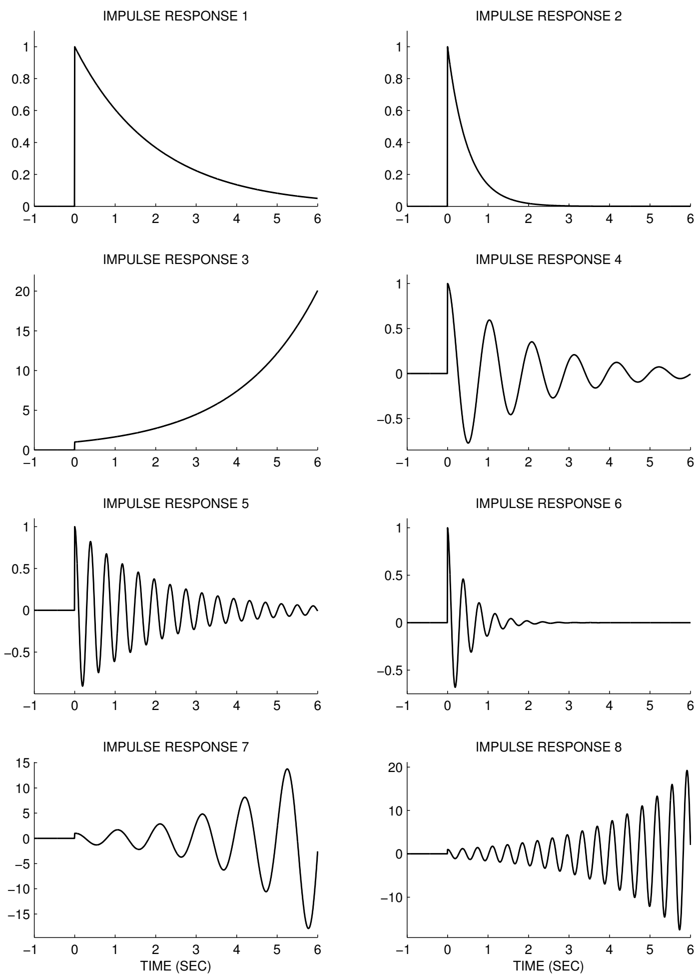
   (b) Accurately sketch the frequency response phase $\angle H^f(\omega)$.

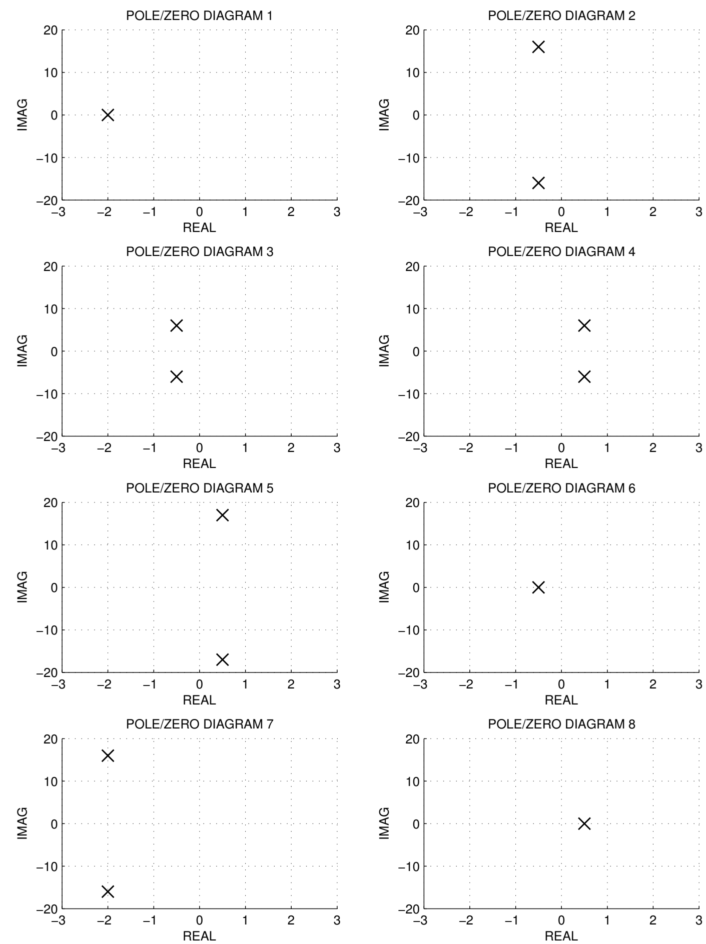
   (c) Find the output signal produced by the input signal

$$x(t) = 1 + 2\,\cos(\pi\,t) + 4\,\cos(3\,\pi\,t).$$

5. The impulse responses of eight causal continuous-time systems are illustrated below, along with the pole/zero diagram of each system. But they are out of order. Match the figures with each other by completing the table.

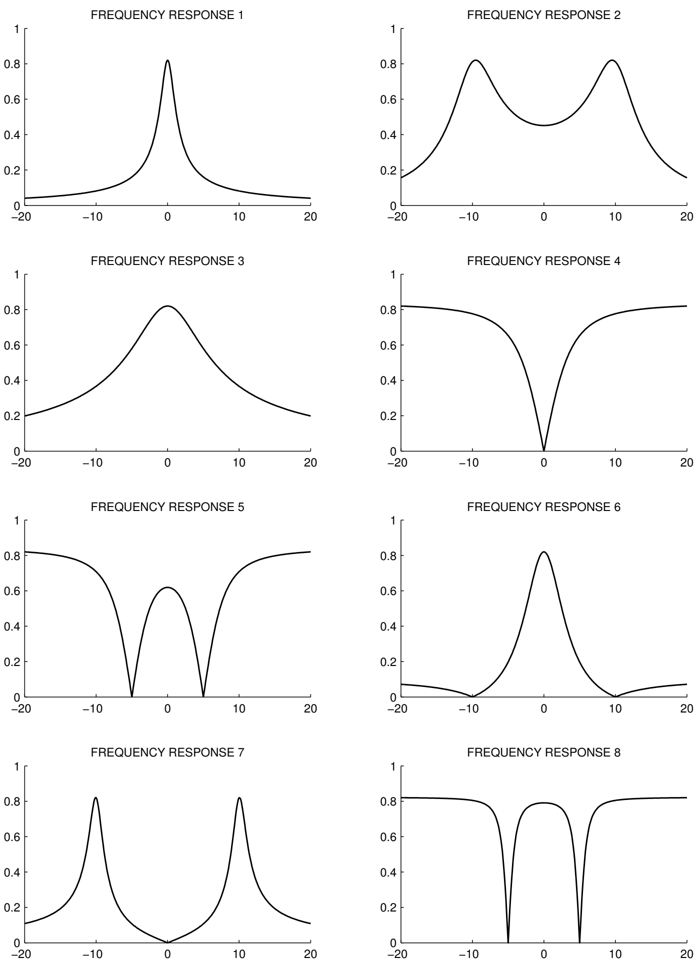
Impulse response	Pole-zero diagram
1	
2	
⋮	
8	

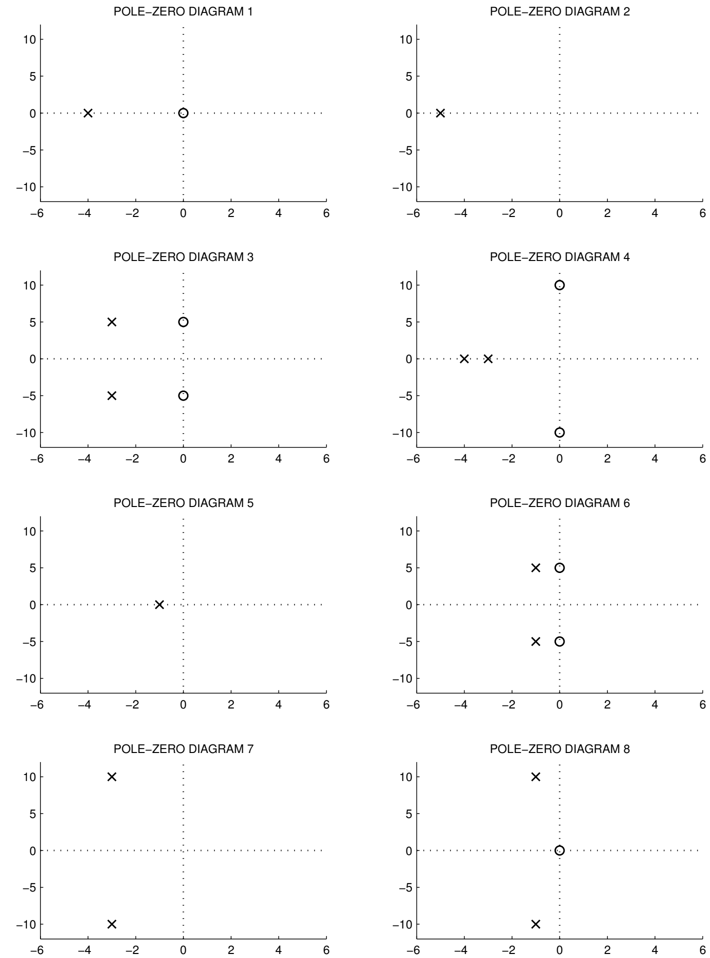

IMPULSE RESPONSE 1
IMPULSE RESPONSE 2
IMPULSE RESPONSE 3
IMPULSE RESPONSE 4
IMPULSE RESPONSE 5
IMPULSE RESPONSE 6
IMPULSE RESPONSE 7
IMPULSE RESPONSE 8
TIME (SEC)
TIME (SEC)


POLE/ZERO DIAGRAM 1
POLE/ZERO DIAGRAM 2
POLE/ZERO DIAGRAM 3
POLE/ZERO DIAGRAM 4
POLE/ZERO DIAGRAM 5
POLE/ZERO DIAGRAM 6
POLE/ZERO DIAGRAM 7
POLE/ZERO DIAGRAM 8
IMAG
REAL

6. The frequency responses of eight causal continuous-time systems are illustrated below, along with the pole/zero diagram of each system. But they are out of order. Match the figures with each other by completing a table.

Frequency response	Pole-zero diagram
1	
2	
⋮	
8	





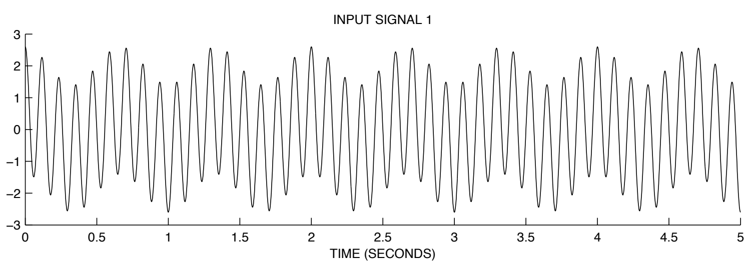
7. Each of the two continuous-time signals below are processed with each of four LTI systems. The two input signals, illustrated below, are given by:

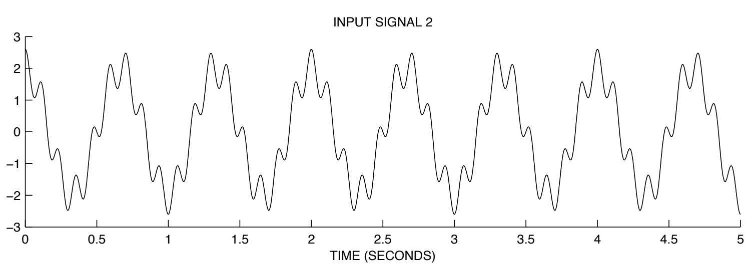
$$\text{Input signal 1:} \quad 0.6\cos(3\pi t) + 2\cos(17\pi t)$$

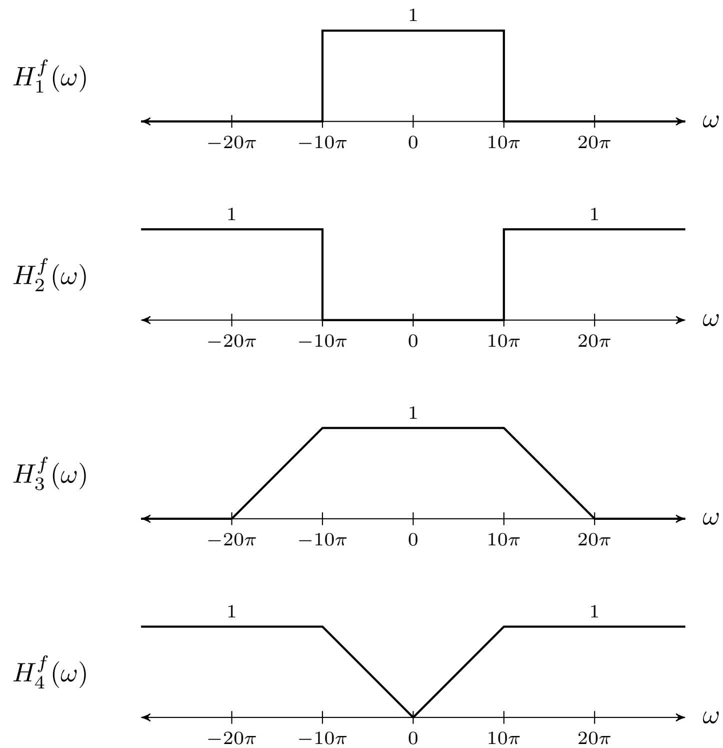
$$\text{Input signal 2:} \quad 2\cos(3\pi t) + 0.6\cos(17\pi t)$$

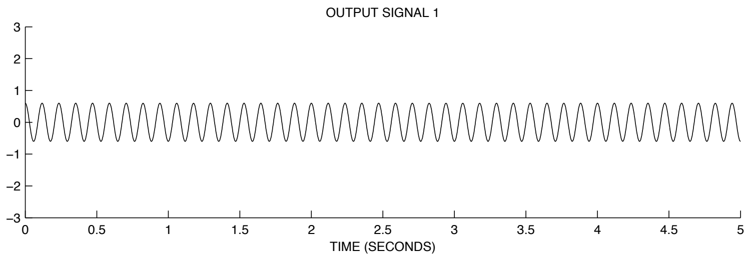
The frequency responses $H^f(\omega)$ are shown below. Indicate how each of the output signals are produced by completing the table below (copy the table onto your answer sheet). Note: one of the output signals illustrated below will appear twice in the table (there are seven distinct output signals).

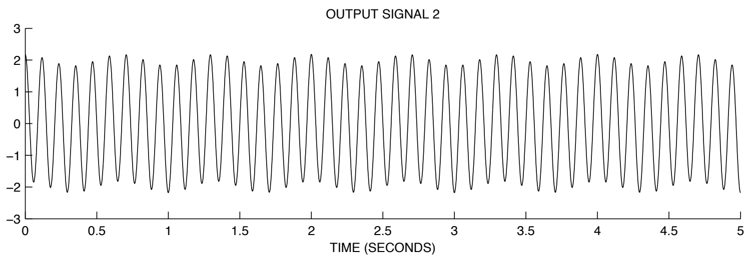
Input signal	System	Output signal
1	1	
1	2	
1	3	
1	4	
2	1	
2	2	
2	3	
2	4	

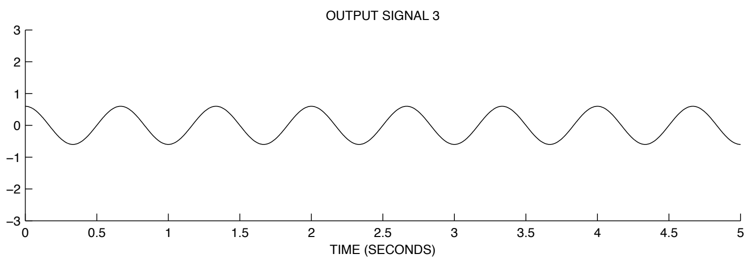


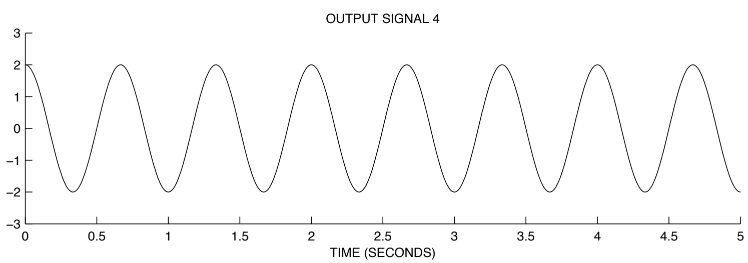


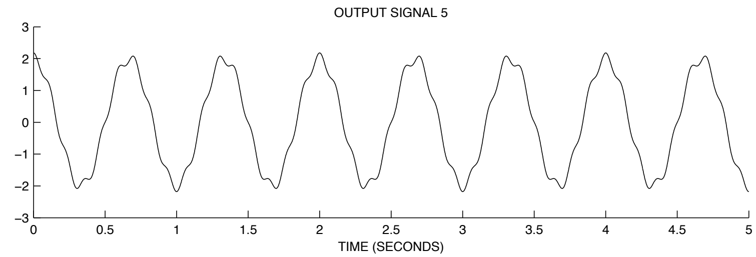


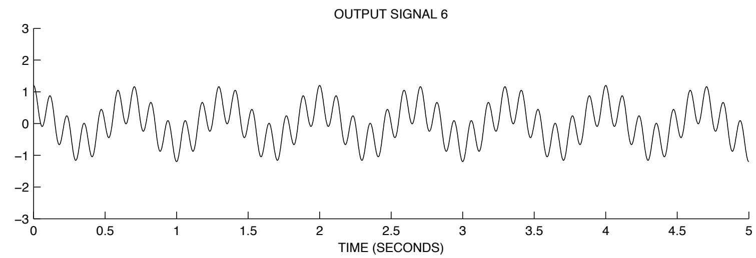

OUTPUT SIGNAL 1
3
2
1
0
−1
−2
−3
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
TIME (SECONDS)

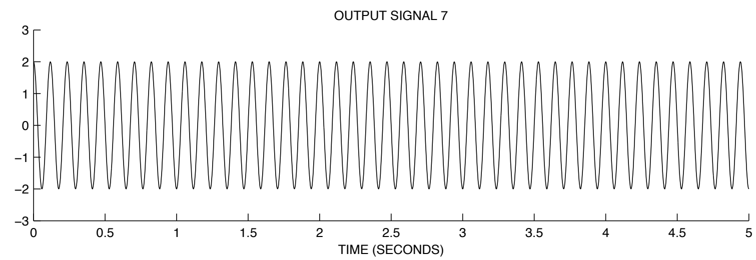

OUTPUT SIGNAL 2
3
2
1
0
−1
−2
−3
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
TIME (SECONDS)


OUTPUT SIGNAL 3
3
2
1
0
−1
−2
−3
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
TIME (SECONDS)


OUTPUT SIGNAL 4
3
2
1
0
−1
−2
−3
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
TIME (SECONDS)


OUTPUT SIGNAL 5
3
2
1
0
−1
−2
−3
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
TIME (SECONDS)


OUTPUT SIGNAL 6
3
2
1
0
−1
−2
−3
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
TIME (SECONDS)


OUTPUT SIGNAL 7
3
2
1
0
−1
−2
−3
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
TIME (SECONDS)

useful formulas

name	formula
Euler's formula	$e^{j\,\theta} = \cos(\theta) + j\,\sin(\theta)$
...for cosine	$\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
...for sine	$\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2\,j}$
sinc function	$\mathrm{sinc}(\theta) := \dfrac{\sin(\pi\,\theta)}{\pi\,\theta}$

formulas for continuous-time LTI signals and systems

name	formula
area under impulse	$\int \delta(t)\,dt = 1$
multiplication by impulse	$f(t)\,\delta(t) = f(0)\,\delta(t)$
...by shifted impulse	$f(t)\,\delta(t-t_o) = f(t_o)\,\delta(t-t_o)$
convolution	$f(t) * g(t) = \int f(\tau)\,g(t-\tau)\,d\tau$
...with an impulse	$f(t) * \delta(t) = f(t)$
...with a shifted impulse	$f(t) * \delta(t-t_o) = f(t-t_o)$
transfer function	$H(s) = \int h(t)\,e^{-st}\,dt$
frequency response	$H^f(\omega) = \int h(t)\,e^{-j\omega t}\,dt$
...their connection	$H^f(\omega) = H(j\omega)$ provided $j\omega$-axis $\subset$ ROC

selected Laplace transform pairs

$x(t)$	$X(s)$	ROC
$x(t)$	$\int x(t)\,e^{-st}\,dt$ (def.)	
$\delta(t)$	1	all s
$u(t)$	$\dfrac{1}{s}$	$\mathrm{Re}(s) > 0$
$e^{-a\,t}\,u(t)$	$\dfrac{1}{s+a}$	$\mathrm{Re}(s) > -a$
$\cos(\omega_o t)\,u(t)$	$\dfrac{s}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$\sin(\omega_o t)\,u(t)$	$\dfrac{\omega_o}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$e^{-a\,t}\cos(\omega_o t)\,u(t)$	$\dfrac{s+a}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$
$e^{-a\,t}\sin(\omega_o t)\,u(t)$	$\dfrac{\omega_o}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$

Note: a is assumed real.

Laplace transform properties

$x(t)$	$X(s)$
$a\,x(t) + b\,g(t)$	$a\,X(s) + b\,G(s)$
$x(t) * g(t)$	$X(s)\,G(s)$
$\dfrac{dx(t)}{dt}$	$s\,X(s)$
$x(t-t_o)$	$e^{-s\,t_o}\,X(s)$