Continuous-Time Signals and Systems

1. Use the Laplace transform to find the convolution of $f(t)$ and $g(t)$.

$$f(t) = 2\,u(t), \qquad g(t) = 3\,\mathrm{e}^{-2t}u(t)$$

2. An LTI system is defined by the differential equation

$$3y''(t) + 4y'(t) + 5y(t) = 6x'(t) + 7x(t)$$

   (a) Find the transfer function $H(s)$.
   (b) Find output signal $y(t)$ when the input is $x(t) = 3$.

3. Find the Laplace transform of the signal

$$x(t) = 5\,\mathrm{e}^{-2t}\,u(t-3)$$

   Also find and show the region of convergence (ROC) of $X(s)$.

4. A causal LTI system is defined by the differential equation

$$y''(t) + y'(t) - 2y(t) = x'(t) - 2x(t)$$

   (a) Find the impulse response $h(t)$.
   (b) Sketch the pole-zero diagram of the system.
   (c) Classify the system as stable/unstable.

5. The impulse response of an LTI system is given by

$$h(t) = 2\,\delta(t) - 3\,\mathrm{e}^{-t}\,u(t) + \mathrm{e}^{-2t}\,u(t)$$

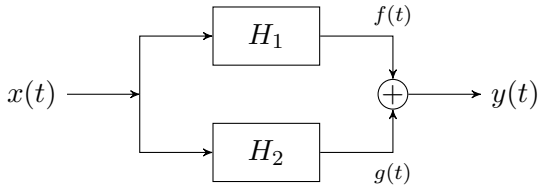
   Find the differential equation of the system.

6. The impulse response of an LTI system is given by

$$h(t) = 7\,\delta(t) + 8\,\mathrm{e}^{-2t}\,u(t) + 9\,\mathrm{e}^{-4t}\,u(t)$$

   Find the poles of the system.

7. Two LTI systems, H_1 and H_2, are connected in parallel.



   The systems H_1 and H_2 are described by the differential equations:

$$H_1: \; f'(t) + 2\,f(t) = x(t)$$

$$H_2: 2\,g'(t) + 3\,g(t) = x'(t) + 2\,x(t)$$

   Find the differential equation for the total system [from $x(t)$ to $y(t)$].