EE 3054 – Fall 2012 Quiz 3 (Discrete-time systems)

1. **Convolution.** Sketch the convolution of $y(n) = h(n) * x(n)$

   (a) $h(n) = \delta(n-3)$
   $x(n) = \delta(n) - \delta(n-5)$

   (b) $h(n) = \delta(n-3) - 2\,\delta(n-6)$
   $x(n) = \delta(n) - \delta(n-5)$

   (c) $h(n) = \delta(n-3) - 2\,\delta(n-6)$
   $x(n) = u(n) - u(n-5)$

   (d) $h(n) = \delta(n) + 2\,\delta(n-1) + 3\,\delta(n-2)$
   $x(n) = \displaystyle\sum_{k=0}^{\infty} \delta(n - 4\,k)$

2. An LTI system has impulse response

$$h(n) = 2\left(\frac{1}{3}\right)^n u(n).$$

Find the system output $y(n)$ produced by the input signal

$$x(n) = 2.$$

Note: This input signal starts at 'minus infinity'.

3. ***Z*-transforms.** The Z-transform of the signal $x(n)$ is

$$X(z) = 2\,z^3 + z^{-1} - z^{-2}.$$

   (a) Accurately sketch the signal $x(n)$.
   (b) Define $F(z) = z^{-2}\,X(z)$. Sketch the signal $f(n)$.
   (c) Define $G(z) = X(-z)$. Sketch the signal $g(n)$.
   (d) Define $H(z) = z^{-2}\,X(-z)$. Sketch the signal $h(n)$.
   (e) Define $Y(z) = z^{-2}\,X(1/z)$. Sketch the signal $y(n)$.

4. Find the Z-transform $X(z)$ of the signal

$$x(n) = 3\left(\frac{1}{2}\right)^n u(n).$$

Also find and sketch the region of convergence of $X(z)$.

5. Two LTI systems are connected in series

$$x(n) \longrightarrow \boxed{H_1(z)} \longrightarrow \boxed{H_2(z)} \longrightarrow y(n)$$

with transfer functions

$$H_1(z) = 1 + 3\,z^{-1} + 4\,z^{-2}$$
$$H_2(z) = 2 - z^{-1} + z^{-2}.$$

   (a) Find the impulse response $h(n)$ of the total system.
   (b) Find output signal $y(n)$ produced by input signal $x(n)$.