STEADY STATE NETWORK EQUIVALENTS
FOR LARGE ELECTRICAL POWER SYSTEMS

THESIS

Submitted in Partial Fulfillment
of the REQUIREMENTS for the

Degree of
MASTER OF SCIENCE (Electrical Engineering)
at the

POLYTECHNIC UNIVERSITY

by

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June 2008

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Load-flow and short-circuit studies are currently performed using explicit representation of each section and line. When systems are studied for planning and operation, in general, engineers are interested in a few buses. It is necessary to reduce the system under study to a manageable size. The thesis proposes and validates a methodology to obtain equivalent circuits for large radial and meshed power systems for short-circuit and load-flow studies. The method is applicable to systems with any number of terminals. The reduced equivalent is exact for an operating point at which it is obtained and valid for a wide range of load variation. The reduction in load-flow calculation time is proportional to the reduction in the size of the system. Efficiency gain is significant for stability studies where load-flow is performed repeatedly. Examples are shown for validation, illustration and efficiency assessment.
I was born in Yangon, Myanmar on March 22, 1987. I graduated high school in Myanmar. I came to United States in 2004 to pursue Bachelor of Science degree in Electrical Engineering at Polytechnic University. After freshman year at Poly, I was accepted into Honors College which allows me to take summer classes at no cost. I then decided to pursue both BS and MS degrees at the same time in four years by getting accepted into BS/MS Honors program. I have started the thesis research in the beginning of Fall 2007, and completed the simulations by the end of Spring, 2008. Honors College and Dean’s scholarship from Polytechnic University made this thesis possible.
Dedicated to My Parents
ACKNOWLEDGMENTS

First and foremost, I would like to offer my greatest gratitude to my parents, Mr. Thant Zin and Mrs. Moe Thu Zar. Without their unparalleled love, unmatched support and encouragement, this thesis would not have been written. They have sacrificed for my education at Polytechnic University. Their merit is supreme.

I would like to thank my wife, Hnin Haymar, for her unconditional love and unfailing support. She has been very understanding and patient. She, too, made this thesis possible.

I offer my sincerest gratitude to my advisor, Dr. Francisco de Leon, who has supported me throughout the thesis with his knowledge and patience. I greatly appreciate his prompt responses to my questions and doubts. Without his guidance, I would have been lost. I simply could not wish for a better and friendlier advisor.
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Chapter 1

INTRODUCTION

1.1. Background

Load-flow and short-circuit studies for distribution system planning and operation are presently performed with the explicit representation of every section (line, cable and transformer). When the information is available, the secondary network and every load are also modeled in detail. Consequently, steady state studies for distribution systems frequently include many thousands nodes and sections. This is even true for distribution systems of developing countries [1]. In developed countries, it is not unheard of analysis of systems with over 100,000 buses. Load-flow studies are performed very efficiently for radial distribution systems using forward and backward sweeps [2]. When the system is lightly meshed good efficiency is obtained with the compensation method [3].

With the modern trend to include distributed generation in the distribution systems, stability studies for large systems will become a necessity. However, since transient stability programs were conceived for the analysis of highly meshed transmission systems, the solution is obtained operating with matrixes. For stability studies, explicit modeling of every load and section is not necessary. Conversely, it is important to share a common database with static studies.
1.2. Objective

This thesis presents a methodology proposed to obtain equivalent for large distribution systems for load-flow and short-circuit studies. The method is applicable to both radial and meshed systems. The main objective of the equivalents is to reduce the number of sections to a manageable size to study the effects of DGs, controllable devices and load changes in large distribution systems. Yet, the method is equally applicable to transmission systems with any number of terminals. Fig. 1 shows the underlying principle applied to radial systems and Fig. 2 and Fig. 3 for meshed systems.

Fig. 1. Application of the technique to obtain equivalents for a radial system
Fig. 2. Application of the technique to obtain equivalents for a two-terminal meshed system

Fig. 3. Application of the technique to obtain equivalents for a multi-terminal meshed system
1.3. **Scopes and Limitations**

The main features of the new technique are:

- The methodology applies to both radial and meshed systems.
- It is applicable for the representation of passive networks between terminals of the sub network.
- All types of loads can be reduced: constant power, constant current and constant impedance.
- The technique can be used to substitute systems with any number of phases, sections and terminals.
- The sending and receiving ends do not need to have the same number of phases.
- The resulting equivalent, which may substitute hundreds or thousands sections, is a new section (or sections in the case of multi-terminal meshed systems) with the appropriate impedance and loading.
- The reduced system is adequate for load-flow, short-circuit and stability studies.
- The equivalent circuit is exact for a given operating point, but it covers a wide range of operating conditions with acceptable accuracy.
- The equivalent circuits exclude all controllable and switching devices (generators, ULTC transformers, switched capacitors, regulators, etc.), but equivalents can be obtained at their terminals.
Chapter 2
THEORETICAL BACKGROUND

2.1. Studies for Planning and Operation

Load flow and short-circuit studies are performed for distribution system planning and operation. Load flow studies are very important in planning and designing the expansion of power systems as well as in determining the best operation of existing systems. Short-circuit studies are crucial for determining suitable relays and protection mechanism.

2.2. Load flow

Power system operation is considered successful under normal balanced three-phase steady state conditions when (1) generation supplies the load demand and the losses, (2) bus voltage magnitudes are close to rated values (within 5%), (3) transformers and transmission lines are not overloaded, (4) generators operate within specified real and reactive power limits [4]. The load flow program is the tool to investigate above-mention conditions or requirement for successful system operation. Solutions of the load flow program include voltage magnitude and angle, real and reactive power flows. Two popular solution methods are Gauss-Seidel and Newton-Raphson. Current load flow programs assume balanced systems.
2.3. Stability

Power system stability refers to the ability of synchronous machines to move from one operating point to another point following a disturbance, without losing synchronism. Power system stability can be differentiated into three types: steady-state, transient, and small-signal.

2.3.1. Steady-state stability

Steady-state stability studies are usually performed with a load-flow program. The purpose is to ensure that, following gradual changes in operating points, bus voltages remain close to rated values, that phase angles across transmission lines are not too large, and that generator, transmission lines, and transformer and other equipments are not overloaded.

2.3.2. Transient Stability

Following major disturbances such as loss of generation, faults, and load changes, synchronous machine frequencies undergo transient deviations from synchronous frequency of 60 Hz and machine power angles change. The system is considered transiently stable if following a large disturbance, the system attain a significantly different but acceptable steady-state operation condition i.e. if the machines return to synchronous frequency with new steady-state power angles. For a two synchronous machine system, the system is transiently stable if following a major disturbance, the difference in power angles of the two machines does not diverge.
2.3.3. Small-signal Stability

Small signal stability is defined as the ability of the power system to maintain synchronism when subjected to small disturbances due to small variations in loads and generation. Instability may be in the form of (1) steady increase in generator rotor angle due to lack of synchronizing torque, or (2) rotor oscillations of increasing amplitude due to lack of sufficient damping torque [7].
Chapter 3

EQUIVALENCING METHOD

3.1. Methodology for Radial Systems

This section describes the methodology to obtain an equivalent for a radial system, which is useful load-flow and short-circuit studies.

Consider a radial network with many sections that is to be reduced to one equivalent section as shown in Fig 4. The original network consists of series impedances $Z_i$ and shunt loads $L_i$; the loads include the shunt capacitors of lines and cables. The equivalent section is a $pi$ circuit comprised of impedance in series and two shunt loads. The series impedance of the equivalent circuit is computed to match the series impedance of the original network to assure that the equivalent circuit accurately represents the original circuit during short circuit.

The compensating shunt loads $L_s$ and $L_r$ of the equivalent circuit are computed in such a way that the load flow solution of the original and equivalent circuits match (voltages, currents and powers) for a given operating point.
3.1.1. Matching Short Circuit

For the equivalent to work properly for short circuit studies, the impedance of the equivalent section should be the sum of the series impedances of all the sections of the original circuit. Therefore,

$$Z_{eq} = Z_s + Z_{s+1} + Z_{s+2} + ... + Z_{s+n}$$

(1)

Equation (1) applies equally to sequence impedances and phase impedance matrixes. For sequence impedances (1) is used twice, once for the positive sequence and then for the zero sequence impedance. In per phase calculations all impedances in (1) including $Z_{eq}$ are $3 \times 3$ complex matrixes when ground wires and neutral conductors have been eliminated (through Kron's reduction, for example). However, with the method of this paper explicit representation of neutrals and ground are permitted in (1); see a method to resolve general networks in [5].
3.1.2. Matching Load Flow

Since the impedance (or impedance matrix) of the equivalent must match the impedance of the sections, two equivalent loads \( L_s \) and \( L_r \) will be used to compensate for all other elements in the system (loads and shunts) that have been eliminated and demand current and therefore produce voltage drop in the reduced sections.

The procedure to determine equivalents requires the knowledge of the load flow solution for a given operating point. Therefore, the terminal voltages and currents at the sending and receiving nodes are assumed known. The voltage and current at the sending end are \( V_s \) and \( I_s \), while the voltage and current at the receiving end are \( V_r \) and \( I_r \). All variables described here are complex matrices or vectors. Applying Kirchhoff Voltage Law (KVL) to the equivalent circuit we have:

\[
V_s = Z_{eq} I_i + V_r \tag{2}
\]

Therefore, the current that produces the same voltage drop across the equivalent impedance is computed from:

\[
I_i = Z_{eq}^{-1} (V_s - V_r) \tag{3}
\]

In general, \( I_i \) will not match the receiving terminal current \( I_r \), but a compensating load \( L_r \) can be conveniently computed for that purpose. The current that \( L_r \) must demand (or inject) such that the \( I_r \) of the equivalent circuit is equal to that of the real circuit is computed from Kirchhoff Current Law (KCL) as:

\[
I_{Lr} = I_i - I_r \tag{4}
\]
Similarly, the sending end current $I_s$ will not match $I_i$ and thus a second compensating load $L_s$ needs to be computed. The equation for the compensating load current is:

$$I_{ls} = I_s - I_i$$  \hspace{1cm} (5)

The current injections given in (4) and (5) are necessary for the equivalent to match with the terminal currents $I_s$ and $I_r$. Although the use of current sources is common in compensation methods, impedance or a constant $P, Q$ load can also be used. In fact, the best equivalent load is the one that best matches the actual load variations with voltage of the original circuit.

The current injections $I_{lr}$ and $I_{ls}$ represent the reduced loads $L_i$ and capacitive shunts, whose values and compositions are known from the original system. Therefore, the compensating loads should have the same composition as the reduced loads in percent of constant current, constant impedance and constant power; see Fig. 5.

![Compensating loads with constant impedance, constant current and constant power components](image-url)
The values (in power units) of $Z_1$, $Z_2$, $I_1$, $I_2$, $S_1$ and $S_2$ are computed through the ZIP coefficients as described in [6]. We first compute the equivalent real and reactive total powers for both the sending and the receiving ends as:

$$S_{Ls} = V_s I_{ls}^* = P_{s0} + jQ_{s0}$$
$$S_{Lr} = V_r I_{lr}^* = P_{r0} + jQ_{r0}$$

(6)

The values, in power units [VA], rather than in current [A] or impedance [$\Omega$] units, for the loads in Fig. 3 are:

$$Z_s = Z_p P_{s0} + jZ_q Q_{s0}$$
$$I_s = I_p P_{s0} + jI_q Q_{s0}$$
$$S_s = P_p P_{s0} + jP_q Q_{s0}$$
$$Z_r = Z_p P_{r0} + jZ_q Q_{r0}$$
$$I_r = I_p P_{r0} + jI_q Q_{r0}$$
$$S_r = P_p P_{r0} + jP_q Q_{r0}$$

(7)

, where $Z_p$, $Z_q$, $I_p$, $I_q$, $P_p$, $P_q$ are the dimensionless ZIP coefficients and correspond to the percent of active (sub $p$) and reactive (sub $q$) powers that have been reduced for the three types of load: constant impedance ($Z_p$, $Z_q$), constant current ($I_p$, $I_q$), and constant power ($P_p$, $P_q$). The sums of the real and imaginary ZIP coefficients add to 1 in per unit or 100%.
Chapter 4

RESULTS

Three different test systems are developed to validate and illustrate the methodology. An 8-bus radial network is used to illustrate numerically the method applied to radial systems. A 101-bus network studied for transient stability is for efficiency assessment.

4.1. 8-Bus Radial Network

The following radial 8-bus system is for numerical illustration purpose. All loads are constant P-Q loads with the value of 0.5 + j 0.25 pu. All lines have the same series impedance of Z = 0.001 + j 0.01 pu and shunt admittance of 0.005 pu. The computed voltage magnitudes are displayed for each bus.

Fig. 6. Illustration of the process to obtain the equivalent circuit
To obtain the equivalent, the sending end and receiving end buses are first chosen. In this illustration, Bus 2 and 8 are designated as the sending end bus and the receiving end bus, respectively. The solution of the initial load-flow of the original system provides voltage and current at each bus. Thus, $V_s = 0.9874 \angle -1.104^\circ$, $I_s = 1.716 \angle -28.46^\circ$, $V_r = 0.9688 \angle -2.813^\circ$ and $I_r = 0.5766 \angle -28.22^\circ$. The equivalent impedance, $Z_{eq}$, is computed as:

$$Z_{eq} = 3Z = 3(0.001 + j 0.01) = 0.003 + j 0.03$$  \hspace{1cm} (13)

The current across the equivalent impedance is computed as

$$I_i = \frac{V_s - V_r}{Z_{eq}} = \frac{(0.9874 \angle -1.104^\circ) - (0.9688 \angle -2.813^\circ)}{(0.003 + j 0.03)}$$

$$I_i = 1.147 \angle -28.74^\circ$$  \hspace{1cm} (14)

The current that the compensating loads at the sending end and receiving end must demand are computed as:

$$I_{L_r} = I_i - I_r = (1.147 \angle -28.74^\circ) - (0.5766 \angle -28.22^\circ)$$

$$I_{L_r} = 0.5691 \angle -27.89^\circ$$  \hspace{1cm} (15)

and

$$I_{L_s} = I_s - I_i = (1.716 \angle -28.46^\circ) - (1.147 \angle -28.74^\circ)$$

$$I_{L_s} = 0.4994 \angle -27.89^\circ$$  \hspace{1cm} (16)

The actual compensating loads, $L_r$ and $L_s$, are calculated as:

$$S_{L_s} = V_s I_{L_s}^* = (0.9874 \angle -1.104^\circ)(0.5691 \angle 27.89^\circ)$$

$$S_{L_s} = 0.5016 + j 0.2533$$

$$S_{L_r} = V_r I_{L_r}^* = (0.9688 \angle -2.813^\circ)(0.4241 \angle 28.24^\circ)$$

$$S_{L_r} = 0.4994 + j 0.2374$$
Note that the total compensating loads are close to, but not the same as, the loads that they substitute. The maximum error found in all variables is in the order of $10^{-6}$. This error is attributable to round-off errors since the method is in theory exact for the operating point at which the equivalent was obtained.

In above illustrative example, all loads were P-Q loads and they were compensated by P-Q loads in the equivalent circuit. To determine the validity range of the technique, load variation experiments were performed on three cases with different load modeling.

Case 1: All loads and compensating loads are P-Q loads.

Case 2: All loads and compensating loads are ZIP loads.

Case 3: All loads are P-Q loads, but compensation is by current source.

ZIP load is a combination of constant impedance ($Z$), constant current ($I$) and constant power ($P$) loads. All ZIP loads had 33% of each component. Current source compensation is modeled as ZIP load with 100% constant current component.

All loads were doubled, i.e. extreme load variation, and for each case, the equivalent was obtained by doubling the compensating loads accordingly. The load-flow solution of the equivalent circuits matched that of the original circuits with the following maximum % errors.

Case 1: $10^{-4}$  
Case 2: $10^{-3}$  
Case 3: $10^{-2}$

It is evident that if the compensating loads are the same type or have the same composition, the results are better. In case 3, P-Q loads were compensating by current sources, and the equivalent circuit was less accurate.
4.2. Transient Stability Study of 101-Bus Radial Network

The 101-bus radial system consists of two identical generators, one at each end of the radial network. This test system is used to assess the efficiency gain of the equivalent method. Transient stability study is done on the 101-bus radial network with explicit representation of each section. The studies are done in MATLAB using SimPowerSystem toolbox. The system is simulated for 120 seconds. The simulation measures the difference in the rotor angles of the two machines. The time it takes to complete the simulation and the results are recorded. Then the equivalent of the network is obtained. All sections are reduced to one equivalent section. The reduced network is, therefore, of one hundredth the size of the original. The simulation time required to complete is expected to reduce to one percent.

4.2.1. Description of the study

The system, shown in Figure 7, consists of two synchronous machines both operating in generator mode. Both machines are rated 5000 kVA. There are 100 sections. Each line has the same impedance. Except at Bus 1 and 101, each bus has the same load of 75 + j30 kVA. At Bus 1 and 101 are 10 kW loads. The total load demand at 101 buses is 7445 + j2970 kVA. M1 is set as the swing generator and M2 as PV generator loaded 40 percent of the rated value. Single-line-to-ground fault occurs at Bus 1 at \( t = 60 \) seconds and clears after 0.1 seconds. The difference in rotor angles of M1 and M2 are observed for full and reduced systems. Bus 1 and Bus 101 are designated as the sending end bus and receiving end bus, respectively. Every section in
between is reduced to one equivalent section with properly computed compensating loads at both ends. The equivalent impedance, $Z_{eq}$, is 100 times the impedance of each line. Compensating loads, $L_s$ and $L_r$, are computed using equations (2) to (6). Computed values of $S_{Ls}$ and $S_{Lr}$ are 2941 + j2097 kVA and 3370 + j1415 kVA, respectively.

Fig.7. 101-bus radial system with two synchronous machines

Fig.8. Reduced, equivalent system
Fig. 9. The rotor angle difference between two machines; original system

Fig. 10. The rotor angle difference between two machines; zoomed around fault instance; original system
Fig. 11. The rotor angle difference between two machines; both original system and reduced system.

Fig. 12. The rotor angle difference between two machines; zoomed at steady state result; both original system and reduced system.
Fig. 13. The rotor angle difference between two machines; zoomed at the transient following the fault; both original system and reduced system
Chapter 5

CONCLUSIONS

5.1. Observation

5.1.1. 8-Bus Radial Network - Illustration

The methodology was thoroughly illustrated. Effects of different type of compensation on accuracy of solutions were analyzed. If the compensating loads are the same type or have the same composition, the results are better. Validity range was determined. As long as the compensating loads are adjusted, the methodology is valid for a wide range of operating conditions. Adjusting the compensating loads consists of only multiplication. Note that the compensating loads are not recalculated using equations (2) to (7). They are only increased or decreased in proportion to the load change. Therefore, it is still very efficient to use the same equivalent for a wide range of operating conditions with adjusted compensating loads.

5.1.2. 103-Bus Radial Network – Efficiency Assessment

The system is transiently stable as the angle difference returns to the initial value i.e. the two machines remain in synchronism after the SLG disturbance. See Figure 9 and 10. It takes an average of 21 seconds to complete 120-second simulation of the original system. The system is then reduced using the proposed method. Then 120-second simulation is performed again. This time, with the smaller number of sections, it only
takes an average of 7 seconds to complete the simulation. It shows that the
efficiency is not directly proportional to the number of section reduced. The
fact that the equivalent has only 1\% of the number of sections in the original
does not imply that the simulation takes only 1 percent of the time it takes to
complete the simulation of the original. The relationship between the
reduction in size and the reduction in CPU time is unclear. However, it is not
linear.

The steady state solution of the equivalent network matches that of
the original network with maximum error of 0.001\%. Those errors are due to
round-off errors. The error in steady state rotor angle difference is in the
order of 10^{-2}\%. Steady state values of the rotor angle difference for the full
system and the reduced system are 37.2835 and 37.2795, respectively. Figure
11 shows the rotor angle difference of the full and reduced systems on a
single plot. The difference is too small to be visible in full picture. In Figure
12, the plot is zoomed in to make steady state error visible. The error in the
steady state results before the fault and after the transient dies out is 10^{-2}\%.
Figure 13 zooms around the instance of fault. The difference is visible here.
During the transient following the fault, the result of the equivalent system
does not match that of the full system. This is expected because the
methodology works only for the steady state and for operation around the
point at which the equivalent is initially obtained.

The transient during the first 40 seconds should not exist. Since the
machines were initialized using load flow solution before the simulation, the
plot is expected to start with steady state value and stay flat. The tool,
Matlab, is assumed to be at fault for producing such transient at the beginning.

5.2. Contribution

The methodology presented here allows engineers to perform faster simulation studies on large electrical power systems. The methodology for obtaining smaller network equivalent is valid for short-circuit, load flow and stability studies. The methodology can help engineers achieve noticeable efficiency gains in time domain simulations. The methodology is suitable for studies of both transmission and distribution systems with any number of terminals and phases. It is applicable for both radial and meshed systems.

5.3. Future Work

With further work, the results of this thesis will be submitted to the IEEE Transactions. Applying the methodology for a meshed system will be illustrated. Efficiency gain for mesh systems will also be assessed. The methodology will also be validated with another simulation software.
REFERENCES
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