A SIMPLE REPRESENTATION OF DYNAMIC HYSTERESIS LOSSES IN POWER TRANSFORMERS

Francisco de León
Instituto Politécnico Nacional - E.S.I.M.E.
Edificio No. 5, 3rd Piso
07738 - México, D.F., México

Adam Semlyen
Department of Electrical and Computer Engineering
University of Toronto
Toronto, Ontario, Canada, M5S 1A4

Abstract - The paper describes a procedure for the representation of hysteresis in the laminations of power transformers in the simulation of electromagnetic transient phenomena. The model is based on the recognition that in today’s transformers, the hysteresis loops are not circular and therefore the modeling details are only important in relation to the incurred losses and the associated attenuation effects. The resultant model produces losses proportional to the square of the flux density, as expected from measurement data. It is formulated as a simple linear relationship between the variation $B - B_{ref}$ of the magnetic flux density $B$ after a reversal point $B_{ref}$ and the resulting additional field intensity $H_{loss}$. This idea can be easily implemented in existing transformer models with or without frequency dependent modeling of eddy currents in the laminations. It has been found that in many simulation tests the representation of hysteresis is not necessary and those situations have been described where the modeling of hysteresis appears to be more meaningful.

Keywords: Transformer modeling, Electromagnetic transients, Hysteresis, Ferroresonance.

INTRODUCTION

Transformer modeling for the simulation of electromagnetic transients has made significant advances in the last decade. A fairly complete list of references in this field can be found in [1]. This reference summarizes our contributions to the field with the remark that it covers all the major phenomena that are relevant for transformer modeling with the exception, however, of the dynamic representation of hysteresis in the iron core. The main reason why this has been left out in the complexity of the phenomena, where problems appear to be many of linear or non-linear nature of saturation is coupled with the complicated dependence of the magnetic field intensity on the present and past values of the flux density, characteristic to hysteresis. Numerous studies exist, however, related to hysteresis [2][5] and successful achievements have been reported in the implementation of some models in the representation of transformers. Because of their significance, we present a fairly extensive overview of these models in the Appendix at the end of the paper.

A general characteristic of most existing hysteresis models is their sophistication and complexity. This may slow down the computer simulation of transients. A careful examination of the rationale for the representation of hysteresis in transformer models and of the published results of measurements of dynamic hysteresis has led us to the conclusion that a very simple hysteresis model could be adequate for achieving the correct representation of the attenuation of transients that can be attributed to hysteresis. We are thus in the position of presenting a simple and efficient hysteresis model to supplement the fairly complete transformer model we have previously described [1].

We make from the outset the following clarification regarding the terminology we use: the word “dynamic” in relation to hysteresis is used to indicate and emphasize that the phenomena are history dependent, rather than to include – as done in many classical texts – the effects of eddy currents in the laminations.

Review of Existing Models

There are basically three types of vaguely defined approaches and origins in modeling of hysteresis in ferromagnetic materials. In the first group we have the physicists. They primarily look at the physical properties of the material, i.e., domain alignments, wall movements, spin rotations, etc. In the second group are those working in designing based on electromagnetic fields. They prefer a macroscopic description of hysteresis using mathematical models to predict the $B-H$ curve but without completely neglecting the physics of the material. In the third group we have power system engineers. They need equivalent circuits to be introduced in existing computer programs. Their base for modeling is the $B-H$ curve obtained by tests. The circuits should predict the losses in transient and steady state conditions. The purpose of the paper is to contribute to this last approach.

The bibliographic review presented in the Appendix is mainly devoted to models of the second two groups and especially to the last one. After 1970 Most of the publications pre-1970 can be found in the references of [3].

In the following we introduce, justify, and describe the new hysteresis model. Then we show its effects on different types of transformer transients.

DYNAMIC HYSTERESIS MODELING

Fundamental Remarks

Hysteresis is a very complex phenomenon. Curves showing the dynamic relation between $B$ and $H$ illustrate that the hysteresis related component $H_{loss}$ of the magnetic field intensity $H$ is strongly dependent on the magnetization history. In figure 1, we show a measured hysteresis characteristic showing minor loops taken from reference [26], Figure 7. It is not our purpose to analyze or describe this problem. We make however two basic observations relative to the problem of hysteresis as it applies to power transformers:

- As a result of technological improvements, the iron core laminations have at present much reduced losses compared to past constructions. These are generally only a fraction of one percent (based on transformer rating). Therefore, the figures that describe hysteresis should be viewed as having increased scales for $H$ in order to exhibit the details; when, however, the magnetization curve is displayed with a sufficient portion of the saturated branches, then the hysteresis loops narrow down to a very thin strip so that their details become immeasurable and only the associated losses and attenuation remain relevant for the simulation of transients (see Figure 2). Figure 6 of reference [26] presents a measured full cycle that shows the described features (very narrow cycle). Therefore, in what follows, we shall focus primarily on an adequate reproduction of the hysteresis related losses and give preference to simplicity over precision as the latter has only negligible influence on the magnitude of the magnetizing current.

- Magnetizing curves have branches with asymptotically finite slopes at increasing flux densities. It may therefore appear that hyperbolic approximation [12] would be the most appropriate for their fitting. While they have been examined in great detail and implemented for the modeling of hysteresis loops [54], they do not have a flexibility comparable to polynomial approximations for improved fitting of the magnetization characteristic. Since, as discussed above, the precise representation of hysteresis loops is not of primordial importance, full freedom remains for the representation of the basic magnetization curve, including polynomial fitting.
The New Dynamic Hysteresis Model

In order to build our hysteresis model we have found it convenient to postulate the existence of a “basis” magnetization curve

\[ H_{\text{basis}} = f_0(B) \]  

(1a)

This curve is related to the standard magnetization curve for the real magnetic material (i.e., in the presence of hysteresis) through the hysteretic losses, as reflected by the model described below. It should not be identified with the magnetization curve for the idealized behavior of the same magnetic material without hysteresis. The term “basis” simply reflects the fact that in our model hysteretic effects are assumed to originate and to end on this curve. It is a “reset” curve for hysteresis before any reversal of \( B \).

As the hysteresis loop is very thin (as mentioned above), we will use a polynomial approximation for the basis curve with a very steep initial slope

\[ H_{\text{basis}} = K_{\text{basis}} B + K_{\alpha_1} B^{-\alpha_1} + K_{\alpha_2} B^{-\alpha_2}. \]  

(1b)

In Figure 3 we show a basis curve for \( n_1 = 17 \) and \( n_2 = 21 \) with \( K_{\text{basis}} = 0, K_{17} = 0.2181, \) and \( K_{21} = 0.1353 \) (for S.I. units; see [47]).

Therefore,

\[ H = H_{\text{basis}} + K_{\text{hyst}} (B-B_{\text{rev}}) \]  

(2b)

(see Figure 4; here, for better illustration the basis curve is different from that of Figure 3). Reversal means that the time derivative of \( B \) changes sign.

\[ \Delta B = B - B_{\text{rev}} \]

(a)

\[ \Delta B = B - B_{\text{rev}} \]

(b)

Figure 4. Basic idea of hysteresis modeling
(a) \( H_{\text{hyst}} \) component to be added, equation (2a)
(b) \( H_{\text{hyst}} \) added to basis curve, equation (2b)

If at the point \( A \) there is again a reversal, we return to the basis curve \( f_0 \). If the process is now duplicated with descending \( B \), we have the loop shown in Figure 5. The resulting area of the loop is

\[ \text{AREA}_{\text{loop}} = H_{\text{hyst}} (B - B_{\text{rev}}) = K_{\text{hyst}} (B - B_{\text{rev}})^2 \]  

(3)

Figure 5. Asymmetrical loop

This indicates that even in minor loops the losses are proportional to \( \Delta B^2 \), see reference [55]. In 1892 Steinmetz [56] proposed an empirical equation that relates the hysteresis losses to frequency and flux density:

\[ P_{\text{hyst}} = K_{\text{hyst}} f B^n \]  

(4)

Steinmetz computed an exponent \( n = 1.6 \) which, however, for modern steels used in transformers varies between 1.5 and 2.5 and may not be constant [55]. Although an expression of the form (4) is not fully accurate for general use, as an approximation we have selected an exponent equal to two.
Consider now the symmetrical loop of Figure 6. Then

\[ B_{rev} = -B \tag{5} \]

and (3) yields

\[ \text{AREA}_{rpm} = K_{loss} B^3 \tag{6} \]

where

\[ K_{loss} = 4K_{hyst} \tag{7} \]

The area of (6) corresponds to the hysteresis losses with symmetrical magnetization. Values of \( K_{loss} \) are available from measurements. Thus we also know

\[ K_{hyst} = \frac{K_{loss}}{4} \tag{8} \]

Figure 6. Symmetrical loop

If symmetrical loops of different amplitude are repeated, we get the picture of Figure 7. This appears to be a generalization of idealized hysteresis loops presented in the literature. In [57], for example, straight line loops are proposed (Fig. 2.21), similar to those in the central, unsaturated part of Figure 7. Such idealized, symmetrical hysteresis loops shown in the literature are, however, the starting point for simulations, while in the approach of the paper they are the result of a more general, dynamic model (equation 2a) valid for any type of transient and not restricted to linear magnetization curves.

By definition, the magnetization curve

\[ H = f(B) \tag{9} \]

is the locus of the return points \( A \) and \( A' \) in Figure 7. It is obtained from \( f_0 \) of (1) by adding \( H_{hyst} \) of (2a) corresponding to \( 2B \), according to (5): \[ H_{hyst} = K_{hyst} \times 2B \tag{10} \]

This yields

\[ H = H_{hyst} + K_{hyst} 2B = K_1 B + K_2 B^{n_1} + K_3 B^{n_2} \tag{11} \]

where, by (7),

\[ K_1 = K_{hyst} + 2K_{hyst} = K_{hyst} + \frac{K_{loss}}{2} \tag{12} \]

Accordingly,

\[ K_{Sat} = K_1 - \frac{K_{loss}}{2} \tag{13} \]

IMPLEMENTATION OF THE Hysteresis MODEL

Due to the model’s simplicity, the computer implementation is straightforward. It is based on equation (2b). We start by computing \( K_{loss} \) from hysteresis loss measurements. Then, for a given approximation (11) of the magnetization curve, we use (13) to obtain \( K_{hyst} \). For time simulations one only needs to keep track of \( B \) for the present time and the two previous integration steps. Using a very simple logic (only one if statement) one can control the program flow. If the direction of change in \( B \) is unchanged (i.e., the point where we are is not a reversal point), we continue using equation (2b). When a reversal point is encountered, then we first reset \( H \) of (2b) to \( H_{ext} \) and continue with equation (2b). What we do at a reversal point is, geometrically speaking, displacing the operating point horizontally to the hysteresis curve (see Figure 8). The displacement at reversal points is horizontal in our case, consistently with the asymptotes of the polynomial describing the saturation characteristics, but primarily for simplicity. In a transformer model the above procedure is implemented in all magnetic branches, including those possibly used in the discretized representation of the magnetization for the purpose of eddy current modeling [1]. However, the focus of the following simulations is on the effect of hysteresis itself.

In order to illustrate how the model works, we present in Figure 8 the simulation of an iron core driven by a sinusoidal voltage source with increasing amplitude. Figure 8a shows the excitation voltage and Figure 8b shows the response when the iron core has no remanent magnetization.

SIMULATION RESULTS

In this section we perform a number of transient studies to find the effect of including hysteresis in the simulations. To start, we note however that only transients involving a single winding may lead to dominance of iron core phenomena. This excludes all “longitudinal” transients. Moreover, even in open circuit, at high frequencies the magnetizing flux will be small and, therefore, modeling of both saturation and hysteresis becomes unimportant. Consequently, only a few situations which are suspected to remain of significance (although there may be more) are analyzed below.

**Inrush Currents**

When a transformer is energized, a large (inrush) current may be drawn from the source. There are a great number of references dealing with this problem; see, for example, the book by Greenwood [58] or reference [59]. To illustrate the effect of hysteresis in the inrush current, we use the simplest representation for the source, i.e., an ideal sinusoidal voltage source with constant amplitude. In Figure 9 we present the simulation of the inrush current for phase C of the three-
phase three-legged transformer presented in reference [1]. The figure actually shows two cases: with and without hysteresis in the simulation. We note that there is no difference between the two cases in the magnitude and damping of the inrush current.

![Figure 9. Inrush current with and without hysteresis](image)

We believe, the explanation of this negative result is as follows: if we imagine the circuit representation of hysteresis as a resistance in parallel with the inductor, then the voltage source will absorb directly the losses caused by hysteresis. Since the magnitude of the inrush current is only dependent on how much the material becomes saturated, the effect of hysteresis is important solely in establishing the point from where the flux starts building up.

From our simulations we conclude:

- Hysteresis does not add noticeable damping to the inrush current.
- Hysteresis only affects the magnitude of the inrush current when there is remanent magnetization (which sets the initial condition).

**Magnetically Current Chopping**

The chopping of magnetizing currents may lead to large transient overvoltages. This subject has also received very much attention in the literature; see for example [58], [60] and [61]. In this section we analyze the effects that hysteresis has on the disconnection of a transformer. The magnetizing current is abruptly chopped by a circuit breaker before its zero crossing, leaving a capacitance $C \left(=10^{-10} F\right)$ connected to the terminals of the transformer of [1]. In Figure 10 we show the transient voltage (without restrikes) when the starting point is well into saturation. Figure 10a corresponds to the simulation with the hysteresis model presented in this paper. We can observe that as the amplitude decreases, the frequency of the oscillations becomes smaller, as expected. If the damping due to hysteresis is represented by a shunt resistance calculated from the losses at 60 Hz, then the transient is excessively damped. If, on the other hand, we increase the value of the shunt resistance to give the correct damping at the highest frequency of the transient, then the damping is insufficient at lower frequencies; the results are shown in Figure 10b.

![Figure 10. Transient following magnetizing current chopping](image)

(a) Damping due to hysteresis model

(b) Damping due to high frequency equivalent resistance

**Discussion**

The frequency variation of the transient voltage is due to the fact that the effective inductance of the transformer varies with the saturation conditions. In saturation the inductance is smaller giving faster transient oscillations. Figure 11 gives the phase portrait (flux versus voltage) for the transient of Figure 10a. We note that the external contours, corresponding to the beginning of the transient, reflect the distortion due to saturation. As the transient attenuates, the contours describe more perfect ellipses, corresponding to non-saturated conditions.

![Figure 11. Flux versus voltage](image)

The differences in the amount of damping seen in the simulations of Figure 10 can be explained by examining a periodic oscillation. Then the voltage (gradient) is related to the flux density by

$$ E = \mu B $$

Our model predicts the correct losses, proportional to $B^2$ and $f$. Thus, according to (4),

$$ P_{loss} = K'_{loss} \alpha B^2 $$

(where $K'_{loss} = K_{loss} \alpha (2\pi)$). From (14) and (15) we have

$$ P_{loss} = \frac{K'_{loss}}{\alpha} E^2 $$

The losses in a constant resistance are

$$ P_R = \frac{1}{R} E^2 $$

From (16) and (17) we see that for a given sinusoidal voltage the hysteresis losses vary inversely proportional with frequency, while for a shunt resistance the losses do not depend on frequency. To properly represent the hysteresis losses the equivalent resistance should be

$$ R_{equs} = \frac{\alpha}{K'_{loss}} $$

From our observations we conclude:

- Hysteresis has a significant effect in the damping of transients due to magnetizing current chopping.
- For the calculation of electromagnetic transients, hysteresis losses cannot be adequately represented by a constant resistance connected in parallel with the nonlinear inductance.

**Ferroresonance**

The phenomenon is a series resonance between the nonlinear inductor of an iron core transformer and the capacitance of the cable connected to it [58]. A very large voltage can appear across the inductor or capacitor even if the applied voltage is within reasonable bounds. There has been a considerable amount of work in this area; see for example the recent publications [62]-[64]. The circuit for the analysis is shown in Figure 12.

![Figure 12. Circuit for the study of ferroresonance](image)

Often, the situation in which ferroresonance occurs [62], [64] is when one or two phases of the feeder to an open-circuited transformer are disconnected from the supply source so that the capacitance to ground of these conductors appears in series with the magnetizing inductance of the transformer. The winding involved may have any connection, but if it has a star point, it should be isolated from ground. Any load or loss-producing element may prevent the appearance of a resonance condition.
Since the circuit of Figure 12 has no unique natural frequency (see Figure 10), one cannot analyze the phenomenon of ferroresonance using the simple concepts and approaches applied to the examination of resonance in linear circuits. In particular, one cannot separate a transient part from a steady state solution. The latter may not even exist and when it does it may and will often take many seconds (real, not simulation time), or even longer, to reach it. This long duration dynamic is very complex and extremely sensitive to small variations of all parameters of the problem. These include: \( C \), \( V \), the initial conditions (which can be considered as contributors to the initial stored energy in the system), and last but not least, the damping due to resistances and hysteresis.

While all our simulations eventually converged to a periodic steady state, with or without subharmonics, the long duration transient has often shown significant overvoltages, multiples of the peak of the source voltage. Therefore, the reliable simulation of ferroresonance is of great practical importance. Since we are dealing with a transient, it is inaccurate to assign a single frequency to it. Often subharmonic oscillations of different orders have been noticed in our simulations; see, for example, Figure 13. These results were obtained with the transformer used above for \( v = 100 \sin(\omega t) \) and \( C = 10^{-2} \text{ F} \). However, due to space limitations we can only show a single sample of the many interesting results we have obtained. Figure 14 represents the peak values of the voltage oscillations across the capacitance in Figure 12, as a function of time. It corresponds to the transient shown in Figure 13 for a ten times longer time. A subharmonic of order 2 is clearly visible during most of the transient. This type of display is similar to those in \([62]\) and is also related to the ideas of Poisson sections, except that here we show maximum values (rather than periodically taken samples) because of their practical importance. We emphasize the significance of the special display used in our analysis: it gives directly the maximum values relevant for insulation coordination while any regular patterns can also be distinguished.

Figure 14. Maxima of the voltage of Figure 13

Since under transient conditions an equivalent resistance for the representation of hysteresis losses, chosen for the single frequency of 60 Hz, is inadequate, it was suspected that the simulation of ferroresonance, under otherwise equal conditions, would give different results with a hysteresis model than with an "equivalent" resistance. Figure 15a shows that indeed with hysteresis damping the simulation converges to a voltage of 60 Hz base frequency, while with an equivalent resistance (for 60 Hz), the steady state voltage, shown in Figure 15b, has a base frequency of 30 Hz (subharmonic of order 2). These simulations were obtained using the same transformer, as above, with \( C = 10^{-2} \text{ F} \) and \( v = 245 \sin(\omega t) \).

We conclude by noting that even a small change in the hysteresis loss coefficient \( K_{\text{hy}} \) may significantly change the results. For practical purposes it is therefore useful to condense the results of simulations as in Figures 14 and 15a with \( K_{\text{hy}} \) varied over a reasonable range, by displaying only the maxima of the peak values, i.e., their upper envelope.

Figure 15. Ferroresonance with (a) hysteresis, (b) resistance damping

**CONCLUSIONS**

The paper describes a simple procedure for the representation of hysteresis in the lamination of power transformers for the simulation of electromagnetic transients. The model produces losses proportional to frequency and to the square of the flux density, as expected from measurements.

The main characteristics of the model are, besides its simplicity, the fact that it is dynamic (i.e., it is not restricted to symmetrical hysteresis loops or, in fact, any closed loops at all), and that it can be applied to any magnetization characteristic (described by polynomial, hyperbolic, or other types of functions). It deforms insignificantly the magnetization characteristic and affects a transient only through the incurred damping.

While the damping can be obtained by an equivalent parallel resistance, the frequency dependence of the two is different. Therefore, in cases where the dynamics of the phenomena is very sensitive to the losses and to speed and frequency, as in studies of ferroresonance and magnetizing current chopping, the dynamic modeling of hysteresis appears to be particularly important. We have found, however, that hysteresis does not add damping to the inrush currents. This indicates that the mere existence of losses may be of no practical importance in situations where they are directly covered from the power source.

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**APPENDIX**

**Review of Existing Models**

References [2] to [53] represent a list of the most cited (or used) publications. It is of course not exhaustive. The first classic model for the prediction of hysteresis was by Preisach [2] in the 1930's. In this model materials are comprised by a number of magnetic dipoles each one exhibiting a square loop. Many researchers have followed Preisach's approach. It is the preferred approach of those developing finite elements programs. In 1970 Chua and Stromsmoe [3] presented the first attempt for the computer modeling of hysteresis with an electric circuit. Their model consists of a nonlinear resistor in parallel with a nonlinear inductor following a series of complicated function compositions. In 1971 Swift [4] states that eddy currents losses are much more important than hysteresis losses for power transformers. Bone [5] in 1971 presents a form of functional to give a mathematical description of hysteresis. Chua and Lass [6] improve on the model of reference [3] in 1972 to account for the d.c. loop with a still more complicated model. In 1974 Gersz and et al. [7] present static, dynamic and transient models based on the theory of Preisach [2]. The static model is obtained from the magnetization curve and the largest static hysteresis loop they could measure. The dynamic model is suitable for steady state a.c. conditions. Although, there are no details for their
REFERENCES


