MULTI-WINDING MULTI-PHASE TRANSFORMER MODEL WITH SATURABLE CORE

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Abstract This paper presents a model for multi-winding multi-phase transformer developed by the nodal inverse inductance matrix (NIIM). To obtain the NIIM the inverse of the inductance matrix is needed. Due to the strong magnetic coupling among the windings, the inductance matrix of a transformer is ill-conditioned. A new way is described to find the inverse of the inductance matrix for transformers. Core saturation and hysteresis effect are also considered. The model developed by this method is best suitable for digital computer simulation. It is applicable for both steady state and transient analysis. The winding capacitance matrix can be easily combined with the transformer model to calculate the transients of the windings.

1 Introduction

The nodal inverse inductance matrix (NIIM) has been applied in past for modeling of single-phase transformer without considering the core saturation and the hysteresis effect [1,2]. Three-phase transformer with saturable core is modeled in [3] using leakage- and mutual inductances. Single-phase multi-winding transformer is modeled in [4] taking into account the core saturation and the hysteresis effect.

In order to determine the NIIM for a network containing inductances with mutual coupling, the inverse of the inductance matrix is needed. For the reason of strong magnetic coupling among the windings on a limb, the inductance matrix for a transformer is ill-conditioned and is usually not known. Based on the physical interpretations of the elements of the triangular matrices obtained through decomposition of the inductance matrix, the inverse of the inductance matrix for multi-winding multi-phase transformer is thus evaluated.

In the NIIM the core saturation appears in the form of additional non-linear terms. This fact is used to decompose the whole network into two sub-networks, one with linear branches and the other with non-linear branches.

The hysteresis effect is considered as an ampere-turn in a fictitious winding. The value of the ampere-turn depends on the flux history in the limb. The fictitious winding presents itself to be a one layer winding with a thin conductor which lies directly on the limb. The fictitious winding is also used to determine the short circuit inductance matrix [5].

The model developed in this paper is primarily used to calculate voltage stresses on the transformer insulations, particularly in case of winding oscillations. The model is developed to calculate the transients in the combination of transformer, generator and transmission lines due to switching operations in the network [6,7]. It is extended now for multi-phase transformer considering the core saturation and the hysteresis effect.

Because of the generality of the method the model can be used in various cases. For instance, it can be used to calculate ferroresonance or inrush currents. It is very convenient to combine the above transformer model with other networks which are presented in the nodal form, for instance to combine it with the winding capacitance matrix to calculate their natural frequencies and their transients or with the bus admittance matrix to calculate voltages and currents in complicated windings arrangements and network configurations.

1.1 An Inductance Network without Mutual Coupling

For a network containing only inductances without a mutual coupling, the relationship between the node voltages and the derivatives of the node currents is given by:

\[ \mathbf{\dot{i}} = \mathbf{H}\mathbf{V} \]  

(1)

\( \mathbf{i} \) and \( \mathbf{V} \) are column matrices. The symmetrical matrix \( \mathbf{H} \) is the nodal inverse inductance matrix (NIIM). The element \( H_{ij} \) of the matrix \( \mathbf{H} \) is the negative inverse of the inductance which is connected between the nodes \( i \) and \( j \). The diagonal element \( H_{ii} \) is the sum of inverses of all inductances which are connected to the node \( i \) at one end. Equation (1) is the nodal formulation developed in [8], adapted now to an inductance network without mutual coupling.

1.2 An Inductance Network with Mutual Coupling

For a network with windings which are mutually coupled, the relationship between the branch voltages and the branch currents is given by:

\[ \mathbf{V}_b = \mathbf{M}\mathbf{I}_b \]  

(2)

\( \mathbf{M} \) is the symmetrical positive definite inductance matrix. The diagonal element \( M_{ii} \) of \( \mathbf{M} \) represents the self inductance.
of the winding $i$ and the off diagonal element $M_{ij}$ is the mutual inductance between the windings $i$ and $j$. $V_s$ and $I_s$ are column matrices.

Suppose the inverse of the inductance matrix $M$ is known. It can be shown that the following equation is valid [1,2]:

$$\mathbf{I} = (\mathbf{AM}^{-1}\mathbf{A}^T)\mathbf{V} \tag{3}$$

where $\mathbf{A}$ is the incidence matrix [8,9]. The equations (1) and (3) represent the relationship between the nodal voltages and the derivatives of the nodal currents.

The matrix $H$ in (1) can be considered as the matrix $(\mathbf{AM}^{-1}\mathbf{A}^T)$ in (3) as shown in [2]. It is therefore possible to find an equivalent circuit containing inductances without a mutual coupling. Of course the inverse of the inductance matrix of the original circuit must be known.

2 The Inverse of the Inductance Matrix for Single-Phase Transformer

The inverse of a non-singular matrix $M$ can be performed by the triangular decomposition or LU-factorization technique [8,10].

$$M = LU \tag{4}$$

$$L = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

$$U = \begin{pmatrix}
U_{11} & U_{12} & U_{13} & U_{14} & U_{1n} \\
0 & U_{22} & U_{23} & U_{24} & U_{2n} \\
0 & 0 & U_{33} & U_{34} & U_{3n} \\
0 & 0 & 0 & U_{44} & U_{4n} \\
0 & 0 & 0 & 0 & U_{nn}
\end{pmatrix}$$

The inverses of $L$ and $U$ can be found easily and the inverse of $M$ is:

$$M^{-1} = U^{-1}L^{-1} \tag{5}$$

The elements of the matrices $L$ and $U$ are [8,10]:

$$U_{ik} = M_{ik} - \sum_{j=1}^{i-1} L_{ij}U_{jk} \quad i \leq k \tag{6}$$

$$L_{ki} = (M_{ki} - \sum_{j=1}^{i-1} L_{kj}U_{ji})/U_{ii} \quad i \leq k \tag{7}$$

In our case, the matrix $M$ is symmetrical and $L$ can be found as follows [10]:

$$L^T = D^{-1}U \tag{8}$$

$D$ is a diagonal matrix with the elements equal to the diagonal elements of $U$. From (5) and (8) it follows:

$$M^{-1} = U^{-1}D(U^{-1})^T \tag{9}$$

2.1 Physical Interpretations of the Elements of the Triangular Matrices $L$ and $U$

The physical interpretations of the elements of the triangular matrix factors $L$ and $U$ obtained by the triangular decomposition of the inductance matrix $M$ are very interesting and forms the basis of the analysis in this paper. As is shown in [11], the element $L_{ki}$ ($i < k$) of the matrix $L$ is the voltage ratio of the winding $k$ to the winding $i$ when all the windings $i$ through $i - 1$ are short circuited (resistances neglected). The element $U_{ik}$ ($i < k$) is the mutual inductance of the windings $i$ and $k$ provided all the windings $i$ through $i - 1$ are short circuited. The element $U_{ii}$ is the self inductance of the winding $i$ when all the windings $i$ through $i - 1$ are short circuited.

For a single-phase single-limb transformer with a highly magnetic coupling among the windings, the elements of the first column of the matrix $L$ and the elements of the first row of the matrix $U$ are given by:

$$L_{i1} = N_i/N_1 \tag{10}$$

$$U_{1i} = SN_i/N_1 \tag{11}$$

where $S$ is the reciprocal value of the reluctance of the magnetic core and $N_i$ is the number of turns of the winding $i$.

The elements of the triangular matrices obtained by decomposition of the complex impedance matrix $Z$ have similar physical interpretations.

2.2 Calculation of the Triangular Matrix $U$

As stated earlier, due to the strong magnetic coupling among the windings on a limb, the inductance matrix for a transformer is ill-conditioned. However, it is possible to calculate or to measure the self- and mutual inductances of the windings, if one winding (say number 1) is short circuited [5]. The inductance matrix for the case that the winding 1 is short circuited shall be noted as $M_1$. Factorization of this matrix gives $U_1$.

Assume the matrix $M$ is known. Factorization of $M$ gives $U$. Because of the physical interpretations of the elements of $U_1$ and $U$, the corresponding elements of these two matrices are equal besides the first row of $U$ which does not exist in $U_1$. The first row of $U$ can be calculated from (11). Adding this row to $U_1$ gives $U$. The lower triangular matrix $L$ can be obtained from (8).

2.3 Calculation of the Inverses of the Matrices $U$ and $M$

The elements of the inverse of the matrix $U$ can be calculated beginning from the last row of the matrix $U$ and continuing by the second last row and so on [10]. The first row of $U$ is not needed at the beginning of the calculation and the elements of $U^{-1}$ can be calculated from the last row up to the second row by inverting the matrix $U_1$. The first row of the matrix $U^{-1}$ can be calculated as follows (Appendix a):

$$u_{11} = 1/(SN_1/N_1) \tag{12}$$

$$u_{1i} = -\sum_{j=1}^{i} u_{ji}N_j/N_1 \tag{13}$$
\( u_{ij} \) is an element of the matrix \( U^{-1} \).

Only the element \( u_{11} \) of \( U^{-1} \) depends on the reluctance of the core. It follows from (8) that no element of \( L \) depends on the core reluctance, and it follows from (9) that only one element of the inverse of the inductance matrix depends on the core saturation (Appendix b). That is \( m_{11} \) and is equal to:

\[
m_{11} = 1/(SN_1N_2) + \sum_{j=2}^{n} \frac{u_{1j}^2}{u_{jj}}
\]

(14)

The first part of this element depends on the core reluctance and the second part does not depend on the core saturation.

At the first step the reluctance of the core can be assumed to be zero. Thus the value of \( S \) is infinite and the first term in (14) is zero. Depending on the core flux the reluctance of the core can be calculated and if necessary the first term of (14) can be added to the elements of the matrix \( H \) which depend on the reluctance of the core.

3 Resistances of the Windings

The resistances of the windings can be inserted in the equivalent circuit evaluated for self- and mutual inductances of the windings. For this purpose the incidence matrix \( A \) has to be taken for the case that the ends of all windings are floating. The resistances of the windings inserted to the equivalent circuit for the inductances connect the windings to each other and to the external nodes as in real practice.

For the solution of the steady state sinusoidal excitation the complex matrix \( Z = (j\omega M + Re) \) can be taken instead of \( M \). \( Re \) is a diagonal matrix with the element \( Re_i \) equal to the resistance of the winding \( i \). Taking \( Z^{-1} \) instead of \( M^{-1} \) gives the complex nodal admittance matrix. Separation of the real and the imaginary parts of this matrix gives:

\[
Y = jB + G
\]

(15)

Each part of the matrix \( Y \) describes a network. One network contains only inductances without a mutual coupling and the other one contains only resistances.

4 Multi-Phase Transformer

In the case of several single-phase transformers the inverse of the inductance matrix for each phase can be obtained and put together as follows:

\[
M^{-1} = \begin{pmatrix}
M_{11}^{-1} & 0 & 0 & 0 \\
0 & M_{22}^{-1} & 0 & 0 \\
0 & 0 & M_{33}^{-1} & 0 \\
0 & 0 & 0 & M_{44}^{-1}
\end{pmatrix}
\]

The saturation of the core for each transformer can be taken into consideration by changing one element of the corresponding matrices \( M_{11}^{-1}, M_{22}^{-1} \) etc.

In the case of a multi-phase transformer with a magnetic coupling between windings of different limbs the problem is more complicated. With some assumptions the problem becomes simpler. These assumptions are:

4.1 Assumptions

1. The magnetic coupling between two windings of different limbs is only given by the flux through the core and not through the air.

2. The short circuit impedances of windings on each limb do not depend on the core saturation.

3. If the winding 1 is energized and all other windings on the same limb are open the no load voltage ratio of the windings 1 and 1 on the same limb is equal to the corresponding turn ratio and does not depend on the core saturation.

4. If one or more windings on a limb are short circuited and one winding on the same limb is energized a magnetic field figure arises in the space between the windings and the core. In this case a certain portion of the flux of the energized winding reaches the yoke and closes itself through the other limbs. It is assumed that the flux which reaches the yoke is a certain percentage of the flux of the energized winding and does not depend on the core saturation.

The first three assumptions are easily comprehended as good approximations. The following equation should show that even the last assumption is a good approximation.

The flux \( \Phi \) through the energized winding, while some other windings on the same limb are short circuited, closes itself in two ways. One part finds its way through the air (\( \Phi_a \)) and the other part finds its way through the yoke (\( \Phi_y \)). The last part reaches the yoke through an air gap. The equivalent magnetic circuit is shown in figure 1.

\[
\Phi_y = \frac{R_a}{R_a + R_y + R_c}
\]

(16)

Figure 1:

The reluctance \( R_a \) in figure 1 is for the portion of the flux through the air. \( R_y \) is the reluctance of that part of the flux which reaches the yoke, before reaching it and \( R_c \) is the reluctance to the flux \( \Phi_y \) after reaching the yoke. Only \( R_c \) depends on the core saturation. The ratio \( \Phi_y/\Phi \) can be found as follows:

\[
\frac{\Phi_y}{\Phi} = \frac{R_a}{R_a + R_y + R_c}
\]

\( R_c \) is usually small in comparison with \( R_a \) and \( R_y \). Therefore the change in \( R_c \) does not change this ratio considerably.

4.2 Evaluation of the Matrix \( U \) for Multi-Phase Transformer

With these assumptions the matrix \( U \) (and \( L \)) can be evaluated for a multi-phase transformer with saturable core. For
this purpose the magnetic circuit of the core must be considered simultaneously with the electric circuit. For a three-phase transformer a simple magnetic circuit is shown in figure 2. A more complicated circuit could be also taken into consideration too.

![Diagram](image)

**Figure 2:**

In figure 2, $R_1$, $R_2$, $R_3$ are the reluctances of the three wounded limbs. $R_4$ is the reluctance to the flux from the upper yoke to the lower yoke through the air and $R_5$ is the same but through further limbs which are not wounded. The following factors are needed to build up the matrices $L$ and $U$:

1. $S_1$ The inverse of the reluctance of the magnetic circuit if the winding 1 on the limb 1 is energized and all other windings are left open.
2. $S_2$ The inverse of the reluctance of the magnetic circuit if all the windings on the limb 1 are short circuited ($R_1 = \infty$) and the winding 1 on the limb 2 is energized. All other windings are left open.
3. $S_3$ The inverse of the reluctance of the magnetic circuit if all the windings on the limbs 1 through $i - 1$ are short circuited ($R_1,...,R_{i-1} = \infty$) and the winding 1 on the limb $i$ is energized. All other windings are left open.

For the case that the winding 1 on the limb 1 is energized and all other windings are left open the flux $\Phi_i$ generated in the limb 1 closes itself through other limbs. The factor $a_{ii}$ is the ratio of the flux $\Phi_i$ in the limb $i$ to the flux $\Phi_i$.

For the case that all windings on the limbs 1 through $i - 1$ are short circuited and the winding 1 on the limb $i$ is energized the flux $\Phi_i$ arises in the limb $i$ and closes itself through other limbs. The factor $a_{ij}$ is the flux in the limb $j$ divided by the flux in the limb $i$.

For the case that the windings 1 through $i - 1$ on a certain limb are short circuited and the winding $i$ on the same limb is energized and all other windings on this limb are left open, the total flux of the energized winding is $\Phi_i$ and a part of this flux reaches the yoke. The factor $b_i$ is the ratio of the flux which reaches the yoke to the flux $\Phi_i$. For $n$ windings on a limb there exist $n - 1$ such factors: $b_2, b_3, \ldots, b_n$.

The factors $b_i$ and $a_{ij}$ depend on the core saturation. The factors $b_i$ are assumed not to depend on the core saturation.

The factors $S_1$ and $a_{ij}$ can be found by magnetic field calculation or by measurement.

For a transformer with three wounded limbs and with the magnetic circuit as shown in figure 2 the factors $S_1$ , $S_2$ , $S_3$, $a_{12}$ , $a_{13}$ and $a_{23}$ can be calculated as given below:

$$S_1 = \frac{R_2 R_3 R_4 + R_2 R_5 R_5 + R_2 R_4 R_5 + R_3 R_4 R_5}{R_1 R_2 R_3 (R_4 + R_5) + R_1 R_2 R_4 R_5 + R_3 R_4 R_5 (R_1 + R_2)}$$

$$S_2 = \frac{R_3 R_4 + R_3 R_5 + R_4 R_5}{R_2 R_3 R_4 + R_2 R_3 R_5 + R_2 R_4 R_5 + R_3 R_4 R_5 (R_1 + R_2)}$$

$$S_3 = \frac{R_4 + R_5}{R_3 R_4 + R_3 R_5 + R_4 R_5}$$

$$a_{12} = \frac{R_3 R_4 + R_3 R_5 + R_2 R_4 R_5}{R_2 R_3 R_4 + R_2 R_3 R_5 + R_2 R_4 R_5 + R_3 R_4 R_5}$$

$$a_{13} = \frac{R_2 R_3 R_4 + R_2 R_3 R_5 + R_2 R_4 R_5}{R_2 R_3 R_4 + R_2 R_3 R_5 + R_2 R_4 R_5 + R_3 R_4 R_5}$$

$$a_{23} = \frac{R_3 R_4 + R_3 R_5 + R_4 R_5}{R_3 R_4 + R_3 R_5 + R_4 R_5}$$

Considering the physical interpretations of the elements of the matrices $L$ and $U$, the factors defined above and assumptions made earlier, the elements of the matrices $L$ and $U$ can be evaluated. For a $n$-winding $m$-phase transformer there exist $(m \cdot n)$ windings which should be numbered from 1 to $(m \cdot n)$. The windings, one on each limb, are included in $n$. The first winding on the first limb is number one. The corresponding winding on the limb 2 is numbered as $(n + 1)$ and on the next limb as $(2 \cdot n + 1)$ and so on.

The triangular matrices $L$ and $U$ have $(n \cdot m)$ rows and columns. For the first column of the matrix $L$ the no load voltage ratios of all windings to the winding number 1 are necessary. The no load voltage ratios for windings to the winding number 1 on the same limb were assumed to be equal to the corresponding turn ratios. For windings on separate limbs it depends on the corresponding turn ratio and the flux distribution in the various limbs. The flux distribution is given by the magnetic core equivalent circuit. The ratio of the flux in the limb $i$ to the flux in the limb 1 is given by $a_{ii}$. The voltage ratio of the winding $j$ on the limb $i$ to the winding 1 on the limb 1 is therefore:

$$\frac{V_{ji}}{V_{11}} = \frac{N_j}{N_1} a_{ij}$$

(17)

The first element of the first row of the matrix $U$ is:

$$U_{11} = S_1 N_1^2$$

(18)

The other elements of the first row of the matrix $U$ are gained by multiplying this element with the corresponding elements of the first column of the matrix $L$.

The elements of the second column of the matrix $L$ are the voltage ratios of windings to winding 2 on limb 1 if winding 1 on limb 1 is short circuited. All the other windings are left open. For the windings on limb 1 the voltage ratios can be found by magnetic field figure. For the windings on the other limbs the factor $b_2$ is needed. This factor can be found by the same field figure and gives the flux reaching the yoke. The distribution of this flux in various limbs is given by the equivalent circuit of the magnetic core. For this purpose the factor $a_{ii}$ can be used. The voltage ratio of the winding $j$ on
the limb i to the winding 2 on the limb 1 if the winding 1 on
the limb 1 is short circuited, is:

\[
\frac{V_{i1}}{V_{21}} = \frac{N_j}{N_k} b_2 a_{1i}
\]  

(19)

The self inductance of the winding 2 on the limb 1 if the
winding 1 on the same limb is short circuited can be found
from the magnetic field figure in the transformer window.
For the third column of the matrix \( L \) the voltage ratios of
all windings to the winding 3 on limb 1, when the windings 1
and 2 on the same limb are short circuited, are necessary. For
the windings on the limb 1 the voltage ratios can be found
by the triangular factorization of the matrix \( M_1 \), as for
the single-phase transformer. If the windings are on separate
limbs the voltage ratios can be found as follows:

\[
\frac{V_{i1}}{V_{31}} = \frac{N_j}{N_k} b_2 a_{1i}
\]  

(20)

That is the voltage ratio of the winding j on the limb i to the
winding 3 on the limb 1.
In this way all elements of the matrices \( U \) and \( L \) can be
calculated. The inverse of the matrix \( U \) is very interesting.
For a three-winding three-phase transformer the upper tri-
angular matrix \( U \) and its inverse are shown above.

The inverse of \( M \) can be obtained using (9). To separate
the linear and the non-linear terms, the matrix \( U^{-1} \) can be
written as the sum of two matrices. One contains only the
linear terms and the other one only the non-linear terms.
First forming (9) after Cholesky [10]:

\[
M^{-1} = (U^{-1} D^{-1/2})(U^{-1} D^{-1/2})^T
\]  

(21)

The matrix \( D^{-1/2} \) is a diagonal matrix with elements equal to
the square roots of the corresponding elements of \( D \). Then
separating the linear and the non-linear terms:

\[
U^{-1} D^{-1/2} = (B + C)
\]  

(22)

Matrix \( B \) contains only the linear terms. It follows:

\[
M^{-1} = BB^T + BC^T + CB^T + CC^T
\]  

(23)
or

\[
M^{-1} = M_{i}^{-1} + M_{n}^{-1}
\]  

(24)

It is interesting that the linear part is put together from the
inverse of the inductance matrix for each phase, neglecting
the core reluctance. Every wounded limb is considered as one
phase.
With the help of (3) with the same incidence matrix, two
inductance networks can be found. One contains only con-
stant inductances and the other only non-linear inductances.

5 Hysteresis Effect

The hysteresis effect in the core is due to alignment of the
domains and orientation of the electron spins in the direction
of the external magnetic field, and remaining in this direction
after cancelling the external field. The electron spins can be
considered as small current loops and the sum of them acts
as a current sheet on the limb surface [12].

A good method to model the hysteresis effect in a trans-
former limb seems to be an ampere-turn flowing in a ficti-
tious winding which lies directly on the limb and has no resist-
ance. Figure 3 shows a hysteresis curve. \( \Phi \) is the flux in the limb
and \( I \) is the corresponding no load current of a winding on
this limb with \( N \) turns. The flux versus the no load current
without considering the hysteresis effect is shown as a dashed
curve.

The idea is to use this dashed curve all the time without
considering the hysteresis effect by adding an ampere-turn to
the fictitious winding. This ampere-turn varies with the time
and depends on the flux which flowed previously in the limb
and on the flux value at each instant. The ampere-turn in the
fictitious winding, modeling the hysteresis effect depends on
the flux history in the limb. The fictitious winding, used in
[5] as a help to calculate the short circuit inductance matrix and used in this paper to find the triangular matrices from the short circuit inductances, can be useful in this case too. This winding is assumed to be connected to a current source. The amount of the current at each instant is so high that it produces the ampere-turn $I_h N$ in figure 3. The number of turns for the fictitious winding can be chosen equal to $N$. The current source should produce the current $I_h$.

\[ W = \int_0^T N I_h \frac{d\Phi}{dt} dt = \int_0^T I_h V dt \]  

(25)

$V$ is the induced voltage in the fictitious winding. The hysteresis power loss is therefore:

\[ P = I_h V \]  

(26)

That means that the hysteresis loss is equal to the power consumed by the current source.

For simulation of the magnetic field and the electrical losses of the eddy current in the core, a resistance connected to the ends of the fictitious winding can be used, as in usual equivalent circuit of the transformer.

6 Procedure to obtain the Transformer Model

The procedure to obtain the multi-winding multi-phase transformer model with saturable core is as follow:

1. The matrix $M_i$ must be evaluated for each limb. The elements of this matrix are the self- and mutual inductances of the windings on the limb if winding number 1 on the same limb is short circuited. The winding number 1 should be preferably a fictitious winding which is assumed to lie directly on the limb. The method to calculate the matrix $M_i$ for the case that a fictitious winding is short circuited is discussed in [5].

2. Factorization of $M_i$ can be performed by existing computer programs to find the triangular matrices $U_i$ and $L_i$.

3. The inverse of the matrices $U_i$ and $L_i$ can be calculated without numerical complications.

4. The first row of $U^{-1}$ can be calculated using (12) and (13). The other elements of $U^{-1}$ are the same as of $U_i^{-1}$.

5. The inverse of $M$ can be calculated from $U^{-1}$ considering the equation (9). For the first step, the core reluctance can be assumed to be zero.

6. This procedure must be done for all the limbs. The combination of the inverses of these matrices gives the linear part of the matrix $M^{-1}$. The nonlinear part of the matrix $M^{-1}$ must be calculated from (23). The nonlinear part of the inverse of the inductance matrix must be newly calculated whenever the resistances are changed.

7. The nodal inverse inductance matrix (NIIM) has likewise a linear and a non-linear part. The linear part of the NIIM represents a network containing constant inductances and the nonlinear part of the NIIM contains inductances which vary with the core reluctances. Both parts are to be obtained from (3) taking the linear part and the non-linear part of $M^{-1}$ respectively.

8. To model the hysteresis effect a current source has to be connected to the ends of a fictitious winding for each limb. The current of this winding depends on the flux history in the corresponding limb. A resistance connected to the ends of the fictitious winding models the eddy current losses in the limb.

9. The calculation of the voltages, currents and fluxes can be done using the step by step integration methods. During the calculation the reluctances can change depending on the fluxes through the limbs.

7 Conclusion

An equivalent circuit for a multi-winding multi-phase transformer with saturable core is developed. The equivalent circuit contains inductances without mutual coupling. Some of the inductances are constant and some of them depend on the core reluctance. The equivalent circuit is described by the nodal inverse inductance matrix. The hysteresis effect and the eddy current losses in the core are simulated.

It is very convenient to combine the model with other networks which are presented in the nodal form and is of great advantages if the number of the windings is high.

The model can be used to calculate the transients in the windings, ferroresonance, inrush currents and steady state analysis in complicated winding arrangements and network configurations.

8 Appendix

a) The matrix $U$ has the following form:

\[
U = \begin{pmatrix}
S N_1 N_1 & S N_1 N_2 & S N_1 N_i & S N_1 N_n \\
0 & U_{22} & U_{2i} & U_{2n} \\
0 & 0 & U_{ii} & U_{in} \\
0 & 0 & 0 & U_{nn}
\end{pmatrix}
\]
and the matrix $U_1$ has the form:

$$U^{-1} = \begin{pmatrix} u_{11} & u_{12} & u_{14} & u_{1n} \\ u_{21} & u_{22} & u_{24} & u_{2n} \\ u_{41} & u_{42} & u_{44} & u_{4n} \\ u_{n1} & u_{n2} & u_{n4} & u_{nn} \end{pmatrix}$$

Multiplication of these two matrices gives the unit matrix. Thus the elements of the last row up to the second row of $U^{-1}$ can be calculated. The elements of the first row of this matrix are given by:

$$u_{11} = 1/(SN_1N_1), \quad u_{12} = -u_{22}N_2/N_1$$

$$u_{14} = -(\sum_{j=2}^{i} u_{j4}N_j)/N_1$$

Only one element of $U^{-1}$ depends on the core reluctance. That is the element $u_{11}$.

b) The following matrix multiplication gives $M^{-1}$:

$$M^{-1} = U^{-1}L^{-1}$$

No element of the matrix $L^{-1}$ depends on the core reluctance. Writing the last equation in detail:

$$X = \begin{pmatrix} 0 & 0 & + & + & + \\ 0 & 0 & + & + & + \\ 0 & 0 & 0 & + & + \\ 0 & 0 & 0 & 0 & + \end{pmatrix}, \quad L^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ + & + & 1 & 0 & 0 \\ + & + & + & 1 & 0 \end{pmatrix}$$

$$M^{-1} = X \cdot L^{-1}$$

X represents the element which depends on the core reluctance and + the elements which do not depend on the core reluctance. It can be easily shown that only the first element of the first row of the matrix $M^{-1}$ depends on the core saturation.

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References


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Discussion

Francisco de Leon (University of Toronto): The author has presented an interesting method for computing the inverse of the inductance matrix directly for which he should be congratulated. His method avoids the numerical inversion of an ill-conditioned matrix and, simultaneously, he introduces the iron-core saturation and the resistance of the windings in a very nice and easy manner. This discussion will focus on the relation of the matrix $M^{-1}$ (linear case or part) with the leakage inductance or impedance and the method used to evaluate its elements.

The model described in [A] is based on a $\Gamma$ matrix (inverse inductance) with its elements calculated from short-circuit tests. The same is done by many other authors, for example see [B] and [C]. Models using the leakage inductances as a calculation base accurately represent the transfer characteristics of the transformer. One can see from the author’s paper that the leakage inductance between windings was not considered explicitly. Could the author explain how his model accounts for the transfer characteristics?

I would like to suggest two easy tests to corroborate the transfer characteristics of this model:

(a) If the reluctance of the core is assumed to be zero in equation (14), the resulting $M^{-1}$ matrix should be identical to the $\Gamma$ matrix of the reference [A].

(b) The matrix $M^{-1}$ in equation (24) should also be equal to the $\Gamma$ matrix in [A] for the multi-phase case.

It is known from [D] and [E] that the accurate evaluation of the inductance matrix does not guarantee that the leakage effects (or transfer characteristics) in a transformer are also accurate. Thus, assuming that the matrix $U$ is calculated or measured accurately does not imply that leakage related quantities are also accurate.

The comments of the author on the previous points would be very much appreciated.


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Hossein Mohseni: The author appreciate the interest in the paper and the comments by Mr. de Leon. It is true that the matrix $M^{-1}$ in the paper is identical to the matrix $\Gamma$ in reference [A] if the reluctance of the core is assumed to be zero. The matrix $H$ is in this case identical to the matrix $\Gamma_N$ in the same reference.

In case of a multi-phase transformer it is not possible to get all informations if the reluctances are assumed to be zero. For example it is not possible to find the flux distribution in the limbs. For multi-phase transformer the reluctances of the limbs must be considered in equation (24). The values for the reluctances can be assumed to be constant.

If the matrix $U$ is known the matrix $M^{-1}$ can be calculated without numerical difficulty. If now one winding is short-circuited the corresponding branch voltage is zero. Using the following equation

$$M^{-1} U_B = i_B$$

it can be shown that $M^{-1}_{sc}$ is the same as $M^{-1}$ only if the corresponding row and column of $M^{-1}$ does not exist in $M^{-1}_{sc}$. The matrices $M^{-1}$ and $M^{-1}_{sc}$ are not ill-conditioned for transformer.

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