

MULTI-PORT EQUIVALENCING OF EXTERNAL SYSTEMS FOR SIMULATION OF SWITCHING TRANSIENTS

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Abstract – A multi-port external system equivalent model to simulate switching transients is developed based on direct time domain techniques, as opposed to the currently available models based on frequency domain methods. The model comprises discrete-time filters, which are built using the external system’s response at each of the M ports to a multistep excitation signal and solving a system identification problem. At each port, the filters obtained are converted into an equivalent Norton circuit consisting of M current sources (in parallel). A unique feature of the model is that it explicitly incorporates the travel time delay between the ports. The complete M-port model, having M decoupled equivalent Norton circuits, is easily integrated into the discrete-time representation of the study subsystem. The proposed model is validated by simulating the line energization and post-fault transients in a study subsystem connected at two ports to an external system.

Keywords: Network Equivalents, Electromagnetic Transients, System Identification, Modeling, Simulations.

I. Introduction

Power system electromagnetic transients are simulated using time domain methods implemented in most Electromagnetic Transients Programs (EMTPs) [1, 2]. Therefore, even though linear system elements like transmission lines, transformers, etc. exhibit significant frequency dependence over typical bandwidths of interest in electromagnetic transients studies, researchers have strived to obtain accurate time domain models for them [3, 4]. Also, time domain simulation permits the representation of non-linear elements and operation of switches in the system in a fairly straightforward and relatively easy way (compared to frequency domain). The resultant overwhelming computational efficiency of EMTPs has rendered them as practical system planning tools.

However, since EMTPs use very detailed models of system components, modeling the entire system for a transients study can require prohibitive computing resources in terms of memory space and CPU time. In most transients studies such as static/stochastic switching overvoltages (for insulation coordination), post-fault transients, AC/DC system’s dynamic analysis, the event of interest takes place in a small part of the entire power system. Thus, the use of proper equivalents to reduce the system model complexity will significantly reduce the overall study time.

The conventional power frequency short circuit equivalents cannot accurately represent the external network behavior when the simulated transients have a broad frequency spectrum. Various researchers have recognized the need for accurate network equivalents valid over a wide frequency range (frequency-dependent equivalents) since many years [5, 6, 7, 8, 9]. However, all of the research efforts have concentrated on the frequency domain approach. The objective was to synthesize a (passive) network of lumped R-L-C components whose optimal values are determined such that it exhibits approximately the same frequency response as the external system over the frequency range of interest.

Recently, new techniques of external network equivalencing were proposed. A Hybrid EMTP has been developed by interfacing the frequency domain transfer function of the single-port external system with the time domain model of the study system, provided they are connected through a long transmission line [10]. The authors have developed a technique of directly obtaining the time domain model to represent the external system in form of a discrete-time Norton equivalent using system identification methods [11]. Good results were obtained using a single-port external system equivalent.

In modern interconnected power systems, the area identified for a switching transients study will be typically connected to the remaining system at two or more boundary buses (ports), as shown in Fig. 1. Therefore, any equivalencing technique proposed must be amenable to enhancement for representation of multi-port external systems. For frequency domain equivalencing techniques, this has been accomplished by synthesizing R-L-C networks for each branch of a multi-port π-equivalent circuit [12] and is available as the FDNE feature of EPR/DCG EMTP v2.0 [13].

This paper presents the theory, implementation and validation of the time domain multi-port frequency dependent equivalent by augmenting the core idea and concepts developed in [11]. The resultant discrete-time equivalent at each port is a Norton circuit consisting of multiple current sources (equal to number of ports). The equivalent connected at a port is decoupled from that at another port due to the finite travel time between them. This is very similar to the discrete-time model used for (frequency dependent) transmission line in EMTPs. The proposed equivalent is easy to obtain, preserves the integrity of port-interconnections through appropriate travel delays, and lends itself to easy implementation in any existing EMTP.
II. Multi-Port Discrete-Time Model

Consider the system shown in Fig. 1 where the part of the network selected for detailed modeling to study transients (study subsystem) is connected to the rest of the system (external system) at M ports. It is required that the M-port equivalent of the external system produces an accurate response at the boundary buses, over a wide frequency range, to the signals incoming from the study system.

The passive external system may be described by expressing the injected current at a port in terms of the voltages at the M ports and the short circuit (s.c.) admittances, as given below:

\[
\begin{align*}
I_1 &= Y_1 V_1 + Y_{12} V_2 + \ldots + Y_{1M} V_M \\
I_2 &= Y_2 V_1 + Y_{22} V_2 + \ldots + Y_{2M} V_M \\
&\vdots \\
I_M &= Y_M V_1 + Y_{M2} V_2 + \ldots + Y_{MM} V_M
\end{align*}
\]

or equivalently,

\[
[I]_{M \times 1} = [Y]_{M \times M} [V]_{M \times 1}
\]

The elements of the s.c. admittance matrix \([Y]\) are:

\[
\begin{align*}
Y_{ii} &= \frac{1}{V_i} \left| V_{i0} \right| \quad m = 1, 2, \ldots, M; \quad m \neq i \\
Y_{ij} &= \frac{1}{V_i} \left| V_{j0} \right| \quad m = 1, 2, \ldots, M; \quad m \neq i
\end{align*}
\]

where superscript \(sc\) indicates that this current injection corresponds to the short circuit current (to ground) at port \(j\). Since \(Y_{ii} = Y_{ii}\) for a passive network consisting of linear, bilateral, lumped or distributed circuit elements, therefore, all the entries of \([Y]\) can be known by evaluating only the lower (or upper) triangular part of \([Y]\). For an M-port network, this involves \(M + (M-1) + \ldots + 1 = M(M+1)/2\) calculations from \(M\) different network short circuit configurations that correspond to the \(i = 1, 2, \ldots, M\) cases where \(V_i = 0\), when all \(V_{i0} = 0; m \neq i\). For example, for a 2-port network, \(2(2+1)/2 = 3\) calculations (for lower triangular entries of \([Y]\) need to be done using \(M = 2\) different short-circuit configurations; \(Y_{11}\) and \(Y_{22}\) are calculated when \(V_2 = 0\) (port 2 shorted), while \(Y_{21}\) is calculated when \(V_1 = 0\) (port 1 shorted).

For simplicity, the derivation of discrete-time model is explained for a 2-port external system. The same technique is applicable for an \(M\)-port external system. In general, the s.c. admittances are functions of complex frequency \(s\), and can be expressed as a ratio of polynomials in \(s\), as given below:

\[
\begin{align*}
I_{1i}(s) &= Y_{1i}(s) V_i(s) = K_{1i} \frac{N_{1i}(s)}{D_{1i}(s)} V_i(s); \quad V_2 = 0 \\
I_{2i}(s) &= Y_{2i}(s) V_i(s) = K_{2i} e^{-r_{D_{1i}(s)}} \frac{N_{2i}(s)}{D_{2i}(s)} V_i(s); \quad V_2 = 0 \\
I_{12}(s) &= Y_{21}(s) V_2(s) = K_{21} \frac{N_{21}(s)}{D_{21}(s)} V_2(s); \quad V_1 = 0
\end{align*}
\]

Note that the transfer functions \(Y_{ii}(s)\) are expected to have a transportation lag or (travel) time delay, \(r_{D_{1i}(s)}\), since they relate the response at port \(i\) to the excitation at port \(j\) and, ports \(i\) and \(j\) are generally connected through distributed-parameter elements (transmission lines) in power system networks. Also, in Eq. (1) - (3), both \(N_i(s)\) and \(D_i(s)\) are assumed to be polynomials in \(s\) of same degree \(d\); this does not violate any properties of the driving point or transfer functions [14]. However, the value of \(d\) will be different in each of the three equations.

Cross multiplication in each Eq. (1) - (3) results in an s-domain equation with polynomials of degree \(d\) on both sides, or equivalently, a linear differential equation of degree \(d\) in continuous-time domain, and a linear difference equation of degree \(d\) in discrete-time domain. The difference equations corresponding to Eq. (1) - (3) are, therefore, as follows [11]:

\[
\begin{align*}
i_1(t) &= -\sum_{k=0}^{d} a_{1k}(1) i_1(t - k\Delta t) + \sum_{l=0}^{d} b_{1l}(1) v_l(t - l\Delta t) \\
i_2(t) &= -\sum_{k=1}^{d} a_{2k}(2) i_2(t - k\Delta t) + \sum_{l=0}^{d} b_{2l}(2) v_l(t - l\Delta t) \\
i_2(t) &= -\sum_{k=1}^{d} a_{2k}(2) i_2(t - k\Delta t) + \sum_{l=0}^{d} b_{2l}(2) v_l(t - l\Delta t)
\end{align*}
\]

where \(T_{12} \approx r_{D_{12}}/\Delta t\) in Eq. (5). Since \(Y_{12}(s) = Y_{21}(s)\), the difference equation for \(i_1(t) = Y_{12}(s) v_2(s)\) will have the same parameters as in Eq. (5). That is, \(p_{12} = p_{21}\), \(a_{1k}(1) = a_{2k}(2)\), \(b_{1k}(2) = b_{2k}(2)\), and \(T_{12} = T_{21}\).

Therefore, Eq. (4), (5) and (6) define the complete model for a 2-port (passive) external system in time domain. Consequently, for an \(M\)-port external system, \(M(M+1)/2\) difference equations will completely define its discrete-time model. The unknowns in the model: \(p_{11}, p_{21}\) and \(p_{21}\), travel delay \(T_{12}\), and parameters \(a_{1k}(1), b_{1k}(2)\) can be determined by system identification techniques, as explained in the next section.

III. Model Identification and Parameter Estimation

The (injection) current samples in equations such as Eq. (4) or (6) represent the response of the external network at the same port to which the excitation signal is applied, and those in equations such as Eq. (5) represent the response(s) at the other port(s) of the external system due to the same excitation. Both types of responses are obtained when all the ports, except the one to which the excitation is applied, are shorted to ground. Since the objective is to excite all modes of the external system within a selected broad range of frequencies, the multisine signal is an appropriate excitation for this purpose [15].

Model identification refers to determining the model order \(p\). Once the order is specified, then the delay \(T\), and parameters \(a_{1k}(1), b_{1k}(2)\) of the model have to be estimated and this constitutes the parameter estimation step. For a particular \(p\), the model's parameters are estimated and the error \(e_{\text{est}}\) of the fitted model is computed using

\[
\sigma_{\text{est}} = \frac{1}{N - p} \sum_{k=p+1}^{N} (i(k\Delta t) - i(k\Delta t))^2
\]

where:

- \(N\) is the total number of time-steps in the external system response,
- \(i(k\Delta t)\) is the current injection at time step \(k\Delta t\),
- \(\hat{i}(k\Delta t)\) is the estimated current injection at time step \(k\Delta t\) using the model.

When \(\sigma_{\text{est}}\) is smaller than a certain threshold, \(\epsilon\), the system identification is complete.

The following steps outline the system excitation, model identification and parameter estimation procedure:

1. Disconnect the study subsystem and render the external system passive by turning OFF all the sources.
2. Connect a periodic multisine voltage source as excitation at one boundary bus (say, port \(m\)) and short the remaining \(M - 1\) ports of the external system to ground. Extract the time-domain steady-state response of the external system of duration \(T_{\text{max}}\) seconds from the EMTP simulation of system excitation. The measurements set comprises of \(N\) samples of each: the voltage, \(v\) and injected current, \(i^n\), at port \(m\) and the short circuit currents, \(i^n\), at other \(M - m\) ports. The criteria for the choice of \(N\), \(T_{\text{max}}\) and \(\Delta t\) are explained in the Appendix.
3. Obtain the complete measurements set required for external system identification by repeating step 2 for \( m = 1, 2, \ldots, M \). It will consist of \( M \) sequences of \( v \) and \( i^m \), and \( M(M+1)/2 \) difference equations of the type Eq. (4) and \( M(M+1)/2 \) difference equations of the type Eq. (5).

4. For each of the \( M(M+1)/2 \) difference equations, obtain the least squares (LS) estimates for the model parameters \( a_k \) and \( g_k \) based on the current value of model order, using the \( N \) samples of relevant voltage and current. The LS formulation is as given in Appendix B of [11]. Compute the rms error of the fitted model by using Eq. (7). If \( e_{\text{rms}} > \epsilon \), then increase the model order and re-estimate the parameters. The best minimum-order model is the one identified in the iteration in which \( e_{\text{rms}} \leq \epsilon \) is first satisfied.

5. Estimate the travel time delay parameter \( T_{ij} \) by setting it to zero until the true model order \( p_{ij} \) is identified and the parameters \( a_k^{(i)} \) and \( g_k^{(i)} \) are available from step 4. For \( \ell < T_{ij} \), the estimated values of \( g_k^{(i)} \) are negligibly small compared to those for \( \ell \geq T_{ij} \) and may be dropped from the model without affecting \( e_{\text{rms}} \). This results in an updated model using the estimated \( T_{ij} \).

The insignificant \( g_k^{(i)} \) are identified by inspection; however, if needed, the elimination procedure can be easily automated so that those having extremely small normalized values are rejected.

IV. Model Implementation in EMTP

A. Single-Phase Equivalents

The discrete-time model corresponding to the diagonal entries of \( [Y(m)]_{M \times M} \) for an \( M \)-port network can be rewritten by collecting all the delay terms together and denoting them by \( h \) (history term):

\[
i_m^{(m)}(t) = g_0^{(m)} v_m(t) + \sum_{k=1}^{M} g_k^{(m)} v_m(t-k\Delta t) - a_k^{(m)} i_m(t-k\Delta t)
\]

\[
= g_0^{(m)} v_m(t) + h^{(m)}; \quad m = 1, 2, \ldots, M
\]

(8)

Since the discrete-time model corresponding to the off-diagonal entries of \( [Y(m)]_{M \times M} \) is a summation of delayed terms only, it is essentially a history term:

\[
i_m^{(n)}(t) = \sum_{k=1}^{M} g_k^{(m)} v_n(t-k\Delta t) - a_k^{(m)} i_m(t-k\Delta t); \quad g_0^{(m)} = \delta_1^{(m)}, \ldots, g_{M-1}^{(m)} = 0
\]

\[
= h^{(m)}; \quad m, n = 1, 2, \ldots, M \quad m \neq n
\]

(9)

Observe that Eq. (8) represents a discrete-time Norton equivalent circuit consisting of an ideal current source \( h^{(m)} \) in parallel with a conductance \( g_0^{(m)} \) connected between port \( m \) and ground, while Eq. (9) represents a current injection \( h^{(m)} \) into port \( m \) that is dependent on the voltage at another port \( n \) delayed by \( T_{mn} \). Therefore, the single-phase discrete-time equivalent of a passive external system at any port \( m \) will consist of one Norton equivalent in parallel with \( M - 1 \) ideal current sources, as shown in Fig. 2. Note that the equivalent is decoupled from all the other \( M - 1 \) ports due to the presence of the time delay \( T_{mn} \) in the history currents \( h^{(m)} \). The circuit represents the frequency-dependent s.c. admittances of the M-port external system as seen from port \( m \) (row \( m \) of \( [Y(m)] \)) in the time domain.

The active voltage sources in the external system can be accounted for by connecting an appropriate Thévenin voltage source, or its dual — a Norton current source, in the discrete-time passive system equivalent of Fig. 2. Here, the latter is adopted to achieve compatibility with the existing current sources in the equivalent. The resultant single-phase active system equivalent to be attached to the study subsystem at port \( m \) is, therefore, as shown in Fig. 3. Here, \( i_m(t) \) is the net current injection at port \( m \).

The phasor value of the independent Norton source can be easily evaluated for each port of the system using:

\[
[\hat{I}_n]_{M+1} = [\hat{Y}]_{M+1} [\hat{V}_n]_{M+1}
\]

(10)

where

\[
[\hat{Y}] = \text{vector of Norton current phasors},
\]

\[
[\hat{V}_n] = \text{vector of open-circuit voltage phasors at the M ports},
\]

\([\hat{Y}] = \text{complex s.c. admittance matrix of external system evaluated at power frequency.}\]

The conventional short-circuit analysis programs can readily provide \( [\hat{V}_n] \) and \( [\hat{Y}] \) for a power system.

Consequently, for a 2-port external system, the discrete-time frequency dependent equivalent will be as shown in Fig. 4. Note that until a study subsystem is actually connected between ports 1 and 2, the net current injections \( i_1 \) and \( i_2 \) are zero, and the voltages \( v_1 \) and \( v_2 \) are the time-functions corresponding to the steady-state open-circuit phasors \( \hat{V}_{o1} \) and \( \hat{V}_{o2} \).

B. Three-Phase Equivalents

For a three-phase external system, each of the entries of the s.c. admittance matrix \( [Y] \) will be sub-matrices of dimension 3. Correspondingly, the difference equations representing them in discrete-time are matrix versions of Eq. (8) and (9), as given below:

\[
i_m^{(m)}(t) = G_0^{(m)} v_m(t) + \sum_{k=1}^{M} G_k^{(m)} v_m(t-k\Delta t) - A_k^{(m)} i_m(t-k\Delta t)
\]

\[
= G_0^{(m)} v_m(t) + H^{(m)}; \quad m = 1, 2, \ldots, M
\]

(11)

\[
i_m^{(n)}(t) = \sum_{k=1}^{M} G_k^{(m)} v_n(t-k\Delta t) - A_k^{(m)} i_m(t-k\Delta t); \quad G_0^{(m)} = G_1^{(m)}, \ldots, G_{M-1}^{(m)} = 0
\]

\[
= H^{(m)} = H^{(m)}; \quad m, n = 1, 2, \ldots, M \quad m \neq n
\]

(12)
Here, $I_{\text{m,mode}}^m(t) = [I_{\text{in,mode}}^m(t) I_{\text{in,mode}}^m(t) I_{\text{in,mode}}^m(t)]^T$, injected phase currents vector, $I_{\text{m,mode}}^m(t) = [I_{\text{in,mode}}^m(t) I_{\text{in,mode}}^m(t) I_{\text{in,mode}}^m(t)]^T$, short circuit phase currents vector, $V_{\text{n}}(t) = [V_{\text{n}}(t) V_{\text{n}}(t) V_{\text{n}}(t)]^T$, phase voltages vector, $H_{\text{m,mode}} = [h_{\text{in,mode}}^m h_{\text{in,mode}}^m h_{\text{in,mode}}^m]^T$, phase history terms vector, $G_{\text{m,mode}} = [g_{\text{in,mode}}^m g_{\text{in,mode}}^m g_{\text{in,mode}}^m]^T$, phase coefficients matrix, $A_{\text{m,mode}} = [a_{\text{in,mode}}^m a_{\text{in,mode}}^m a_{\text{in,mode}}^m]^T$, phase coefficients matrix, $A_{\text{m,mode}} = [a_{\text{in,mode}}^m a_{\text{in,mode}}^m a_{\text{in,mode}}^m]^T$, phase coefficients matrix.

where $m, n = 1, 2, \ldots, M$ and $k = 0, 1, \ldots, P_{\text{m}}$.

For a three-phase fully balanced (transposed) system, the coefficient matrices $G_{\text{m,mode}}$ and $A_{\text{m,mode}}$ are in the form:

$$
\begin{bmatrix}
    s & m & m \\
    m & s & m \\
    m & m & s
\end{bmatrix}
$$

which can be diagonalized by using Clarke’s or Karrenbauer’s transformation matrix $[T]$, for which $[T]^{-1} = [T]^T$. Therefore, the vectors $I_n, V_n$ and $H_{\text{m,mode}}$, and the coefficient matrices $G_{\text{m,mode}}$ and $A_{\text{m,mode}}$ can be transformed to modal quantities:

$$
I_{\text{m,mode}}^m = [T]^T I_n \text{ } I_{\text{m,mode}}^m = [T]^T I_n \text{ } V_{\text{m,mode}} = [T]^T V_n \text{ } H_{\text{m,mode}} = [T]^T H_{\text{m,mode}} \text{ } G_{\text{m,mode}} = [T]^T G_{\text{m,mode}} \text{ } A_{\text{m,mode}} = [T]^T A_{\text{m,mode}}
$$

The three decoupled modal equations that result from transforming Eq. (11) are:

$$
i_{\text{m,mode}}^m(t) = \dot{g}_{\text{m,mode}}^m v_{\text{n}}(t) + h_{\text{m,mode}}^m
$$

$$
i_{\text{m,mode}}^m(t) = \dot{g}_{\text{m,mode}}^m v_{\text{n}}(t) + h_{\text{m,mode}}^m
$$

$$
i_{\text{m,mode}}^m(t) = \dot{g}_{\text{m,mode}}^m v_{\text{n}}(t) + h_{\text{m,mode}}^m
$$

where the modal history terms are:

$$
h_{\text{m,mode}}^m = \sum_{k=1}^{P_{\text{m}}} \left[ k_{\text{m,mode}}^m v_{\text{n}}(t - k\Delta t) - a_{\text{m,mode}}^m i_{\text{m,mode}}(t - k\Delta t) \right]
$$

and $s = 0, \alpha, \beta$. Note that each of Eq. (13), (14) and (15) corresponds to Eq. (8) for single-phase case and, therefore, represents a Norton equivalent circuit for a mode at port $m$. As a set, they represent the discrete-time models corresponding to the three entries in each modal $V_{\text{n}}$ (diagonal) sub-matrix, and, $M$ such sub-matrices make the block-diagonal of the modal $[Y]$, for the external system.

The transformation of Eq. (12) results in three decoupled modal current injections $h_{\text{m,mode}}^m, h_{\text{m,mode}}^m, h_{\text{m,mode}}^m$ into port $m$, each of which corresponds to the result obtained in Eq. (9) earlier with modal time delay $\tau_{\text{m,mode}}$.

Therefore, the three-phase discrete-time multi-port equivalent at any port $m$ is obtained in the modal domain, where the mode for each mode is as shown in Fig. 2 and 3. Consequently, for a 2-port external system, the discrete-time equivalent for each mode will be as shown in Fig. 4. Once the models for the modal domain equivalent are available, conductances $g_{\text{m,mode}}^m, h_{\text{m,mode}}^m$, and $h_{\text{m,mode}}^m$ are converted to phase domain to yield the $3 \times 3$ matrix $G_{\text{m,mode}}^m$ of Eq. (11). Also, the modl current injections at port $m$, $[i_{\text{m,mode}}^m i_{\text{m,mode}}^m i_{\text{m,mode}}^m]$, are transformed to the injected phase currents, $[i_{\text{m,mode}}^m i_{\text{m,mode}}^m i_{\text{m,mode}}^m]$. Thus, for a 2-port external system, the model structure of the discrete-time equivalent in phase domain will be as shown in Fig. 5.

The following procedure identifies the modal domain models:

- Run EMTP using the multisine excitation signal applied only to phase 'a' of port $m$ of the external system, while all phases of the remaining $M - 1$ ports are shorted to ground. This will ensure that all modes are excited due to the unbalanced three-phase excitation.

- Obtain $N$ samples of vectors $V_n$ and $I_n^m$ at port $m$, and of vectors $I_n^m$ at $M - m$ other ports. Transform them into modal vectors $V_{\text{m,mode}}, I_{\text{m,mode}}^m$ and $V_{\text{m,mode}}^m$. Repeat these steps for each port $m = 1, 2, \ldots, M$.

- Identify the $M$ models of type Eq. (8) and $M(M-1)$ models of type Eq. (9) independently for each mode, as described in Section III earlier for the single-phase equivalent.
C. Integrating Three-Phase Equivalents in an EMTP

The 3 x 3 matrix, \( G^{(m)} \), is directly added to the 3 x 3 sub-matrix which is the block diagonal element corresponding to port \( m \) in the discrete-time conductance matrix \( [G] \) for the study subsystem. This modification is done only once, for all the \( M \) ports, before the EMTP's time-step integration is started. The only computational burden during the time-step integration due to the external system equivalent is the updating of the history vectors \( H^{(m)} \); \( m, n = 1, 2, \ldots, M \). This is carried out in the modal domain by first updating the elements of vector \( H^{(m)} \) independently, and then converting it to \( H^{(m)} \) of Eq. (11) - (12) using the transformation matrix \([T]\) at each time-step.

V. Validation of Equivalent

The multi-port external system equivalencing technique is validated by simulating two types of switching transients: Line Energization and Short Circuit. For each event, the equivalent is evaluated using two criteria: accuracy and cpu time of simulation. The three-phase test system considered is shown in Fig. 6 where the study subsystem consists of a 200 mile transmission line between buses 1 and 5. The remaining part of the network then constitutes the 2-port external system.

The 2-port modal domain equivalent for the external system was obtained by following the procedure outlined in Section IV-B. The model orders \( p_{11}, p_{12} \) and \( p_{35} \) identified for the \( (\alpha, \beta) \) mode are 150, 160 and 210 respectively, and those for the ground mode are 120, 130 and 150 respectively. Also, the travel time delay between the two ports, \( T_{35} \), is \( 16.2\mu s \) and \( 18.2\mu s \) for the aerial and ground modes respectively, where \( \Delta t = 50\mu s \) as obtained in the Appendix. A Small Transients Program (STP) for time domain simulation of electromagnetic transients was developed, wherein the equivalent is interfaced to the study subsystem at buses 1 and 5, as explained in Section IV-C. The discrete-time conductance matrix \([G]\) for the study subsystem created in STP is modified by adding matrices \( G^{(1)}_{21} \) and \( G^{(5)}_{25} \) to the diagonal blocks corresponding to buses 1 and 5. The test system then effectively reduces to Fig. 7 for solution within the STP.

Simultaneous three-phase switching in the study subsystem, whether for line energization or for fault creation, with only balanced sources in the external system will not excite the ground \( (\alpha, \beta) \) mode. Therefore, in order to validate the equivalent for phenomena which excite all modes of the system, the switching transients are simulated using an unbalanced source in the network. In EMTP simulation, all the lines in the external system are represented by frequency-dependent models and that in the study subsystem by constant-parameter model. Also, the source at bus 4 is rendered unbalanced by forcing the phase 'c' voltage magnitude to zero. Simultaneous closing of the three switches at both bus 1 and bus 5 now amounts to unbalanced energization of the 200 mile line. The same event is simulated in STP and the line energization transients at the two ends of the line are obtained. Comparison of the results obtained from EMT and STP is given in Fig. 8 and Fig. 9, which show the phase 'a' voltage transients at buses 1 and 5 respectively.

Next, an unsymmetrical fault is simulated by simultaneous closing of three switches to ground at the middle of the line (bus 0). The post-fault voltage and current transients that will be seen by any protective relays at the two ends of the line are obtained using both EMT and STP. The results are compared in Fig. 10 and 11 for the phase 'a' voltage waveforms at buses 1 and 5 respectively, and the corresponding current waveforms at each end of the line are shown in Fig. 12 and 13 respectively. It can be easily seen that the signals obtained using the 2-port equivalent in STP match very well with the corresponding ones obtained using the full system representation in EMT. This validates the accuracy of the equivalent.

On a VAX 9000-210V computer, the model preparation for the aerial and ground mode 2-port equivalent took 7.25 and 3.9 minutes of cpu time respectively. The line energization simulation run for 3 cycles (50 ms) using the equivalent in STP takes 2.25 seconds of cpu time in the time-step loop, while the corresponding cpu time using the full system in EMT is 5.1 seconds. Similarly, for 3 cycles of post-fault simulation the cpu times are 2.3 and 5.3 seconds using STP and EMT respectively. The timing figures strongly suggest that use of the equivalent reduces the computational burden. However, the actual saving can be rigorously quantified only if the proposed model is implemented in an EMT, or if the full test system is modelled in STP.

The saving in cpu time will be significantly more for large systems since the EMT (full system) simulation time increases directly with network size, but the equivalent complexity is relatively unchanged. Also, use of the equivalent is expected to save computing time when a large number of switching transients simulations in the study subsystem are carried out. This is achieved without loss of accuracy in the transients produced, whereas those generated by reduced order equivalents may not be as accurate for some switching events [16].
Fig. 8. Line energization voltage transients at bus 1

Fig. 9. Line energization voltage transients at bus 5

Fig. 10. Post-fault voltage transients at bus 1

Fig. 11. Post-fault voltage transients at bus 5

Fig. 12. Post-fault current transients at end 1 of line

Fig. 13. Post-fault current transients at end 5 of line
VI. Conclusions

This paper extends the time domain equivalencing method developed earlier for single-port external system to the multi-port case. One of the contributions of the presented method is the incorporation of travel time delay between ports explicitly into the equivalent model. The method is explained in detail and a 2-port equivalent for an 8-bus three-phase test system is obtained to illustrate the implementation. Examples are given for two types of switching transients in the study subsystem. The simulation results obtained using the equivalenced system in STP agree well with those obtained by full system representation in EMTP. The equivalent can be easily integrated into existing EMTPs since it uses discrete-time models similar to those used in EMTP to represent various system components.

The equivalencing technique is based on linear system theory and, therefore, any non-linear device may not be included in the external system. However, by having a port of the external system at the non-linear element’s terminals, it can be incorporated in the study. All power system equipment represented by linear models in EMTP may be considered in the external system for equivalencing using the proposed method.

Appendix

A flat-spectrum multisine signal of bandwidth 5 kHz is generated by implementing Eq. (19) and (21) of Appendix A in [11]. The EMTP simulation parameters $T_{\text{max}}$, $\Delta t$ and $N$ for exciting the external system by $K-1$ equally spaced frequencies from 0 to $f_{\text{max}} = 5kHz$ in steps of $\Delta f$ can now be fixed. $T_{\text{max}}$ is synonymous with the measurement period $T$, which is also the time-period of the multisine signal. Therefore, using $K = 1024$

$$T_{\text{max}} = \frac{1}{\Delta f} = \frac{K}{f_{\text{max}}} = \frac{1024}{5000} = 204.8 \text{ms}$$

To prevent aliasing effect, $\Delta t$ should be selected to satisfy Nyquist’s criterion. Consequently, the sampling frequency in time domain is selected as four times $f_{\text{max}}$, which gives

$$\Delta t = \frac{1}{4f_{\text{max}}} = \frac{1}{20kHz} = 50 \mu\text{s}$$

Evidently, $N = T_{\text{max}}/\Delta t = 4K = 4096$ samples.

In order to get the true (steady-state) response of the external system to the excitation signal, the system response should be stationary during the measurement time [15]. This is assured in the EMTP simulation by exciting the system over three periods of the multisine signal and selecting the response measurement time as the last period. Therefore, the actual EMTP simulation lasts for $3T_{\text{max}}$ duration.

References


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Discussion

Francisco de León (Instituto Politécnico Nacional, Mexico) and Claudio Fuerte (Instituto Tecnológico de Morelia, Mexico): The authors should be congratulated for their very interesting paper in the computation of equivalent circuits for the simulation of electromagnetic transients. This paper is the multi-port extension of their previous work and the validation test shows impressively accurate results.

In general, an external equivalent circuit is useful only when the cpu time invested in the computation of the equivalent circuit plus the time of the simulations is considerably smaller than the time of performing the same simulations with the complete system (assuming, of course, that the equivalent reasonably represents the behavior of the system from which it was originated). For the line energization example presented in the paper and using the cpu times provided we have:

\[ T_s = T_p + N_s T_s \]

where:

- \( T_p \) = cpu preparation time (669 s for STP; 0 s for EMTP)
- \( T_s \) = cpu simulation time (2.25 s for STP; 5.1 s for EMTP)
- \( T_t \) = cpu total time
- \( N_s \) = number of simulations

Thus, a total of 235 simulations using EMTP for the entire system could be performed to consume the same cpu time as the STP for an equal number of simulations. The method presented in the paper will be useful for this example only if a great number of simulations are required. Typically 100 simulations are performed for statistical switching analyses. Will the authors kindly comment of this?

In our experience [A] a relatively small part of the entire system has an important influence in the transient and, therefore, should be modeled to the detail. Another region has some influence and can be treated with a model of reduced order. There is no need to have detailed modeling (as frequency dependency for transmission lines) for the whole "external system". As the transient surges penetrate into the system, the modeling requirements are less demanding. In fact, there is a practical maximum distance away from the switching node where a simple 60 Hz (multi-port three-phase) Norton equivalent would not lead to large errors. Therefore, much larger cpu savings might not be expected for larger systems. The authors comments would be much appreciated.

For the purpose of obtaining their equivalent will they consider all lines in a large system as frequency dependent? Have the authors looked at the problem of the maximum region that should be represented in full detail? Have they encounter stability problems using their method?

The backbone of the equivalencing technique presented in the paper is the calculation of the inverse Fourier transform. Then, the external system has to be considered as linear. This restriction is not imposed with full system simulations. How will the authors treat nonlinear components in the external system?

Finally, the equivalencing procedures described in the paper should be of great value, specially if a more efficient (non-iterative) methodology for the determination the order of the equivalent is devised.


Manuscript received February 17, 1994.

Adam Semlyen (University of Toronto): I would like to congratulate the authors for their new contribution to the equivalencing of external systems in the computation of electromagnetic transients. The most fundamentally interesting feature of their methodology is the fact that their model, while fully reflecting the frequency dependence of the components involved and of the system, is entirely in the time domain. In principle, the discrete-time modeling adopted is the most natural representation of any linear subsystem for time domain simulation. Indeed, in this approach no frequency domain model has to be built and then adequately approximated for time domain calculations.

While, as emphasized above, the difference equations approach is intrinsically appropriate for subsystem modeling, the particular application of equivalencing an external system that includes transmission lines has peculiarities which, due to the time delays and wave reflections involved, are both useful and very disturbing. The travel time delays permit the decoupling of the computations at the individual ports at any given time, as a true reflection of the actual physical independence of the phenomena, much as during the calculation of transients in the EMTP. The authors have shown full advantage of this possibility in their study, as reflected in Figures 2 to 5. It must be scalar, as opposed to matrix, i.e., multi-port network, equivalents. This equivalencing keeps the equivalents conceptually and computationally simple. The only coupling remains in some of the history terms.

The negative effect of wave reflections from nodes that are not part of the connection ports (e.g., nodes 4, 6, 7, and 8 in Figure 6) is the fact that delayed effects are present in the output even many time steps following the application of an input. This affects the order of the model as seen by the very large number of past terms forming the history, several hundreds in the example of the paper. This is by at least one order of magnitude higher than the order of the rational approximations used in the modeling of transmission lines with frequency dependent parameters where a lower order approximation is possible due to the smoothness of the transfer functions. Consequently, the efficiency of an external system equivalent in the form of difference equations, used for simulation of transients in the EMTP, is fundamentally reduced, compared to other applications. The ballast of the related long convolutions present in the history terms. Larger systems would probably lead to even higher order models and only strong frequency dependence, as in the ground mode, is likely to reduce the requirement for high model order.

The high accuracy of modeling adopted in the example of the paper for the purpose of validation may be responsible for the excessively high model order. In larger systems, retarded effects from remote buses may be insignificant and could produce a spurious combined effect after a longer period of simulation. Therefore, particularly for the external subsystem, the model order could be chosen relatively lower, for higher efficiency and at little sacrifice in accuracy.

The authors' comments will be highly appreciated.

Manuscript received February 22, 1994.

Harinderal Singh and Ali Abu: We appreciate the discussers' interest in our paper and their questions and comments pertaining to it. One of the issues raised is of common concern and we will address it first.

Both Dr. Semlyen and Dr. de León et al seek comments on the influence of the size of external system on the model order of the equivalent, though each of them poses the question in a different context. Dr. Semlyen postulates that the model order required for the equivalent of a large external system may not be much higher due to retarded effects of (reflections from) remote nodes of the system on the system's response at the ports.

On the other hand, Dr. de León reaches the same conclusion by pointing out that even in a full system representation, the detailed frequency dependent (fd) modeling of network components identified as remote is not essential to achieve fairly ac-
curate transients in the study subsystem. In other words, the external system's response at the ports is not compromised by identifying the part of the network which has a significant influence on it and using fd models only in that region. We agree with the discussers' observations in this regard. However, we have the following comments on this topic.

- The problem of identifying the optimum size of the external system adjoining the ports that should be modeled in detail is not trivial. Selection of the region depends on the experience and judgment of the engineer and varies with the topology of the system as well as the type of transients study. In fact, a CIGRE study [C1] has recommended the use of detailed representation of the system up to two buses behind the terminal buses (ports).

- In view of the above, a conservative approach to identify the region of the external system requiring fd modeling is usually advocated. Depending on the network topology, this may entail representing a very large number of fd lines, especially if the ports are crowded buses, that is, there are many lines incident on the boundary node. Use of accurate equivalents in such cases helps save significant cpu time without compromising the results.

- In the proposed time domain equivalencing technique, our experience indicates that the model order doesn’t increase appreciably when most of the lines in a large external system are at least one layer of nodes away from the ports. However, presence of many incident lines at the ports influences the model order more significantly. Nevertheless, even in the latter case, use of equivalents requires much lesser cpu time in simulations compared to full representation.

Dr. Semlyen has suggested that the relatively high model order obtained for the equivalent may be explained by the delayed effects present in the system response due to reflections from nodes adjacent to a port. While we concede this to be a plausible reason, it would’ve been intuitively more appealing to us if the system output (response) was due to a shock input. Instead, our technique involves obtaining the “steady-state” response corresponding to a periodic (multisine) input. Based on our later investigation, we are more inclined to explain the high model order due to an oversized model that results from time domain system identification using “noisy” data. The numerical “noise” introduced in computer simulation causes the curve fitting process (parametric identification) to account for both the physical and noise modes, which translates into introduction of incipient “noise” poles and corresponding zeros in the model [C2, C3]. Identification and removal of these spurious pole/zero pairs can help reduce the model order and contribute to improving the computation efficiency of the equivalent. At present, we have not completed investigating this issue.

Due to the large model preparation time required, Dr. de León et al have discovered the use of this equivalencing technique to be inefficient for statistical switching studies. While this may be perceived as a drawback of the equivalencing technique due to its certain aspects (such as iterative model order selection) which require further refinement, it does not limit the usefulness of the equivalent produced. In studies like statistical switching, where the distribution of overvoltage magnitudes over 100 switchings is of sole interest, the equivalents used need not be the most accurate. In fact, with the steady increase in processing power of computers, the use of equivalents in such conventional studies may not be necessary. On the other hand, in many new applications such as real-time protective relay testing [C4, C5] it is imperative to use accurate equivalents to ensure good reproduction of transients in voltage and current relaying signals in real-time. In such cases, the time spent “off-line” in equivalent preparation is inconsequential. Other applications include use of accurate ac system equivalents at hyde converter terminals for ac/dc system disturbance studies.

Finally, in response to the question by Dr. de León et al about dealing with non-linear components in the external system, we would like to draw their attention to the concluding paragraph of the paper where we have provided the explanation.

References


Manuscript received April 25, 1994.