Measurement and Modelling of the Temporal Dependence in Packet Loss

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Abstract— **Understanding and modelling packet loss in the Internet is especially relevant for the design and analysis of delay-sensitive multimedia applications. In this paper, we present analysis of 128 hours of end-to-end unicast and multicast packet loss measurement. From these we selected 76 hours of stationary traces for further analysis. We consider the dependence as seen in the autocorrelation function of the original loss data as well as the dependence between good run lengths and loss run lengths. The correlation timescale is found to be $1000ms$ or less. We evaluate the accuracy of three models of increasing complexity: the Bernoulli model, the 2-state Markov chain model and the k-th order Markov chain model. Out of the 38 trace segments considered, the Bernoulli model was found to be accurate for 7 segments, the 2-state model was found to be accurate for 10 segments. A Markov chain model of order 2 or greater was found to be necessary to accurately model the rest of the segments. For the case of adaptive applications which track loss, we address two issues of on-line loss estimation: the required memory size and whether to use exponential smoothing or a sliding window average to estimate average loss rate. We find that a large memory size is necessary and that the sliding window average provides a more accurate estimate for the same effective memory size.**

I. INTRODUCTION

Packet loss is a key factor determining the quality seen by delay-sensitive multimedia applications such as audio/video conferencing and Internet telephony. These applications experience degradation in quality with increasing loss and delay in the network. Understanding the loss seen by such applications is important in their design and performance analysis. Adaptive audio/video applications adjust their transmission rate according to the perceived congestion level in the network (see [6], [8], [9]). By this adjustment a suitable loss level can be maintained and bandwidth can be shared fairly between connections. For such adaptive applications it is important to have simple loss models that can be parameterized in an on-line manner.

In this paper, we present careful analysis of end-to-end loss using 128 hours of measurements. The measurements are of loss as seen by packet probes sent at regular intervals (of $20ms$, $40ms$, $80ms$ and $160ms$) sent on both unicast and multicast Internet connections. Since significant non-stationary effects are seen in the data, we divide the traces into two-hour segments and check each segment for stationarity. Gradual decrease and increase in the mean loss rate, abrupt and dramatic increase in loss rate for a few minutes and spikes of high loss rate are observed. We analyze 76 hours of stationary trace segments to determine the extent of temporal dependence in the data. The results show that the correlation timescale, the time over which what happens to one packet is connected to what happens to another packet, is approximately 1 second or less. Beyond this timescale, packet losses are independent of each other.

We also consider loss models of increasing complexity and estimate the level of complexity necessary to accurately model the stationary trace segments. The loss process is modelled as a 2^k-state discrete-time Markov chain model where $k = 0$ corresponds to the Bernoulli model and $k = 1$ corresponds to a 2-state Markov chain model. k is refered to as the order of the Markov chain. For the datasets with sampling intervals of $160ms$, the Bernoulli model was found to be accurate for 7 segments, the 2-state Markov model for 10 segments and Markov chain models of orders 2 to 6 for the remaining 16. The Bernoulli and the 2-state Markov chain models are not accurate for the traces with sampling intervals of $20ms$ and $40ms$ and the estimated order of the Markov chain model ranged from 10 to 42.

Adaptive multimedia applications often track the congestion level in the network in order to adjust the rate of transmission to share the bandwidth fairly and to maintain a low enough loss level ([6], [9]). They can use information about loss in the network over a period of time, instead of the traditional per-packet loss information. For such situations of on-line loss estimation, it is useful to know how much past information is necessary to accurately estimate the parameters of the loss models. The non-stationarity observed in the data shows that the loss rate does vary over time, sometimes dramatically, and thus up-to-date information is important.

Because of the need of adaptive applications for loss rate estimates, we address the question of how much memory is necessary for the on-line parameter estimator to provide an accurate enough estimate. In general, it is found that the variance in the estimator due to limited number of samples is high and thus large numbers of samples are required to get an accurate estimate. We also address the question of whether or not exponential smoothing provides a better estimate of the mean loss rate than the sliding window average. Though computationally quicker and requiring less buffer space, exponential smoothing needs more than twice as many samples to provide an estimate with the same accuracy as the sliding window average.

Earlier works on the measurement of unicast and multicast packet loss in the Internet ([1], [4], [5], [13], [14]) have noted that the number of consecutively lost packets is small. In [14] long outages of several seconds and minutes were also observed on the MBone (multicast backbone network overlaid on the Internet). In [12] measurements of voice traffic are ana-

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TABLE I
TRACE DESCRIPTIONS

	Date	Type	Sampling Interval (τ)	Destination	Time	Duration	Loss Rate	Seg-ments	# of stationary segments
1	14Nov97	unicast	80ms	SICS, Sweden	09:52	8hr	2.7%	4	0
2	14Nov97	multicast	80ms	SICS, Sweden	09:53	8hr	11%	4	0
3	20Nov97	unicast	160ms	SICS, Sweden	16:03	2days	1.9%	24	19
4	12Dec97	multicast	160ms	St. Louis	14:23	2days	5.2%	25	14
5	20Dec97	unicast	20ms	Seattle	13:41	2hr 27min	1.7%	1	1
6	20Dec97	multicast	20ms	Seattle	13:49	2hr 27min	3.8%	1	1
7	21Dec97	unicast	20ms	Los Angeles	13:17	2hr	1.4%	1	1
8	21Dec97	multicast	20ms	Los Angeles	13:26	2hr	3.4%	1	0
9	26Jun98	unicast	40ms	Atlanta	16:56	6hr	2.2%	3	3

lyzed to assess the effects of strategies to compensate for varying delay and loss on the quality of the voice connection. In [1] statistical analysis of temporal correlation in loss data has been described and weak correlation has been noted. The use of discrete-time Markov chain models, particularly the 2-state Markov chain model (sometimes called the Gilbert model) has been proposed in [13], [7], [8], [11]. Discrete-time Markov chain models of increasing levels of complexity, including the 2-state Markov chain model have been described in [13]. This paper takes a more careful look at both temporal dependence and the validity of using various models for packet loss using 76 hours of stationary data.

The remainder of this paper is structured as follows. In Section II we describe how the data was collected. In Section III we first describe two ways of representing the data and then go on to discuss our analysis of stationarity and the temporal dependence in the data. The models and an evaluation of their accuracy is presented in Section IV. In Section V we discuss the memory size required for on-line estimation of model parameters and also the relative suitability of exponential smoothing and sliding window averaging as ways of estimating the average loss rate. Finally, the conclusions are in Section VI.

II. MEASUREMENT

Our study of end-to-end loss in the Internet is based on 128 hours of traces. These traces are gathered by sending out packet probes along unicast and multicast end-to-end connections at periodic intervals (of 20, 40, 80 or 160 ms) and recording the sequence numbers of the probe packets that arrived successfully at the receiver. The packets whose sequence numbers were not recorded were assumed to have been lost. Thus the loss data can be represented as a binary time series $\{x_i\}_{i=1}^n$ where x_i takes the value 0 if the i^{th} probe packet arrived successfully and the value 1 if it was lost. The interval at which the probe packets are sent out into the network is referred to as the sampling interval (τ) for the rest of the paper.

The measurements are summarized in Table I. The source was located at Amherst, Massachusetts for all the traces. Note that one multicast trace (not shown in the table) had an extremely high loss rate of 42% and was therefore disregarded for analysis. For the rest of the paper, We refer to the datasets using the notation “Date.Type” (for example, “20Nov97.uni”).

III. ANALYSIS

We consider two ways of representing the loss data. The first is as a discrete-time binary time series $\{x_i\}_{i=1}^n$ taking on values in the set $\mathcal{X} = \{0, 1\}$. The trace can also be divided into portions of consecutive 0s (called good runs) and portions of consecutive 1s (called loss runs). Thus a second way of representing the data is as the interleaving of the two sequences of observations: $\{g_i\}_{i=1}^e$ and $\{l_i\}_{i=1}^e$ where g_i is the i^{th} good run length (expressed in number of packets) and l_i is the corresponding loss run length (also expressed in number of packets).

The remainder of this section is structured as follows. We discuss our analysis of the traces for stationarity in Section III-A. We find that approximately 76 hours exhibit stationarity. In Section III-B we focus on the temporal dependence in these stationary trace segments as shown by the sample autocorrelation function of the binary time series representation of the data. We discuss our analysis of the temporal dependence using the good run and loss run representation of the data in Section III-C.

A. Stationarity

By looking at the smoothed loss data, obvious non-stationarity is found in most of the traces. Hence, the traces are divided into approximately two-hour segments and each segment is checked for stationarity. For the rest of the paper we refer to the trace segment using the notation “Date.Type-Segment#” (for example, “14Nov97.uni-3”). A time series is said to be stationary in the strict-sense, if the statistical properties remain constant over the entire series. A time series is said to be stationary in the wide-sense (also called weak stationarity) if the mean and covariance function remain constant over time. Since there is no good way to rigorously test for stationarity, we check whether the average loss rate varied significantly in the trace segments. We smooth the trace using a moving average filter (of window size 2000 packets) to judge the extent of variation of the average loss over the length of the trace. Abrupt increases of greater than 0.05 in the average loss rate are taken to be an indication of non-stationarity. To test for gradual increase or decrease in the average loss rate, we fit a straight line to the data (which minimizes the least-square error) as an estimate of the linear trend. A total change in the average loss rate of 0.015 or greater, over a 2 hour trace segment, is considered to be an indication of non-stationarity.

There are different kinds of observed non-stationary effects. For example, the smoothed loss for the third segment of trace “14Nov97.uni” shows a slow, linear decay in loss percentage from 2.5% to 1% over one hour. Such slow decays and increases were noticed in other traces as well. Also, there is an abrupt, dramatic increase from 1% loss to 25% loss lasting for approximately 10 minutes before abruptly dropping to a lower

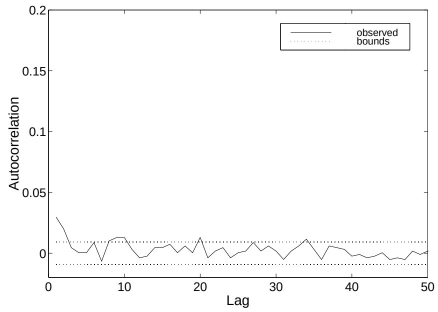

Fig. 1. Sample Autocorrelation Function of Binary Data for 20Nov97.uni-1

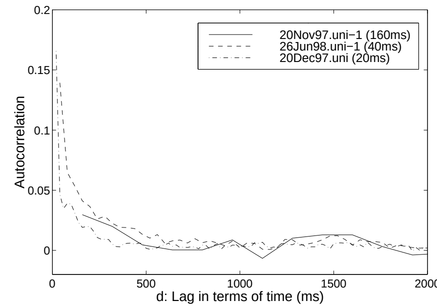

Fig. 2. Sample Autocorrelation Function for trace segments with τ of $160ms$,$40ms$ and $20ms$

loss rate. Such abrupt increases were noticed in other traces although this trace segment shows the most dramatic increases which last the longest time.

Table I summarizes the results of our analysis for stationarity. Traces "20Dec97.multi" and "21Dec97.multi" exhibit periodic behavior (with a period of $500ms$) making them difficult to analyze and model. Thus we identify 38 out of 64 two hour segments from five traces for further analysis in the following sections.

B. Autocorrelation of the Binary Loss Data

The sample autocorrelation function of the loss data for the stationary segments, expressed as the binary time series $\{x_i\}_{i=1}^{n}$, reveals the extent of dependence in the loss experienced by the probe packets over time. In this section, we define the autocorrelation function and the correlation timescale, describe the estimation of correlation timescale from the autocorrelation function and discuss the results for the stationary loss data.

Let $\{Z_i\}_{i=1}^{\infty}$ be a stationary sequence of random variables and let $\{z_i\}_{i=1}^{n}$ be a realization of $\{Z_i\}$. The autocorrelation function for $\{Z_i\}$ is defined as

$$\rho_Z(h) = E[(Z_{i+h} - \mu_Z)(Z_i - \mu_Z)]/E[(Z_i - \mu_Z)^2]$$

where μ_Z is the mean and h is the lag.

The sample autocovariance function, assuming stationarity is

$$\hat{\gamma}(h) = n^{-1} \sum_{i=1}^{n-|h|} (z_{i+|h|} - \bar{z})(z_i - \bar{z}) \qquad -n < h < n$$

where h is the lag and $\bar{z}$ is the sample mean. The sample autocorrelation function is

$$\hat{\rho}(h) = \hat{\gamma}(h)/\hat{\gamma}(0) \qquad -n < h < n \tag{1}$$

The lag can also be expressed in terms of time. That is, let d be the lag in terms of time.

$$d = h\tau \tag{2}$$

where τ is the sampling interval.

Correlation timescale, c is the minimum lag, in terms of time, such that $\{Z_i\}$ is uncorrelated at all lags, d, $d \geq c$.

For an independent stochastic process, the autocorrelation function is zero, and for a stochastic process that is independent at a lag h, the autocorrelation function, $\rho_Z(h)$, is zero at that lag. In the case of an observed sample sequence which is the realization of an independent stochastic process, the sample autocorrelation function could have non-zero, though very small, values. Proposition 1 gives an idea of what constitutes a significant value for the sample autocorrelation function for a given number of samples, n (see [10]).

Proposition 1: For large n, the sample autocorrelations of an IID sequence with finite variance are approximately IID with a normal distribution $N(0, 1/n)$ (mean of 0 and variance of $1/n$)(see [10]). Hence if $\{z_i\}_{i=1}^{n}$ is a realization of such an IID sequence then, approximately 95% of the sample autocorrelations should fall between the confidence bounds $\pm 1.96/\sqrt{n}$.
The correlation timescale is the smallest lag in terms of time, d, at and beyond which the value of the sample autocorrelation function becomes small enough to be considered insignificant.

Using Proposition 1 we can test the following hypothesis:

Hypothesis 1: $\{Z_i\}_{i=1}^{\infty}$ is independent at lag h
The test statistic is calculated from the sample autocorrelation function of the sample sequence, $\{z_i\}_{i=1}^{n}$ at lag h as follows.

$$S = \hat{\rho}(h)/\sqrt{n} \tag{3}$$

If the hypothesis is true, then S has a limiting standard normal distribution. Thus, for example, if $|S| > 1.96$ then the hypothesis is rejected at significance level 0.05.

The results for the loss data show that there is some correlation at small lags, and that the sample autocorrelation function decays rapidly. As an example, consider the first segment of trace "20Nov97.uni", "20Nov97.uni-1" with τ of $160ms$. Fig. 1 shows the autocorrelation function of the binary loss data. A value close to zero of the sample autocorrelation function, for a lag of h, indicates independence between z_i and z_{i+h}. For n samples, the confidence bounds of $\pm 1.96/\sqrt{n}$ (plotted as dotted lines) show the range of values which are close enough to zero to indicate independence. The autocorrelation at lag 1 (that is, the correlation between consecutive packets) is approximately 0.03. It is clear that from a lag of 3 onwards the autocorrelation function becomes insignificantly small except for a few stray points. This means that the data is dependent up to a lag of 2 and that the correlation timescale is $480ms$.

We analyze all of the stationary segments of our traces. The traces, "20Nov97.uni" and "12Dec97.multi" have τ of $160ms$, the traces "20Dec97.uni" and "21Dec97.uni" have τ of $20ms$ and the trace "26Jun98.uni" have a τ of $40ms$. For the trace

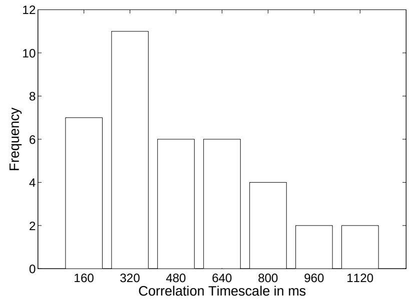

Fig. 3. Frequency Distribution of Correlation Timescale

segments with τ of $160ms$, the sample autocorrelation functions are similar to that described in the example trace segment and the correlation between consecutive packets is found to vary between -0.020 and 0.090 with an average of 0.018.

For the $20ms$ traces, the autocorrelation function at lag 1 is approximately 0.2 which is much higher than that for the $160ms$ trace segments. The $40ms$ trace segments show a similar high autocorrelation of approximately 0.14 at lag of 1. The values of the autocorrelation functions for the $20ms$ traces remain significant for lags of 42 or less, and for the $40ms$ traces, they remain significant for lags of 20 or less. Fig. 2 shows the sample autocorrelation as a function of lag expressed in terms of time, for three trace segments "20Nov97.uni-1", "26Jun98.uni-1" and "20Dec97.uni" with τ of $160ms$, $40ms$ and $20ms$ respectively. The figure shows that the correlation timescale is $1000ms$ or less.

The correlation timescale of a data segment is estimated as the smallest lag (expressed in terms of time) for which the Hypothesis 1 is rejected at significance level of 0.05 (that is, as the smallest lag at which the time series is independent). Note that this method could underestimate the correlation timescale, since the autocorrelation function could show dependence at some higher lag, but we have ignored this effect. The results for all the stationary trace segments are summarized in Fig. 3 as a frequency distribution of the correlation timescale estimated for each data segment. The correlation timescale is less than $1000ms$ except in two cases.

C. Good Run and Loss Run Lengths

The second way of representing the data is as alternating sequences of good run lengths and loss run lengths: that is, $\{g_i\}_{i=1}^{e}$ and $\{l_i\}_{i=1}^{e}$. We check the dependence between the good runs and loss runs by plotting the autocorrelation function of the good run lengths and the loss run lengths and the crosscorrelation function between the good run lengths and the loss run lengths. We use the 95% confidence interval from Proposition 1 to assess whether or not the values of the correlation functions are significant.

The three sample correlation functions indicate independence for all the segments in the trace "20Nov97.uni". Dependence between the good run lengths is seen for trace "12Dec97.multi" in 4 cases (up to lags 4, 5 and 7). The $20ms$ traces ("20Dec97.uni" and "21Dec97.uni") exhibit different results. The autocorrelation function of the loss runs and the crosscorrelation function of the good runs and the loss runs show the independence among loss runs and between good runs and loss runs. However, the good runs lengths do show some dependence (up to lags 25 and 12). The trace "26Jun98.uni" with τ of $40ms$ displays independence for all three correlation functions.

The next step is to look at the distributions of the good run and the loss run lengths. Figures 4 and 5 show the distribution of the good run lengths and the distribution of the loss run lengths for "20Nov97.uni-1". Both figures also show the distributions predicted by the models for comparison with the observed data which will be discussed in Section IV. It is evident that the distribution of the good run lengths decays approximately linearly when a logscale is used for the y-axis, suggesting that good run lengths are geomterically distributed. The loss run lengths are either 1 or 2. The other stationary segments from the $160ms$ traces show similar results. The distribution of the good run lengths for the segments of traces "20Nov97.uni" and "12Dec97.multi", by visual inspection, appear approximately geometric. For some segments there is an excess of short good run lengths (less than 5). The loss run lengths are never greater than 4 for all but 4 segments of "20Nov97.uni" and "12Dec97.multi". This low number of consecutive losses has been noted in earlier work([4], [5]).

The distributions of good run and loss run lengths for the traces "20Dec97.uni", "21Dec97.uni" and "26Jun98.uni" look somewhat different. For example, the distribution of good run lengths in Fig. 6 for the trace "20Dec97.uni" shows a geometric decay except for the high probability of burst lengths of less than 10. The distribution of loss run lengths for this trace shown in Fig. 5, shows approximately geometric decay. Traces "21Dec97.uni" and "26Jun98.uni" show similar results.

Since the good run lengths for "20Nov97.uni" and "12Dec97.multi" appear to be geometrically distributed, we use the Chi-Square goodness-of-fit test to provide more systematic evidence that the distributions are geometric. We compare the empirical distribution of the good run lengths with both an exponential (which is the continuous-valued equivalent of the geometric distribution) and a Gamma distribution. The shape and rate parameters are computed assuming a Gamma distribution for the good run lengths. The shape parameter ($\alpha = (1/\beta)^2$, where β is the coefficient of variation) of the Gamma distribution gives an idea of how close the distribution is to the exponential distribution (value of 1 corresponds to an exponential distribution, which is a special case of the Gamma distribution).The shape parameter for the segments of "20Nov97.uni" varied between 0.407 and 1.620 with an average of 0.806 and that for for the segments of "12Dec97.multi" varied between 0.882 and 1.083 with an average of 0.995.

The Chi-Square goodness-of-fit test is used to check whether to reject the hypothesis that the good run lengths are IID random variables with the given distribution. The following hypothesis is tested for both the Gamma and the Exponential distributions.

Hypothesis 2: The good run lengths are IID random variables with the given distribution function.

For traces "20Nov97.uni" and "12Dec97.multi", the hypothesis is rejected for the exponential distribution for 11 out of 33 segments and for the Gamma distribution it was rejected for 2 out of 33 segments. It is difficult to similarly use the Chi-Square goodness-of-fit test for the loss run lengths because the lengths of the loss run lengths vary within a very small range. The continuous-value distributions were used to make it easier to determine the partitions for the test.

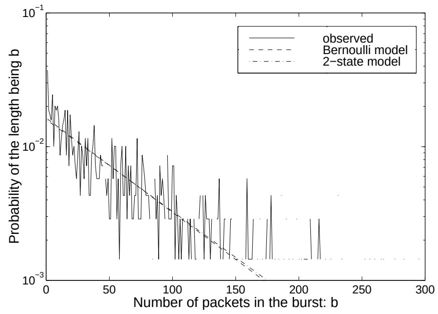

Fig. 4. The Distribution of Good Run Lengths for 20Nov97.uni-1

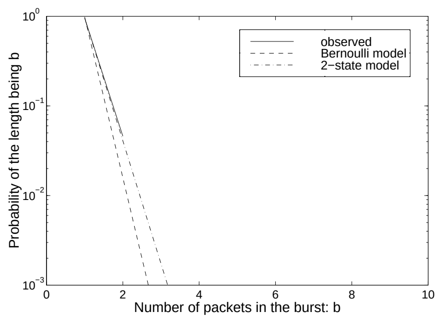

Fig. 5. The Distribution of Loss Run Lengths for 20Nov97.uni-1

IV. Modelling

In this section we discuss the potential of using three models of increasing complexity: the Bernoulli model, the 2-state Markov chain model and the k-th order Markov chain model for modelling the data. The loss process which characterizes the binary time series (as described in Section III) is considered to be $\{X_i\}_{i=1}^{\infty}$ a 2^k-state stationary discrete-time stochastic process. The order of the Markov process, k, is the number of previous values of the process on which the current value depends and is a measure of the complexity of the model. The Bernoulli model is of order 0 and has no states. The 2-state Markov chain model is of order 1. The k-th order Markov chain model is a generalization of these two models and has 2^k states. First, in Section IV-A we describe the models and associated parameter estimators. Then in Section IV-B we assess the validity of the models for the stationary trace segments.

A. Models

A.1 Bernoulli Loss Model

In the Bernoulli loss model, the sequence of random variables, is IID (independent and identically distributed). That is, the probability of X_i being either 0 or 1 is independent of all other values of the time series and the probabilities are the same irrespective of i. This model is characterized by a single parameter, r, the probability of X_i being 1 (corresponds to a lost packet). It is estimated from a sample trace by

$$\hat{r} = n_1/n \tag{4}$$

where n_1 is the number of times the value 1 occurs in the observed time series, $\{x_i\}_{i=1}^{n}$ and n is the number of samples in the time series. Thus $\hat{r}$ is the average loss rate. The good run length distribution for this model is $f(j) = \hat{r}(1-\hat{r})^{j-1}$ for $j = 1, 2, ...\infty$ and the loss run length distribution is $f(j) = (1-\hat{r})\hat{r}^{j-1}$ for $j = 1, 2, ...\infty$.

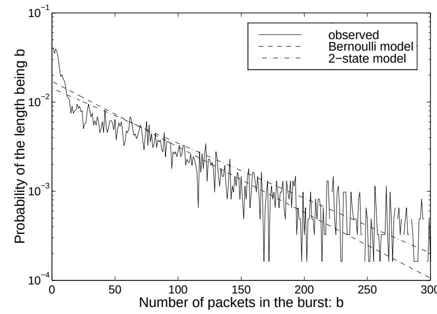

Fig. 6. The Distribution of Good Run Lengths for 20Dec97.uni

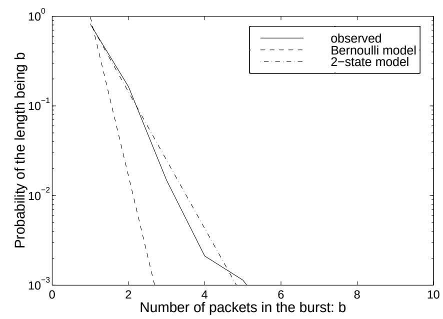

Fig. 7. The Distribution of Loss Run Lengths for 20Dec97.uni

A.2 Two-state Markov Chain Model

In the 2-state Markov chain model the loss process is modelled as a discrete-time Markov chain with two states. The current state, X_i of the stochastic process depends only on the previous value, X_{i-1}. Unlike the Bernoulli model, this model is able to capture the dependence between consecutive losses but has an additional parameter. The two parameters, p and q, are the transition probabilities between the two states.

$$p = P[X_i = 1 | X_{i-1} = 0], \quad q = P[X_i = 0 | X_{i-1} = 1]$$

The maximum likelihood estimators (page 26 of [2]) of p and q for a sample trace are

$$\hat{p} = n_{01}/n_0 \tag{5}$$

$$\hat{q} = n_{10}/n_1 \tag{6}$$

where n_{01} is the number of times in the observed time series that 1 follows 0 and n_{10} is the number of times 0 follows 1. n_0 is the number of 0s and n_1 is the number of 1s in the trace. The good run length distribution for this model is $f(j) = \hat{p}(1-\hat{p})^{j-1}$ for $j = 1, 2, ...\infty$ and the loss run length distribution is $f(j) = \hat{q}(1-\hat{q})^{j-1}$ for $j = 1, 2, ...\infty$.

A.3 Markov Chain Model of k-th order

A class of random processes that is rich enough to capture a large variety of temporal dependencies is the class of finite-state Markov processes. The Bernoulli model and the 2-state Markov chain model are special cases of this class of models. In the Markov chain model, the current state of the process depends on a certain number of previous values of the process which is the order of the process.

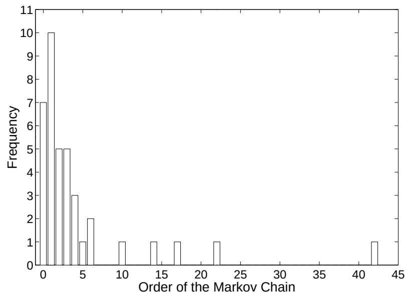

Fig. 8. Frequency Distribution of the Order of The Markov Chain Model

Such a process is characterized by its order k, and by a $k \times 2$ conditional probability matrix P_k whose rows may be interpreted as probability mass functions on $\mathcal{X}$ ($\mathcal{X} = \{0, 1\}$) according to which, the next random variable X_i is generated when the process is in a state $x_{i-k}x_{i-k+1}\ldots x_{i-1}$.

$$P[X_i = x_i | X_{i-1} = x_{i-1}, ..., X_{i-k} = x_{i-k}] \\ = P_k[x_i | x_{i-1}, ..., x_{i-k}]$$

A process $\{X_i\}$ is a Markov chain of order k, if the conditional probability

$$P[X_i = x_i | X_{i-h} = x_{i-h}] \qquad (7)$$

is independent of x_{i-h} for all $h > k$ (see [3]). Note that the Bernoulli and the 2-state Markov chain model are special cases of the Markov chain model corresponding to orders of 0 and 1 respectively.

Let $\{x_i\}_{i=1}^{n}$ be an observed sequence from a Markov source. The k-th order state transition probabilities of the Markov chain can be estimated for all $a \in \mathcal{X}, \underline{b} \in \mathcal{X}^k$ as follows. The number of states is 2^k and the number of conditional probabilities is 2^{k+1}.

Let $\underline{b} = (b_1, ..., b_k)$ be a given state of the chain. Let $n_{\underline{b}a}$ be the number of times state $\underline{b}$ is followed by value a in the sample sequence. Let $n_{\underline{b}}$ be the number of times state $\underline{b}$ is seen. Let $p^k_{\underline{b};a}$ be an estimate of the probability that $x_i = a$, given that $(x_{i-k}, ..., x_{i-1}) = \underline{b}$. Then $p^k_{\underline{b};a}$ estimates the state transition probability from state $\underline{b}$ to state $(b_2, ..., b_{k-1}, a)$. The maximum likelihood estimators of the state transition probabilities of the k-th order Markov chain are

$$p^k_{\underline{b};a} = \begin{cases} \frac{n_{\underline{b}a}}{n_{\underline{b}}} & \text{if } n_{\underline{b}} > 0 \\ 0 & \text{otherwise} \end{cases}$$

B. Results for the Stationary Segments

The autocorrelation function of the binary loss data as discussed in Section III-B shows some correlation for small lags. The Bernoulli model does not capture any of the correlation as seen, for example, in Fig. 1. The 2-state Markov chain model is able to accurately model the correlation only for a lag of 1. The estimated order of the binary loss process indicates the complexity of the Markov chain model necessary to accurately model the data. If the estimated order k is 0, then the Bernoulli loss model is accurate, if it is 1 then the 2-state Markov chain model is accurate, and if it is 2 or greater, then a Markov chain model of order k is required.

TABLE II
THE BERNOULLI AND 2-STATE MARKOV CHAIN MODEL PARAMETERS

Data		$\hat{r}$	$\hat{p}$	$1-\hat{q}$
20Nov97.uni	minimum	0.0005	0.0004	0.0208
	maximum	0.0351	0.0347	0.0909
	average	0.0162	0.0158	0.0471
12Dec97.multi	minimum	0.0376	0.0373	0.0408
	maximum	0.0735	0.0749	0.0636
	average	0.0496	0.0496	0.0513
20Dec97.uni		0.0168	0.0141	0.1732
21Dec97.uni		0.0135	0.0109	0.2085
26Jun98.uni	minimum	0.0204	0.0178	0.1452
	maximum	0.0254	0.0218	0.1608
	average	0.0222	0.0192	0.1546

Testing Hypothesis 1 (that the binary loss data is independent at a given lag h) using the sample autocorrelation function is discussed in Section III-B. An alternative method of testing the same hypothesis is by using the Chi-Square Test for independence. The test statistic for sample sequence, $\{z_i\}_{i=1}^{n}$ and a lag of h is

$$\phi = \sum_{u,v} \frac{(n_{uv} - n_u n_v / n)^2}{n_u n_v / n} \qquad u = 0, 1 \text{ and } v = 0, 1$$

where $n_u, u \in \{0, 1\}$ is the number of times u occurs in the sample sequence and similarly, n_v is the number of times v occurs in the sample sequence. n_{uv} is the number of times $z_i = u$ and $z_{i+h} = v$ (that is, the number of times v follows u after a lag of h). ϕ is distributed as a chi-square distribution with 1 degree of freedom.

The order of the Markov chain process is estimated by estimating the minimum lag beyond which the process is independent. This is similar to the estimation of correlation timescale from the sample autocorrelation function as discussed in Section III-B. Thus we test Hypothesis 1 for lags from 1 to 50. Let l be the first lag at which the hypothesis is not rejected. Then the estimated order is $l - 1$. Fig. 8 shows the frequency distribution of estimated order of the Markov chain for the stationary segments over all the datasets. The two methods of testing the independence hypothesis give the same results. Figure 8 shows that, out of 38 segments, The Bernoulli loss model is accurate for 7 segments, the 2-state Markov chain model is accurate for 10 segments and for the rest of the segments Markov chain models of higher orders are necessary for accurate modelling. The estimated order for datasets "20Nov97.uni" and "12Dec97.multi" ranges from 0 to 6. Out of 33 segments of these traces, the Bernoulli model is accurate for 7, the 2-state Markov model is accurate for 10 and a Markov chain model of order 2 or greater is accurate for the remaining 16. For datasets "20Dec97.uni", "21Dec97.uni" and "26Jun98.uni", the estimated order ranges from 10 to 42 and is much higher than that estimated for the other datasets and hence neither the Bernoulli model, nor the 2-state Markov model are accurate.

Table IV-B summarizes the estimated model parameters (as discussed in Section IV-A). It shows the minimum, maximum and average over all stationary segments for each the dataset. For trace segments where the value of $(1 - \hat{q})$ is higher than the value of $\hat{p}$, the packet loss is burstier than predicted by the Bernoulli model and the 2-state Markov chain model is more accurate.

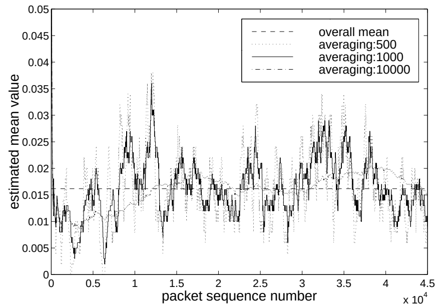

Fig. 9. Mean Loss Rate Estimation for 20Nov97.uni-1 for Memory Size of 500, 1000 and 10000

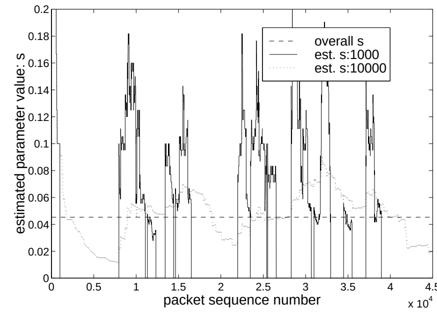

Fig. 10. Estimating the parameter $s = 1 - q$ for 20Nov97.uni-1 for Memory Sizes of 1000 and 10000

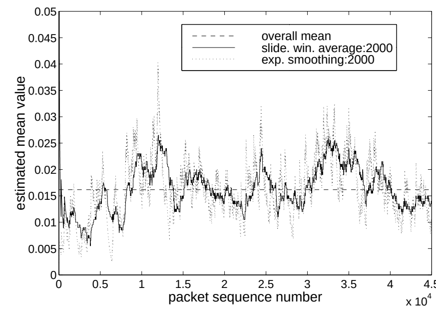

Fig. 11. Comparison of exponential smoothing: $M_e = 2000$ and sliding window average: $M = 2000$ for 20Nov97.uni-1

V. On-Line Parameter Estimation

Adaptive applications monitor the congestion level in the network in order to adjust their transmission rate to keep the loss rate low and to share the available bandwidth fairly with other connections. We address two issues in the on-line estimation of loss for such applications. The first issue is the amount of memory necessary for an accurate estimate of the Bernoulli and the 2-state Markov chain model parameters (discussed in Section V-A). The second issue is whether to use exponential smoothing or a sliding window averaging to get up-to-date and accurate average loss rate estimates (discussed in Section V-B)

A. Memory for the Bernoulli Model and the Two-State Markov Model Parameters

Equation 4 gives an estimator for the Bernoulli loss model parameter, r which is equivalent to the mean loss rate. Assume that M values of the binary loss data, x_i are available for estimation. Henceforth, M will be refered to as the memory size of $\hat{r}$. Fig. 9 shows the estimation of the mean loss rate for the example trace segment "20Nov97.uni-1" for memory sizes of 500, 1000 and 10000 plotted as a function of the packet sequence number. The graphs display the extent of variation in the estimated mean loss rate (it varies between 0.5% and 3.5% for M of 1000). The 95% confidence interval for this estimator is $(\hat{r} - 1.96\sqrt{\hat{r}(1-\hat{r})/M}, \quad \hat{r} + 1.96\sqrt{\hat{r}(1-\hat{r})/M})$ Thus for an accuracy of $\pm b\%$ in the estimator, and a confidence level of 95%, the required $M = (1/\hat{r} - 1)(196/b)^2$. For example, for an accuracy of $\pm 10\%$ in the estimate of $\hat{r}$, the required M is 38032 for a loss rate of 1% and 3457 for a loss rate of 10%.

For the 2-state Markov chain model, the two parameters p and q are estimated as given by (5) and (6). Instead of considering the parameter, q directly we consider the related parameter $s = 1 - q$. For the example trace segment, $\hat{p}$ (not shown here) shows a similar extent of fluctuation as $\hat{r}$ shown in Fig. 9, but $\hat{s}$ fluctuates much more wildly as shown in Fig. 10. The 95% confidence intervals for the estimators for a given M as well as the required M for a given accuracy in the estimators are discussed in [15].

B. Exponential Smoothing versus Sliding Window Averaging

Adaptive applications need to estimate the average loss rate in an on-line manner in order to adjust to the congestion in the network. Exponential smoothing and Sliding Window Averaging are two ways to do so. The sliding window average gives the sample mean over a sliding window, say of size M samples. Thus only the M latest samples contribute to the current estimate ensuring that older information is not used. Restricting the memory size is important because of non-stationarities in the data. Exponential smoothing is a computationally quicker method for estimating the current mean and it requires less buffer space. The question we address is: does exponential smoothing require more memory than sliding window averaging in providing similar quality estimates. As we will see the sliding window average is, indeed, a significantly better estimator. For the same effective memory size, exponential smoothing produces an estimate with more than double the variance of the estimate produced by a sliding window estimate and for the same variance, it requires more than twice as much memory. In our analysis of the variance of the estimate we have assumed, for simplicity, that the loss process is IID. In the presence of correlation, the variance of the sample mean estimator is higher than that calculated assuming independence.

In order to compare the two styles of on-line mean estimation, we first compute the effective memory size in the case of exponential smoothing. For the binary loss data, $\{x_i\}_{i=1}^{n}$, and for a gain of a, the exponential smoothing estimator at time t is

$$\hat{m}_t = ax_t + (1-a)\hat{m}_{t-1} \qquad \hat{m}_1 = x_1$$

Thus all the previous samples are used but the weights given to the old samples decay geometrically with the age of the sample relative to the latest sample.

Let the weight given to an observation, x_i, at estimation time t, where $t \geq i$, be w_i. Let l be a given number of observations immediately older than the observation at time t and let W_l be the sum of the weights given to all observations as old as or older than the observation at time $t - l$.

$$W_l = \sum_{i=1}^{t-l} w_i \approx (1-a)^l \qquad (8)$$

The effective memory, M_e, is defined as the l such that the total weight given to observations as old as or older than $t-l$ is restricted to a small fraction, δ, $0 < \delta < 1$. That is, using (8), M_e is calculated as the l such that $W_l = \delta$.

$$M_e \approx \frac{\ln \delta}{\ln(1-a)} \approx \frac{-\ln \delta}{a} \text{ (for small values of } a) \quad (9)$$

For $\delta = 0.01$, the gain, a for a given M_e, is $a \approx 4.605/M_e$.

To compare the quality of the exponential smoothing estimator to that of the sliding window average we compute its relative efficiency defined as, its variance relative to the sliding window average (which is equivalent to the sample mean estimator). Let the population mean be θ, let the sample mean estimator be $\hat{\theta}$ and let σ^2 be the population variance. Then the variance of the sample mean estimator is $E(\hat{\theta}-\theta)^2 = \sigma^2/M$. The variance of the exponential smoothing estimator $\hat{m_t}$ is

$$E(\hat{m_t}-\theta)^2 = \sigma^2 \frac{a+2(1-a)^{2t-1}}{2-a} \approx \sigma^2 a/2$$

Its relative efficiency, R is the ratio of the variances

$$R = \frac{E(\hat{\theta}-\theta)^2}{E(\hat{m_t}-\theta)^2} \approx \frac{2}{aM}$$

We use (9) to calculate the gain, a ($a \approx 4.605/M_e$) for a given M_e. Thus the relative efficiency, R, of the exponential smoothing estimator, when $M_e = M$ ($\delta = 0.01$) is 43.4%. Hence we see twice as much variance in the exponential smoothing estimator as we do in the sliding window average with the same effective memory. Fig. 11 shows the difference in the variance of the estimators for "20Nov97.uni-1" for a memory size of 2000. The estimator values are plotted against the packet sequence numbers. A similar comparison between the exponential smoothing estimator with effective memory 2000 and the sliding window averaging estimator with memory 1000 shows almost the same variation since the exponential smoothing uses double the memory.

VI. Conclusions and Future Work

We presented measurements of Internet packet loss for unicast and multicast connections. We analyzed the traces for stationarity and identified 76 hours of stationary trace segments for further analysis. The loss data was represented both as a binary time series and as alternating sequences of good runs and loss runs. We checked the data for temporal dependence using both representations.

Our study of the autocorrelation function of the binary time series representation, revealed that the correlation timescale for these traces is $1000ms$ or less. The distributions of the good run lengths and the loss run lengths were found to be approximately geometrically distributed for the $160ms$ traces. For the $20ms$ and $40ms$ traces, the good run lengths were found to be not geometrically distributed, while the loss run lengths were geometrically distributed.

We examined the accuracy of three models: the Bernoulli model, the 2-state Markov chain model and the k-th order Markov chain model, in terms of our analysis of temporal dependence. For the $160ms$ traces, the Bernoulli model was found to be accurate for 7 segments, the 2-state Markov model for 10 segments and Markov chain models of orders 2 to 6 for the remaining 16. The Bernoulli and the 2-state Markov chain models are not accurate for the $20ms$ and $40ms$ traces and the estimated order of the Markov chain model ranged from 10 to 42.

For on-line loss estimation, we computed the memory size for a given accuracy in the parameter estimate for both the average loss rate and the 2-state Markov chain model parameters. Also, we found that using the sliding window average for average loss rate estimation has advantages over exponential smoothing since it varies approximately half as much when the same effective number of past observations are used.

A richer collection of measurements with different sampling intervals and a variety of senders and receivers would allow a better understanding of the temporal dependence in the loss data. And it would be worthwhile to apply the models to protocol design and performance analysis.

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