EL736 Communications Networks II: Design and Algorithms

Class3: Network Design Modelling
Yong Liu
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Polytechnic University

Outline

- Examples
- Basic Problems
- Routing Restriction

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Example: Intra-Domain Traffic Engineering

- IP Routing:
  - Intra-domain: OSPF/IS-IS
  - Inter-domain: BGP
- Intra-domain TE Objective
  - Good end-to-end performance for users
  - Efficient use of the network resources
  - Reliable system even in the presence of failures

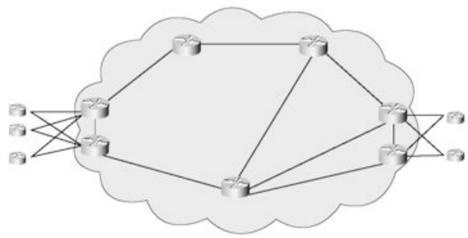

FIGURE 3.1 Intra-Domain IP Network

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TE Optimization: The Problem

- Intradomain traffic engineering
  - Predict influence of weight changes on traffic flow
  - Minimize objective function (say, of link utilization)
- Inputs
  - Networks topology: capacitated, directed graph
  - Routing configuration: routing weight for each link
  - Traffic matrix: offered load each pair of nodes
- Outputs
  - Shortest path(s) for each node pair
  - Volume of traffic on each link in the graph
  - Value of the objective function

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TE Optimization: model

Intra-Domain Traffic Engineering (3.1.1)

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - c_e capacity of link e
  - δ_{edp} = 1, if link e belongs to path p realizing demand d; 0, otherwise
  - h_d volume of demand d
- **variables**
  - w_e metric of link e, $\boldsymbol{w} = (w_1, w_2, ..., w_E)$ (primary)
  - $x_{dp}(\boldsymbol{w})$ flow allocated to path p of demand d determined by the link system w
  - $\underline{y}_e(\boldsymbol{w})$ load of link e determined by the link system w
  - r maximum link utilization variable, r = $\max_{e=1,...,E} \left\{ \underline{y}_e(\boldsymbol{w})/c_e \right\}$
- The problem of minimizing the maximum link utilization can be formulated as

minimize$_{w,r}$ $\quad \boldsymbol{F} = r$

subject to

$$\sum_p x_{dp}(\boldsymbol{w}) = h_d \qquad d = 1, 2, ..., D$$

$$\sum_d \sum_p \delta_{edp} x_{dp}(\boldsymbol{w}) \le c_e r \qquad e = 1, 2, ..., E$$

r continuous

w_e non-negative integers.

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Example: MPLS Networks

- Traditional IP routing
  - packets are forwarded based on their destination IP addresses
  - forwarding on core routers can be bottleneck
- MPLS: multi-protocol label switching
  - recent technique for TE in core IP networks
  - introducing a connection oriented mechanism in the connectionless IP networks
  - packets from a traffic class forwarded along a preset virtual path: Label Switched Path (LSP)

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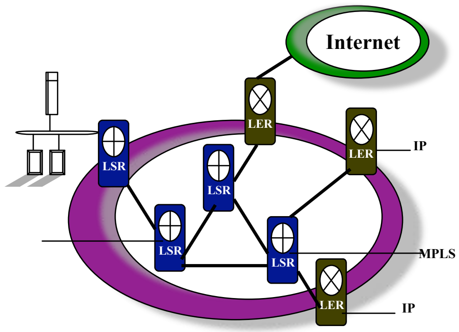
MPLS Basics

- Label Edge Router (LER)
  - analyze the IP header to decide which LSP to use
  - add a corresponding local Label Switched Path Identifier, in the form of a label
  - forward the packet to the next hop
- Label Switched Router (LSR)
  - just forward the packet along the LSP
  - simplify the forwarding function greatly
  - increase performance and scalability dramatically



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MPLS Basics contd..

- New advanced functionality for QoS, differentiated services can be introduced in the edge routers
- Backbone can focus on capacity and performance
- Routing information obtained using a common intra domain routing protocol such as OSPF

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MPLS Design Problem

- *how to carry different traffic classes in an MPLS network through the creation of tunnels in such a way that the number of tunnels on each MPLS router/link is minimized and load balanced?*

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MPLS Design Formulation

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ number of possible tunnels for demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - c_e capacity of link e
  - δ_{edp} = 1, if link e belongs to tunnel p realizing demand d; 0, otherwise
  - h_d volume of demand d
- **variables**
  - x_{dp} fraction of the demand volume d carried over tunnel p
  - ε lower bound on fraction of flow on a tunnel (path)
  - u_{dp} = 1, to denote selection of a tunnel if the lower bound is satisfied; 0, otherwise
  - r maximum number of tunnels over all links.
- The problem of minimizing the number of tunnels on each MPLS router/link and load balancing in an MPLS network can be formulated as

minimize$_{\boldsymbol{x},\boldsymbol{u},r}$ $\boldsymbol{F} = r$

subject to

$\sum_p x_{dp} = 1 \quad d = 1, 2, ..., D$

$\sum_d h_d \sum_p \delta_{edp} x_{dp} \leq c_e \quad e = 1, 2, ..., E$

$\varepsilon u_{dp} \leq h_d x_{dp} \quad d = 1, 2, ..., D \quad p = 1, 2, ..., P_d$

$x_{dp} \leq u_{dp} \quad d = 1, 2, ..., D \quad p = 1, 2, ..., P_d$

$\sum_d \sum_p \delta_{edp} u_{dp} \leq r \quad e = 1, 2, ..., E$

x_{dp} continuous and non-negative

u_{dp} binary, r integer.

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Example: WDM Networks

- Wavelength Division Multiplexing (WDM)
  - 100+ wavelengths (colors) in one optical fiber
  - each wavelength ~ 10Gbps
- Optical Cross-Connects (OXC)
  - switch light from one input fiber to one output fiber
  - with/ w.o. wavelength conversion

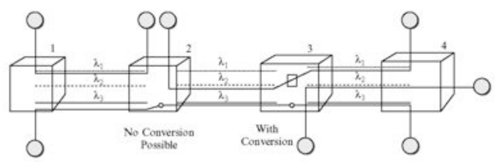

FIGURE 3.9 WDM Network

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WDM Restoration Design Problem

- possible link failure: fiber cuts
- without wavelength conversion: provide enough light paths for all demands under any possible failure scenario
- fiber cost-effective: just enough!

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WDM Restoration Design: formulation

- **indices**
  - $c = 1, 2, ..., C$ colors
  - $e = 1, 2, ..., E$ links
  - $v = 1, 2, ..., V$ nodes
  - $s = 0, 1, ..., S$ failure situations
- **constants**
  - h_{ds} $(d = 1, 2, ..., D)$ volume of demand d to be realized in situation s,
  - ξ_e $(e = 1, 2, ..., E)$ cost of one LCU (i.e., optical fibre) on link e
  - α_{es} = 0 if link e is failed in situation s ; 1, otherwise
  - δ_{edp} = 1 if link e belongs to path p realizing demand d, ; 0, otherwise
  - θ_{dps} =0 if path p of demand d is failed in situation s ; 1, otherwise
- **variables**
  - x_{dpc} flow (number of light-paths) realizing demand d in color c on path p
  - z_{ce} number of times the color c is used on link e
  - y_e capacity of link e expressed in the number of fibers
- The optimization problem for the OXCs without wavelength conversion can be formulated as

minimize$_{x,z,y}$ $\quad F = \sum_e \xi_e y_e$

subject to

$$\sum_p \theta_{dps} \sum_c x_{dpc} \geq h_{ds}, \quad d = 1, 2, ..., D \quad s = 0, 1, 2, ..., S$$

$$\sum_d \sum_p \delta_{edp} x_{dpc} = z_{ce}, \quad c = 1, ..., C, \quad e = 1, 2, ..., E$$

$$y_e \geq z_{ce}, \quad c = 1, ..., C, \quad e = 1, 2, ..., E.$$

x_{dpc}, z_{ce}, y_{es} non-negative integers

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NDP Modeling

- Design for Normal (nominal) operating state
  - average demand volumes, no variation
  - resource fully available, no failure.
- Two time scales
  - uncapacitated design: for a given demand, how much resource needed and how to distribute, medium/long term planning;
  - capacitated design: given demand, resource, how to allocate flows to paths to optimize a network goal, short/medium term design

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Simple Design Problem

Simple Design Problem

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1, if link e belongs to path p realizing demand d; 0, otherwise
  - h_d volume of demand d
  - ξ_e unit (marginal) cost of link e
- **variables**
  - x_{dp} flow allocated to path p of demand d (continuous non-negative)
  - y_e capacity of link e (continuous non-negative)
- **objective**
  - minimize $\boldsymbol{F} = \sum_e \xi_e y_e$ (bandwidth cost)
- **constraints**
  - $\sum_p x_{dp} = h_d, \quad d = 1, 2, ..., D$ (demand constraints)
  - $\sum_d \sum_p \delta_{edp} x_{dp} \leq y_e, \quad e = 1, 2, ..., E$ (capacity constraints).

Shortest path allocation rule: allocate all volume to cheapest path

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Node-Link Formulation I

- **constants**
  - a_{ev} = 1 if link e originates at node v, 0 otherwise
  - b_{ev} = 1 if link e terminates in node v, 0 otherwise
  - s_d source node of demand d
  - t_d sink node of demand d
  - h_d volume of demand d
  - ξ_e unit cost of link e
- **variables**
  - x_{ed} flow realizing demand d allocated to link e (continuous non-negative)
  - y_e capacity of link e (continuous non-negative)
- **objective**
  - minimize $\boldsymbol{F} = \sum_e \xi_e y_e$
- **constraints**

$$\sum_e a_{ev} x_{ed} - \sum_e b_{ev} x_{ed} = \begin{cases} h_d, & \text{if } v = s_d \\ 0, & \text{if } v \neq s_d, t_d, \\ -h_d, & \text{if } v = t_d \end{cases} \quad v = 1, 2, ..., V; d = 1, 2, ..., D$$

$$\sum_d x_{ed} \leq y_e, \; e = 1, 2, ..., E.$$

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Node-Link Formulation II

- **constants**
  - a_{ev} = 1 if link e originates at node v, 0 otherwise
  - b_{ev} = 1 if link e terminates in node v, 0 otherwise
  - $h_{vv'}$ volume of demand d originating at node v and terminating at node v'
  - H_v $= \sum_{v' \neq v} h_{vv'}$ - total demand volume originating in node v
  - ξ_e unit cost of link e
- **variables**
  - x_{ev} flow realizing *all* demands originating at node v on link e
  - y_e capacity of link e
- **objective**
  - minimize $\boldsymbol{F} = \sum_e \xi_e y_e$
- **constraints**

$$\sum_e a_{ev} x_{ev} = H_v, \quad v = 1, 2, ..., V$$

$$\sum_e b_{ev'} x_{ev} - \sum_e a_{ev'} x_{ev} = h_{vv'}, \quad v, v' = 1, 2, ..., V, \ v \neq v'$$

$$\sum_v x_{ev} \leq y_e, \quad e = 1, 2, ..., E.$$

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Model Comparison

- Complexity

TABLE 4.1 Model Comparison

	Number of Variables	Number of Constraints
Link-path formulation	$\bar{P} \times V'(V'-1) + \frac{1}{2}\bar{k} \times V = O(V^2)$	$\bar{P} \times V'(V'-1) + \frac{1}{2}\bar{k} \times V = O(V^2)$
Node-link formulation	$\frac{1}{2}\bar{k} \times V \times V'(V'-1) = O(V^3)$	$V \times V'(V'-1) + \frac{1}{2}\bar{k} \times V = O(V^3)$
Modified node-link formulation	$\frac{1}{2}\bar{k} \times V \times (V'+1) = O(V^2)$	$V'(V+1) + \frac{1}{2}\bar{k} \times V = O(V^2)$

- Flexibility
  - path formulation (PF): pre-compute path,
  - link formulation (LF): implicitly all possible paths
  - path eliminating
    - PF: exclude in path pre-processing, set path flow to zero
    - LF: manipulate link flow to control path flow

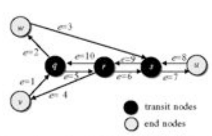

FIGURE 4.1 Network with Separate End and Transit Nodes

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Capacitated Problem

- given link capacities, whether demands are realizable?

Pure Allocation Problem

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1 if link e belongs to path p realizing demand d; = 0 otherwise
  - h_d volume of demand d
  - c_e capacity of link e
- **variables**
  - x_{dp} flow allocated to path p of demand d (continuous non-negative)
- **constraints**
  - $\sum_p x_{dp} = h_d, \quad d = 1, 2, ..., D$
  - $\sum_d \sum_p \delta_{edp} x_{dp} \le c_e, \quad e = 1, 2, ..., E.$

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Modified Link-Path Formulation

- how much additional bandwidth needed on each link to accommodate current demand?

PAP – Modified Link-Path Formulation

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1 if link e belongs to path p realizing demand d; = 0 otherwise
  - h_d volume of demand d
  - c_e capacity of link e
- **variables**
  - x_{dp} flow allocated to path p of demand d
  - z auxiliary continuous variable (of unrestricted sign)
- **objective**
  - minimize z
- **constraints**
  - $\sum_p x_{dp} = h_d, \quad d = 1, 2, ..., D$
  - $\sum_d \sum_p \delta_{edp} x_{dp} \le z + c_e,$

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How many paths needed?

- Proposition: If there is a feasible allocation, then there exists a allocation with at most D+E non-zero flows
  - D flows if all links are unsaturated

- Assign the entire demand volume of each demand to one of its shortest paths, (#hops), if the resulting solution all links are saturated (at least on overloaded), then there is no feasible allocation. (homework).

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Mixed Problem

- with upper bounds on link capacities

Bounded Link Capacities

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1 if link e belongs to path p realizing demand d; 0, otherwise
  - h_d volume of demand d
  - c_e upper bound on the capacity of link
  - ξ_e unit cost of link e
- **variables**
  - x_{dp} flow allocated to path p of demand (continuous non-negative) d
  - y_e capacity of link e (continuous non-negative)
- **objective**
  - minimize $\boldsymbol{F} = \sum_e \xi_e y_e$
- **constraints**
  - $\sum_p x_{dp} = h_d, \quad d = 1, 2, ..., D$
  - $\sum_d \sum_p \delta_{edp} x_{dp} \leq y_e, \quad e = 1, 2, ..., E$
  - $y_e \leq c_e, \quad e = 1, 2, ..., E.$

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Introducing Routing Restriction

- enforce the resulting routes w./w.o. certain properties
  - path diversity v.s. limited split
  - equal splitting v.s. arbitrary splitting
  - modular flows v.s. unmodular flows
- extend the basic formulation by introducing additional routing constraints.

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Path Diversity

- "never put all eggs in one basket"

Generalized Diversity

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1 if link e belongs to path p realizing demand d; 0, otherwise
  - h_d volume of demand d
  - n_d diversity factor for demand d
  - c_e capacity of link e
- **variables**
  - x_{dp} flow allocated to path p of demand d
- **constraints**
  - $\sum_p x_{dp} = h_d, \quad d = 1, 2, ..., D$
  - $\sum_d \sum_p \delta_{edp} x_{dp} \le c_e, \quad e = 1, 2, ..., E$
  - $\sum_p \delta_{edp} x_{dp} \le h_d / n_d, \quad e = 1, 2, ..., E \quad d = 1, 2, ..., D.$

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Lower Bounds on Non-Zero Flows

- the flow volume on a path greater than B if any.
- implicitly limit number of paths

Lower-Bounded Flows

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1 if link e belongs to path p realizing demand d; 0, otherwise
  - h_d volume of demand d
  - b_d lower bound on non-zero flows of demand d
  - c_e capacity of link e
- **variables**
  - x_{dp} continuous flow variable allocated to path p of demand d
  - u_{dp} binary variable corresponding to x_{dp}
- **constraints**

$$\textstyle\sum_p x_{dp} = h_d, \quad d = 1, 2, ..., D$$
$$x_{dp} \le h_d u_{dp}, \quad d = 1, 2, ..., D \quad p = 1, 2, ..., P_d$$
$$b_d u_{dp} \le x_{dp}, \quad d = 1, 2, ..., D \quad p = 1, 2, ..., P_d$$
$$\textstyle\sum_d \sum_p \delta_{edp} x_{dp} \le c_e, \quad e = 1, 2, ..., E.$$

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Limited Demand Split

- only split among k paths

Single-Path Allocation

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1 if link e belongs to path p realizing demand d; 0, otherwise
  - h_d volume of demand d
  - c_e capacity of link e
- **variables**
  - x_{dp} flow allocated to path p of demand d
  - u_{dp} binary variable associated with flow x_{dp}
- **constraints**

$$x_{dp} = h_d u_{dp}, \quad d = 1, 2, ..., D \quad p = 1, ..., P_d$$
$$\textstyle\sum_p u_{dp} = 1, \quad d = 1, 2, ..., D$$
$$\textstyle\sum_d \sum_p \delta_{edp} x_{dp} \le c_e, \quad e = 1, 2, ..., E.$$

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Node-Link Formulation

- Single Path

- **constants**
  - a_{ev} = 1 if node v is the originating node of link e; 0, otherwise
  - b_{ev} = 1 if node v is the terminating node of link e; 0, otherwise
  - s_d source node of demand d
  - t_d sink node of demand d
  - h_d volume of demand d
  - c_e capacity of link e
- **variables**
  - u_{de} binary variable corresponding to flow of demand d allocated to link e
- **constraints**

$$\sum_d h_d u_{de} \le c_e, \quad e = 1, 2, ..., E$$

$$\sum_e a_{ev} u_{de} - \sum_e b_{ev} u_{de} = \begin{cases} 1, & \text{if } v = s_d \\ 0, & \text{if } v \neq s_d, t_d, \\ -1, & \text{if } v = t_d. \end{cases} \quad v = 1, 2, ..., V; d = 1, 2, ..., D$$

Node-Link Formulation

- equally split among k link-disjoint paths

Equal Split Among k Link-Disjoint Paths

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $e = 1, 2, ..., E$ links
  - $v = 1, 2, ..., V$ nodes
- **constants**
  - a_{ev} = 1 if node v is the originating node of link e; 0, otherwise
  - b_{ev} = 1 if node v is the terminating node of link e; 0, otherwise
  - s_d source node of demand d
  - t_d sink node of demand d
  - h_d volume of demand d
  - c_e capacity of link e
  - k_d predetermined number of paths for demand d
- **variables**
  - u_{de} binary variable corresponding to flow of demand d allocated to link e
- **constraints**

$$\sum_d u_{de} h_d / k_d \le c_e, \quad e = 1, 2, ..., E$$

$$\sum_e a_{ev} u_{de} - \sum_e b_{ev} u_{de} = \begin{cases} k_d, & \text{if } v = s_d \\ 0, & \text{if } v \neq s_d, t_d, \\ -k_d, & \text{if } v = t_d. \end{cases} \quad v = 1, 2, ..., V; d = 1, 2, ..., D$$

Integral Flows

- allocate demand volumes in demand modules

Modular Flow Allocation

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d
  - $e = 1, 2, ..., E$ links
- **constants**
  - δ_{edp} = 1 if link e belongs to path p realizing demand d; 0 otherwise
  - L_d demand module for demand d
  - H_d volume of demand d expressed as the number of demand modules
  - h_d demand volume ($h_d = L_d H_d$)
  - c_e capacity of link e
- **variables**
  - u_{dp} non-negative integral variable associated with the flow on path p of demand d
- **constraints**
  - $\sum_p u_{dp} = H_d, \quad d = 1, 2, ..., D$
  - $\sum_d L_d \sum_p \delta_{edp} u_{dp} \le c_e, \quad e = 1, 2, ..., E.$

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Next Lecture

- Non-linear Link Dimensioning, Cost and Delay Functions
- Budget Constraint
- General Optimization Method for NDP

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