EL736 Communications Networks II: Design and Algorithms

Class2: Network Design Problems -- Notation and Illustrations

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Outline

- Link-Path Formulation
- Node-Link Formulation
- Notions and Notations
- Dimensioning Problems
- Shortest-Path Routing
- Fair Networks
- Topological Design
- Restoration Design

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Network Flow Example in Link-Path Formulation

- node: generic name for routing and switching devices
- link: communication channel between nodes, directed/undirected
- path: sequence of links
- demand: source-destination pair
- demand path-flow variables: amount of flow traffic on each path

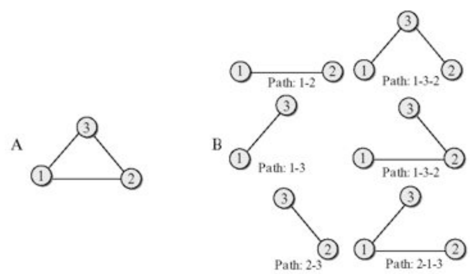

FIGURE 2.1 (A) Three-Node Network Example and (B) All Possible Paths for the Three-Node Example

Constraints on Demand Path-Flow Variables

- Legitimate flow variables

- Demand Constraints (equalities)

- Link Capacity Constraints (inequalities)

- Set of feasible solutions

Objective Function

- Objective function: design goal expressed through a function of design variables
- Routing cost, congestion delay, delay on the most congested link
- unit routing cost of unit flow on each link

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Put it Together

minimize

$$F = \hat{x}_{12} + 2\hat{x}_{132} + \hat{x}_{13} + 2\hat{x}_{123} + \hat{x}_{23} + 2\hat{x}_{213}$$

subject to (constraints)

$$\begin{array}{llllllll} \hat{x}_{12} & +\hat{x}_{132} & & & & & = & 5 \\ & & \hat{x}_{13} & +\hat{x}_{123} & & & = & 7 \\ & & & & \hat{x}_{23} & +\hat{x}_{213} & = & 8 \\ \hat{x}_{12} & & & +\hat{x}_{123} & & +\hat{x}_{213} & \leq & 10 \\ & \hat{x}_{132} & +\hat{x}_{13} & & & +\hat{x}_{213} & \leq & 10 \\ & \hat{x}_{132} & & +\hat{x}_{123} & +\hat{x}_{23} & & \leq & 15 \end{array} \qquad (2.1.2)$$

$$\hat{x}_{12},\ \hat{x}_{132},\ \hat{x}_{13},\ \hat{x}_{123},\ \hat{x}_{23},\ \hat{x}_{213} \geq 0.$$

- Linear programming problem
- Optimal solution/optimal cost, uniqueness?

  $\hat{x}^*_{12} = 5, \hat{x}^*_{13} = 7, \hat{x}^*_{23} = 8, F^* = 20$

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- Multi-commodity network flow problem

Node-Link Formulation

- link flow: traffic of one demand on each link
- flow conservation
  - Source
  - Destination
  - Transit node

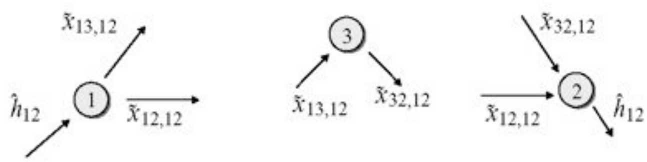

FIGURE 2.2 Flow View for Demand between Nodes 1 and 2

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Optimization in Node-Link Formulation

minimize

$$F = \bar{x}_{12,12} + \bar{x}_{13,12} + \bar{x}_{32,12} + \bar{x}_{12,13} + \bar{x}_{13,13} + \bar{x}_{23,13} + \bar{x}_{21,23} + \bar{x}_{13,23} + \bar{x}_{23,23}$$

subject to

$$\begin{aligned}
\bar{x}_{12,12} + \bar{x}_{13,12} &= \hat{h}_{12} \\
-\bar{x}_{13,12} + \bar{x}_{32,12} &= 0 \\
-\bar{x}_{12,12} - \bar{x}_{32,12} &= -\hat{h}_{12} \\
\bar{x}_{12,13} + \bar{x}_{13,13} &= \hat{h}_{13} \\
-\bar{x}_{12,13} + \bar{x}_{23,13} &= 0 \\
-\bar{x}_{13,13} - \bar{x}_{23,13} &= -\hat{h}_{13} \\
\bar{x}_{21,23} + \bar{x}_{23,23} &= \hat{h}_{23} \\
-\bar{x}_{21,23} + \bar{x}_{13,23} &= 0 \\
-\bar{x}_{13,23} - \bar{x}_{23,23} &= -\hat{h}_{23} \\
\bar{x}_{12,12} + \bar{x}_{12,13} &\le \hat{c}_{12} \\
\bar{x}_{21,23} &\le \hat{c}_{21} \\
\bar{x}_{13,12} + \bar{x}_{13,13} + \bar{x}_{13,23} &\le \hat{c}_{13} \\
\bar{x}_{23,13} + \bar{x}_{23,23} &\le \hat{c}_{23} \\
\bar{x}_{32,12} &\le \hat{c}_{32}
\end{aligned}$$

all $\bar{x}$ non-negative.

(2.2.4)

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Notions and Notations

- demand:
  - source, destination
  - pair label
- link:
  - head, tail
  - link label
- path:
  - node-identifier-based notation
  - link-demand-path-identifier-based notation:
    - labeled paths for each demand

node-identifier-based path identifier	path	link-demand-path-identifier-based path identifier	path
132	1-3-2	12	$\{2,3\}$
213	2-1-3	32	$\{1,2\}$
23	2-3	31	$\{3\}$

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New Formulation

minimize

$$F = x_{11} + 2x_{12} + x_{21} + 2x_{22} + x_{31} + 2x_{32}$$

subject to

$$\begin{array}{llllllll}
x_{11} & + x_{12} & & & & & = & h_1 \\
& & x_{21} & + x_{22} & & & = & h_2 \\
& & & & x_{31} & + x_{32} & = & h_3 \\
x_{11} & & & + x_{22} & & + x_{32} & \leq & c_1 \\
& x_{12} & + x_{21} & & & + x_{32} & \leq & c_2 \\
& x_{12} & & + x_{22} & + x_{31} & & \leq & c_3
\end{array} \quad (2.3.1)$$

$$x_{11},\ x_{12},\ x_{21},\ x_{22},\ x_{31},\ x_{32} \geq 0.$$

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Dimension Problems (DP)

- DP: minimizing the cost of network links with given demand volume between node pairs which can be routed over different paths.
- Illustrative Example

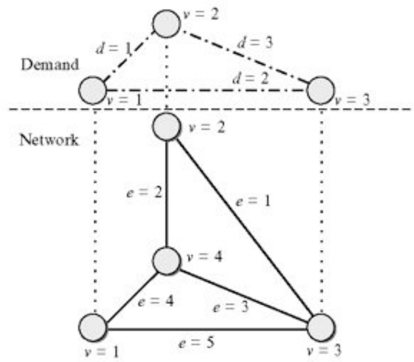

FIGURE 2.3 Four-Node Network Example

DP (cont.d)

- link cost
- list of candidate paths for each demand

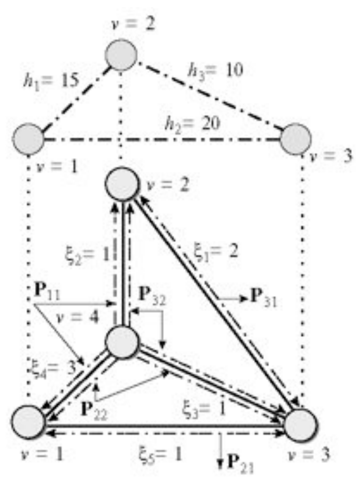

FIGURE 2.4 Four-Node Network Example: Demand Volume and Link Cost

Link-Path Incidence Relation

$\delta_{edp} = 1$ if link e is on path p of demand d, 0 otherwise

TABLE 2.1 **Link-Path Incidence Relation δ_{edp}**

$e \setminus \mathcal{P}_{dp}$	$\mathcal{P}_{11} = \{2,4\}$	$\mathcal{P}_{21} = \{5\}$	$\mathcal{P}_{22} = \{3,4\}$	$\mathcal{P}_{31} = \{1\}$	$\mathcal{P}_{32} = \{2,3\}$
1	0	0	0	1	0
2	1	0	0	0	1
3	0	0	1	0	1
4	1	0	1	0	0
5	0	1	0	0	0

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DP Formulation

minimize

$$F = 2y_1 + y_2 + y_3 + 3y_4 + y_5$$

subject to

$$\begin{array}{llllll} x_{11} & & & & & = 15 \\ & x_{21} & + x_{22} & & & = 20 \\ & & & x_{31} & + x_{32} & = 10 \\ & & & x_{31} & & \le y_1 \\ x_{11} & & & & + x_{32} & \le y_2 \\ & & x_{22} & & + x_{32} & \le y_3 \\ x_{11} & & + x_{22} & & & \le y_4 \\ & x_{21} & & & & \le y_5 \end{array} \qquad (2.4.10)$$

$$x_{11},\ x_{21},\ x_{22},\ x_{31},\ x_{32} \ge 0, \qquad y_1,\ y_2,\ y_3,\ y_4,\ y_5 \ge 0.$$

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General DP Formulation

minimize

objective/cost function (2.4.9): $F = \sum_e \xi_e y_e$

subject to

demand constraints (2.4.3):	$\sum_p x_{dp} = h_d,$	$d = 1, 2, ..., D$
capacity constraints (2.4.7):	$\sum_d \sum_p \delta_{edp} x_{dp} \le y_e,$	$e = 1, 2, ..., E$
constraints on variables:	$\boldsymbol{x} \ge \boldsymbol{0}, \boldsymbol{y} \ge \boldsymbol{0}.$	

(2.4.11)

- Shortest-Path Allocation Rule for DP
  - For each demand, allocate its entire demand volume to its shortest path, with respect to links unit costs and candidate path. If there is more than one shortest path for a demand then the demand volume can be split among the shortest paths in an arbitrary way.

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Variations of DP

- non-bifurcated (unsplittable) flows: each demand only takes single path
  - Pro.s?
  - Con.s?
- Modular link capacity: link capacity only takes discrete modular values
  - combined with single-path requirement
  - complexity?
- Uncapacitated vs. Capacitated

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Shortest-Path Routing

- Link State/Distance Vector Algorithms
  - given a set of link weights, find the shortest path from on node to another
  - how to set up link weights
- Single Shortest-path allocation problem
  - For given link capacities and demand volumes, find a link weight setting such that the resulting shortest paths are unique and the resulting flow allocation is feasible
  - Very complex problem!

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Shortest-Path Routing: complexity

- non-bifurcated flow may not feasible
- difficult to identify single path solution
- difficult to find weight setting to induce the single path solution

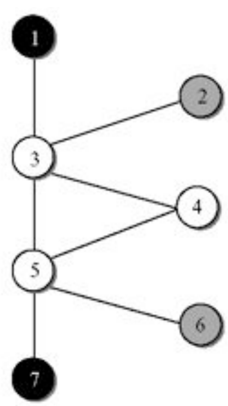

FIGURE 2.6 Infeasible Unique Shortest-Path Case

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Shortest-path Routing with Equal Splitting

- ECMP used in OSPF:
  - For a fixed destination, equally split outgoing traffic from a node among all its outgoing links that belong to the shortest paths to that destination

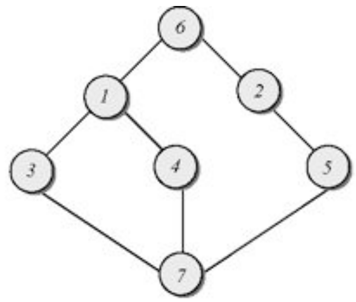

FIGURE 2.7 Equal-Split Rule

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Fair Networks

- Fairness: how to allocate available b.w. among network users.

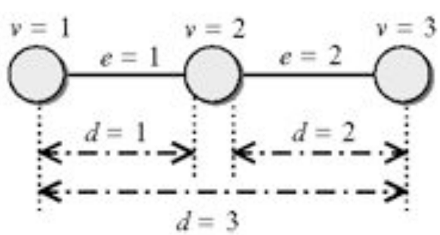

FIGURE 2.8 Two-Link Network

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Max-Min Fairness

- Definition1: A feasible rate vector R^* is max-min fair if no rate R_i can be increased without decreasing some R_k s.t. $R_k < R_i$

- Definition2: A feasible rate vector R^* is an optimal solution to the MaxMin problem iff for every feasible rate vector $\hat{R}$ with $\hat{R}_i > R_i^*$, for some user i, then there exists a user k such that $\hat{R}_k < R_k^*$ and $\hat{R}_k < \hat{R}_i$

- Fairness vs. Efficiency

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Other Fairness Measures

Proportional fairness [Kelly, Maulloo & Tan, '98]

- A feasible rate vector x is proportionally fair if for every other feasible rate vector y

$$\sum w_i \frac{(y_i - x_i)}{x_i} \le 0$$

- Proposed decentralized algorithm, proved properties

Generalized notions of fairness [Mo & Walrand, 2000]

- (α, p)-proportional fairness: A feasible rate vector x is fair if for any other feasible rate vector y

$$\sum w_i \frac{(y_i - x_i)}{x_i^\alpha} \le 0$$

- Special cases: $\alpha = 1$: proportional fairness
  $\alpha \to \infty$: max-min fairness

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Topological Design

- link cost:
  capacity-dependent cost + installation cost
- network cost function: $F = \sum_e \xi_e y_e + \sum_e \kappa_e u_e$
- additional constraint: $y_e \leq \Delta u_e$
- mixed-integer programming

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Restoration Design

- design for the ability to recover from link/node failures

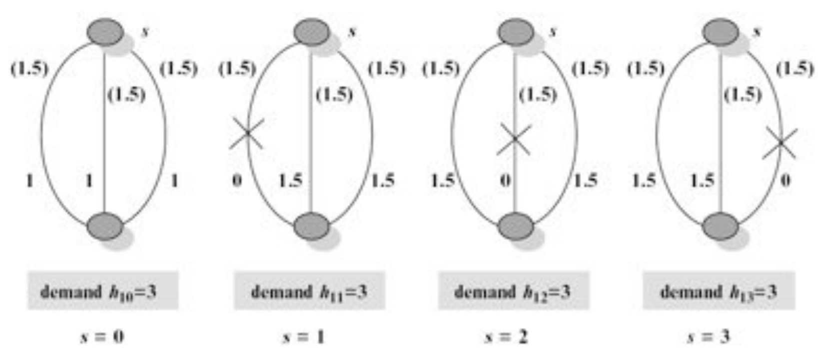

FIGURE 2.10 A Bifurcated Solution

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Restoration Design

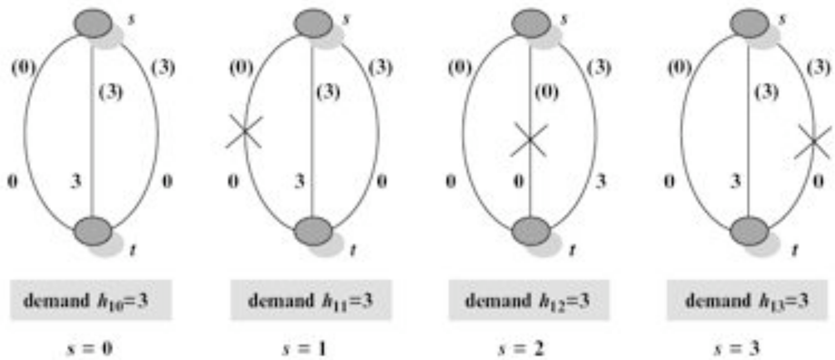

FIGURE 2.11 A Non-Bifurcated Solution

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