EL736 Communications Networks II: Design and Algorithms

Class11: Multi-Hour and Multi-Layer Network Design
12/05/2007
Outline

- Multi-Hour Network Modeling & Design
  - uncapacitated
  - capacitated
  - robust routing

- Multi-Layer Networks
  - modeling
  - dimensioning
  - restoration
Time-of-Day Effect

- Traffic demand varies during hours of a day
- Variations not synchronized

**Figure 11.2** Traffic Variation for a Selected Set of City Pairs During the Day for a 10-Node Network Spanning Continental U.S. (Time is on Eastern Time Zone in U.S.)
Multi-Hour Dimensioning

- how much capacities needed to handle demands at all times?
- rearrange routing when demand changes
- modular link dimensioning

Modular Links, Multi-Hour, Rearrangeable

- **indices**
  
  \[ d = 1, 2, \ldots, D \quad \text{demands} \]
  
  \[ t = 1, 2, \ldots, T \quad \text{traffic busy hours} \]
  
  \[ p = 1, 2, \ldots, P_d \quad \text{allowable paths for flow realizing demand } d \]
  
  \[ e = 1, 2, \ldots, E \quad \text{links} \]

- **constants**
  
  \[ \delta_{edp} \quad = 1, \text{if link } e \text{ belongs to path } p \text{ realizing demand } d, 0 \text{ otherwise} \]
  
  \[ h_{dt} \quad \text{volume of demand } d \text{ at time } t \]
  
  \[ \xi_e \quad \text{cost of one capacity module on link } e \]
  
  \[ M \quad \text{size of the link capacity module} \]

- **variables**
  
  \[ x_{dpt} \quad \text{(non-negative) flow allocated to path } p \text{ of demand } d \]
  
  \[ \text{at time } t \text{ (continuous non-negative)} \]
  
  \[ y_e \quad \text{capacity of link } e \text{ expressed in number of modules (non-negative integer)} \]

- **objective**
  
  minimize \[ F = \sum_e \xi_e y_e \]

- **constraints**
  
  \[ \sum_p x_{dpt} = h_{dt}, \quad d = 1, 2, \ldots, D \quad t = 1, 2, \ldots, T \]
  
  \[ \sum_d \sum_p \delta_{edp} x_{dpt} \leq M y_e, \quad e = 1, 2, \ldots, E \quad t = 1, 2, \ldots, T. \]
Multi-Hour Dimensioning

- unsplittable flows: one path each demand
- non-rearrangeable routing: don’t change routes over time

Modular Links, Multi-Hour, Non-Rearrangeable, Unsplittable

- indices
  - \( d = 1, 2, ..., D \) demands
  - \( t = 1, 2, ..., T \) traffic busy hours
  - \( p = 1, 2, ..., P_d \) allowable paths for flow realizing demand \( d \)
  - \( e = 1, 2, ..., E \) links

- constants
  - \( \delta_{edp} = 1 \), if link \( e \) belongs to path \( p \) realizing demand \( d \), 0 otherwise
  - \( h_{dt} \) volume of demand \( d \) at time \( t \)
  - \( \xi_e \) cost of one capacity module on link \( e \)
  - \( M \) size of the link capacity module

- variables
  - \( u_{dp} \) binary variable corresponding to the flow allocated to path \( p \) of demand \( d \)
  - \( y_e \) capacity of link \( e \) expressed in the number of modules (non-negative integer)

- objective
  - minimize \( F = \sum_e \xi_e y_e \)

- constraints
  - \( \sum_p u_{dp} = 1, \quad d = 1, 2, ..., D \)
  - \( \sum_d h_{dt} \sum_p \delta_{edp} u_{dp} \leq M y_e, \quad e = 1, 2, ..., E \quad t = 1, 2, ..., T. \)
Multi-Hour Routing

- link capacity fixed
- recalculate routing for each time t
- problem separable, optimal routing at each t

Multi-Hour, Rearrangeable, Capacitated

- indices
  - $d = 1, 2, ..., D$ demands
  - $t = 1, 2, ..., T$ time of the day index
  - $p = 1, 2, ..., P_d$ allowable paths for flow realizing demand $d$
  - $e = 1, 2, ..., E$ links

- constants
  - $\delta_{edp}$ = 1, if link $e$ belongs to path $p$ realizing demand $d$, 0 otherwise
  - $h_{dt}$ volume of demand $d$ at time $t$
  - $\zeta_{dpt}$ unit routing cost on path $p$ for demand $d$ in time window $t$
  - $c_e$ capacity of link $e$

- variables
  - $x_{dpt}$ flow allocated to path $p$ of demand $d$ at time $t$ (continuous non-negative)

- objective
  - minimize $F = \sum_d \sum_p \zeta_{dpt} x_{dpt}$

- constraints
  - $\sum_p x_{dpt} = h_{dt}, \quad d = 1, 2, ..., D \quad t = 1, 2, ..., T$
  - $\sum_d \sum_p \delta_{edp} x_{dpt} \leq c_e, \quad e = 1, 2, ..., E \quad t = 1, 2, ..., T$. 
Extension: robust routing under with multiple Traffic Matrices (TM)

- **multiple Traffic Matrices**
  - *dynamic traffic*: demands between routing update period
  - *estimation error*: possible traffic demands

- **Robust routing**: single set of routes achieving good performance under all possible TMs
  - routing reconfiguration too expensive
  - routing: link-path, node-link, destination based, link weight based
  - performance measure
    - good average performance
    - bounded worst-case performance
    - trade-off between two

- **References**
  - “On Optimal Routing with Multiple Traffic Matrices”,
  - “Optimal Routing with Multiple Traffic Matrices: Tradeoff between Average Case and Worst Case Performance”,
    ftp://gaia.cs.umass.edu/pub/Zhang05_tradeofftr.pdf
Multi-Layer Networks

- Traffic vs. Transport Networks
- Technology Example

Cost Component
- cross-layer connection
- physical connection
Dimensioning at two Resource Layers

- demand layer
  - demand between pairs of users
  - to be carried by traffic network

- traffic network layer
  - set of logical links
  - realize each demand through flow allocation
  - capacity of each link realized by transport layer

- transport network layer
  - set of physical links
  - realize each logical link capacity through flow allocation

- dimensioning: how much capacity needed on each logical/physical link?

![Diagram showing two resource layers with demand, traffic network, and transport network layers, along with equations for flow allocation and link capacity constraints.](image-url)
Two-Layer Dimensioning (continuous case)

- **indices**
  
  \[
  d = 1, 2, \ldots, D \quad \text{demands}
  \]
  
  \[
  p = 1, 2, \ldots, P_d \quad \text{candidate paths in upper layer for flows realizing demand } d
  \]
  
  \[
  e = 1, 2, \ldots, E \quad \text{links of upper layer}
  \]
  
  \[
  q = 1, 2, \ldots, Q_e \quad \text{candidate paths in lower layer for flows realizing link } e
  \]
  
  \[
  g = 1, 2, \ldots, G \quad \text{links of lower layer}
  \]

- **constants**
  
  \[
  h_d \quad \text{volume of demand } d
  \]
  
  \[
  \delta_{edp} = 1 \text{ if link } e \text{ of upper layer belongs to path } p \text{ realizing demand } d; \quad 0, \text{ otherwise}
  \]
  
  \[
  \xi_e \quad \text{unit cost of link } e \text{ of upper layer}
  \]
  
  \[
  \gamma_{geq} = 1 \text{ if link } g \text{ of lower layer belongs to path } q \text{ realizing link } e \text{ of upper layer}; \quad 0, \text{ otherwise}
  \]
  
  \[
  \kappa_g \quad \text{unit cost of link } g \text{ of lower layer}
  \]

- **variables**
  
  \[
  x_{dp} \quad \text{(non-negative continuous) flow allocated to path } p \text{ realizing volume of demand } d
  \]
  
  \[
  y_e \quad \text{(non-negative continuous) capacity of upper layer link } e
  \]
  
  \[
  z_{eq} \quad \text{(non-negative continuous) flow allocated to path } q \text{ realizing capacity of link } e
  \]
  
  \[
  u_g \quad \text{(non-negative continuous) capacity of lower layer link } g
  \]

- **objective**
  
  minimize \( F = \sum_e \xi_e y_e + \sum_g \kappa_g u_g \)

- **constraints**
  
  \[
  \sum_p x_{dp} = h_d, \quad d = 1, 2, \ldots, D
  \]
  
  \[
  \sum_d \sum_p \delta_{edp} x_{dp} \leq y_e, \quad e = 1, 2, \ldots, E
  \]
  
  \[
  \sum_q z_{eq} = y_e, \quad e = 1, 2, \ldots, E
  \]
  
  \[
  \sum_e \sum_q \gamma_{geq} z_{eq} \leq u_g, \quad g = 1, 2, \ldots, G.
  \]
Two-Layer Dimensioning (continuous/Integral)

- **indices**
  - \( d = 1, 2, \ldots, D \) demands
  - \( p = 1, 2, \ldots, P_d \) candidate paths in upper layer for flows realizing demand \( d \)
  - \( e = 1, 2, \ldots, E \) links of upper layer
  - \( q = 1, 2, \ldots, Q_e \) candidate paths in lower layer for flows realizing link \( e \)
  - \( g = 1, 2, \ldots, G \) links of lower layer

- **constants**
  - \( h_d \) volume of demand \( d \)
  - \( \delta_{cdp} \) = 1 if link \( e \) of upper layer belongs to path \( p \) realizing demand \( d \); 0, otherwise
  - \( M \) size of the link capacity module in upper layer
  - \( \xi_e \) cost of one \((M\text{-module})\) capacity unit of link \( e \) of upper layer
  - \( \gamma_{geq} \) = 1 if link \( g \) of lower layer belongs to path \( q \) realizing link \( e \) of upper layer; 0, otherwise
  - \( N \) size of link capacity module in lower layer
  - \( \kappa_g \) cost of one \((N\text{-module})\) capacity unit of link \( g \) of lower layer

- **variables**
  - \( x_{dp} \) (non-negative continuous) flow allocated to path \( p \) realizing volume of demand \( d \)
  - \( y_e \) (non-negative integral) \(M\)-module capacity of upper layer link \( e \)
  - \( z_{eq} \) (non-negative integral) flow allocated to path \( q \) realizing capacity of link \( e \)
  - \( u_g \) (non-negative integral) capacity of lower layer link \( g \)

- **objective**
  - minimize \( F = \sum_e \xi_e y_e + \sum_g \kappa_g u_g \)

- **constraints**
  - \( \sum_p x_{dp} = h_d, \quad d = 1, 2, \ldots, D \)
  - \( \sum_d \sum_p \delta_{cdp} x_{dp} \leq M y_e, \quad e = 1, 2, \ldots, E \)
  - \( \sum_q z_{eq} = y_e, \quad e = 1, 2, \ldots, E \)
  - \( \sum_e M \sum_q \gamma_{geq} z_{eq} \leq N u_g, \quad g = 1, 2, \ldots, G \).
Allocation with Two Layers of Resources

- lower layer capacities fixed
- upper layer capacities variable

*LP: A2I/CF/BR/CC*

Two-Layer Allocation Problem

**indices**
- \( d = 1, 2, \ldots, D \) demands
- \( p = 1, 2, \ldots, P_d \) candidate paths in upper layer for demand \( d \)
- \( e = 1, 2, \ldots, E \) links of upper layer
- \( q = 1, 2, \ldots, Q_e \) candidate paths in lower layer for flows realizing link \( e \)
- \( g = 1, 2, \ldots, G \) links of lower layer

**constants**
- \( h_d \) volume of demand \( d \)
- \( \delta_{epd} = 1 \) if link \( e \) of upper layer belongs to path \( p \) realizing demand \( d \); 0, otherwise
- \( \gamma_{eq} = 1 \) if link \( g \) of lower layer belongs to path \( q \) realizing link \( e \) of upper layer; 0, otherwise
- \( c_g \) capacity of lower layer link \( g \)

**variables** (all non-negative continuous)
- \( x_{dp} \) flow allocated to path \( p \) realizing volume of demand \( d \)
- \( y_e \) capacity of upper layer link \( e \)
- \( z_{eq} \) flow allocated to path \( q \) realizing capacity of link \( e \)

**constraints**

\[
\sum_p x_{dp} = h_d, \quad d = 1, 2, \ldots, D \quad (12.1.7a)
\]

\[
\sum_d \sum_p \delta_{epd} x_{dp} \leq y_e, \quad e = 1, 2, \ldots, E \quad (12.1.7b)
\]

\[
\sum_q z_{eq} = y_e, \quad e = 1, 2, \ldots, E \quad (12.1.7c)
\]

\[
\sum_e \sum_q \gamma_{eq} z_{eq} \leq c_g, \quad g = 1, 2, \ldots, G. \quad (12.1.7d)
\]
Two-Layer Mixed Dimensioning Allocation Problem

- lower layer capacities fixed
- upper layer link cost, lower layer routing cost

**Constants**
- $h_d$: volume of demand $d$
- $\delta_{epd}$: $1$ if link $e$ of upper layer belongs to path $p$ realizing demand $d$; $0$, otherwise
- $\gamma_{geq}$: $1$ if link $g$ of lower layer belongs to path $q$ realizing link $e$ of upper layer; $0$, otherwise
- $c_g$: capacity of lower layer link $g$
- $M$: size of the link capacity module in upper layer
- $\xi_e$: cost of one ($M$-module) capacity unit of link $e$ of upper layer
- $\zeta_{eq}$: unit routing cost in the lower layer

**Variables**
- $x_{dp}$: (non-negative continuous) flow allocated to path $p$ realizing volume of demand $d$
- $y_e$: (non-negative integral) capacity of upper layer link $e$
- $z_{eq}$: (non-negative integral) flow allocated to path $q$ realizing capacity of link $e$

**Objective**
- minimize $\sum_e \xi_e y_e + \sum_q \zeta_{eq} z_{eq}$

**Constraints**
- $\sum_p x_{dp} = h_d$, $d = 1, 2, \ldots, D$
- $\sum_d \sum_p \delta_{epd} x_{dp} \leq M y_e$, $e = 1, 2, \ldots, E$
- $\sum_q z_{eq} = y_e$, $e = 1, 2, \ldots, E$
- $\sum_e \sum_q \gamma_{geq} z_{eq} \leq c_g$, $g = 1, 2, \ldots, G$. 
Extension to More than Two Layers

- Example: IP/MPLS/SONET
- Link at layer k+1 is demand for layer k
- Demand considered the top layer
- Joint dimensioning across all layers
- Generalized shortest path allocation rule
  - At layer k, allocate a layer k+1 demand (link $l^{k+1}$) to its cheapest path $p^k$
  - Set link weight at layer k+1 for $l^{k+1}$ using length of $p^k$ at layer k
  - Repeat until find the shortest paths for all demands
Extension: joint optimal routing and capacity design in upper layer

- **Routing:** given demands, link capacities, find the best flow allocation
- **Capacity allocation:** normally done in coarser time scale
- **Exception in wireless/sensor network**
  - no well-defined link capacity
  - links from same node share resource: spectrum, power, timeslot
  - link capacities be adjusted along with routing
- **Joint optimization of rate control, routing and resource allocation**
Multi-Layer Networks for Restoration Design

- Upon failures, path restorations can be done
  - in both upper and lower layers
  - low layer only
  - upper layer only
- Example: IP/SONET
  - upon failure: IP Re-routing/SONET reconfiguration
  - time-scale difference
  - transit loss of link capacity in IP layer
  - transit loss of packets for demands

FIGURE 12.5 Two-Layer Networks with Failures
Two-Layer Restoration Dimensioning with Unrestricted Flow Reconfiguration

- capacity dimensioning to handle all possible failure states
- arbitrary flow reconfiguration at both layers

indices
- \( d = 1, 2, \ldots, D \): demands
- \( p = 1, 2, \ldots, P_d \): candidate paths in upper layer for flows realizing demand \( d \)
- \( e = 1, 2, \ldots, E \): links of upper layer
- \( q = 1, 2, \ldots, Q_e \): candidate paths in lower layer for flows realizing link \( e \)
- \( g = 1, 2, \ldots, G \): links of lower layer
- \( v = 1, 2, \ldots, V \): nodes of upper layer
- \( s = 1, 2, \ldots, S \): failure-demand states (situations) (including the normal state)

constants
- \( h_d \): volume of demand \( d \)
- \( a_{ve} \): 1 if link \( e \) is incident with node \( v \); 0, otherwise
- \( \delta_{edp} \): 1 if link \( e \) of upper layer belongs to path \( p \) realizing demand \( d \); 0, otherwise
- \( \gamma_{geq} \): 1 if link \( g \) of lower layer belongs to path \( q \) realizing link \( e \) of upper layer; 0, otherwise
- \( \zeta_v \): unit cost of the capacity of node \( v \) of upper layer (termination cost and switching cost)
- \( \kappa_g \): unit cost of link \( g \) of lower layer
- \( \chi_{ds} \): demand coefficient of demand \( d \) in state \( s \), \( h_{ds} = \chi_{ds} h_d \)
- \( \beta_{vs} \): binary availability coefficient of node \( v \) of upper layer in state \( s \) (\( \beta_{vs} \in \{0, 1\} \))
- \( \alpha_{gs} \): fractional availability coefficient of link \( g \) of lower layer in state \( s \) (\( 0 \leq \alpha_{gs} \leq 1 \))
Two-Layer Restoration Dimensioning with Unrestricted Flow Reconfiguration

- **variables**  (all variables are continuous and non-negative)
  - $x_{dps}$ flow allocated to path $p$ of demand $d$ in state $s$
  - $Y_v$ capacity of node $v$ of upper layer (state-independent)
  - $yes$ capacity of link $e$ in state $s$ (situation-dependent)
  - $zeqs$ flow allocated to path $q$ realizing capacity of link $e$ in state $s$
  - $u_g$ capacity of link $g$

- **objective**
  - minimize $F = \sum_g \kappa_g u_g + \sum_v \zeta_v Y_v$

- **constraints**
  - $\sum_p x_{dps} = h_{ds}, \quad d = 1, 2, \ldots, D \quad s = 1, 2, \ldots, S$
  - $\sum_d \sum_p \delta_{edp} x_{dps} \leq y_{es}, \quad e = 1, 2, \ldots, E \quad s = 1, 2, \ldots, S$
  - $\sum_e a_{ve} y_{es} \leq \beta_{vs} Y_v, \quad v = 1, 2, \ldots, V \quad s = 1, 2, \ldots, S$
  - $\sum_q z_{eqs} = y_{es}, \quad e = 1, 2, \ldots, E \quad s = 1, 2, \ldots, S$
  - $\sum_e \sum_q \gamma_{geq} z_{eqs} \leq \alpha_{gs} u_g, \quad g = 1, 2, \ldots, G \quad s = 1, 2, \ldots, S$. 
Restoration Dimensioning with reconfiguration only at lower layer

- upper layer link capacities and flows required to be same under any failure state (no rerouting allowed)
- lower layer flow reconfigurable

**additional constants**

\[
H_d = \max \{ h_{ds}, \ s = 1, 2, \ldots, S \} \ \text{maximal value of demand } d
\]

**variables** (all variables are continuous and non-negative)

- \( x_{dp} \) flow allocated to path \( p \) of demand \( d \) in all states
- \( y_e \) capacity of link \( e \) in all states
- \( z_{eqs} \) flow allocated to path \( q \) realizing capacity of link \( e \) in state \( s \)
- \( u_g \) capacity of link \( g \)

**objective**

\[
\text{minimize } F = \sum_g \kappa_g u_g
\]  \hspace{1cm} (12.2.4a)

**constraints**

\[
\sum_p x_{dp} = H_d, \quad d = 1, 2, \ldots, D
\]  \hspace{1cm} (12.2.4b)

\[
\sum_d \sum_p \delta_{ep} x_{dp} \leq y_e, \quad e = 1, 2, \ldots, E
\]  \hspace{1cm} (12.2.4c)

\[
\sum_q z_{eqs} = y_e, \quad e = 1, 2, \ldots, E \quad s = 1, 2, \ldots, S
\]  \hspace{1cm} (12.2.4d)

\[
\sum_e \sum_q \gamma_{eqs} z_{eqs} \leq \alpha_{gs} u_g, \quad g = 1, 2, \ldots, G \quad s = 1, 2, \ldots, S.
\]  \hspace{1cm} (12.2.4e)
Restoration Dimensioning with reconfiguration only at upper layer

- lower layer flows are not reconfigurable, lower layer path may not available after failure
- upper layer link capacities affected by failure, and flows can be reconfigured arbitrarily

variables
- \( x_{dps} \) flow allocated to path \( p \) of demand \( d \) in state \( s \)
- \( y_{es} \) capacity of link \( e \) in state \( s \)
- \( z_{eq} \) flow allocated to path \( q \) realizing capacity of link \( e \) in all state
- \( u_g \) capacity of link \( g \)

objective

\[
\text{minimize } F = \sum_g \kappa_g u_g \quad (12.2.5a)
\]

constraints

\[
\sum_p x_{dps} = h_{ds}, \quad d = 1, 2, \ldots, D \quad s = 1, 2, \ldots, S \quad (12.2.5b)
\]

\[
\sum_d \sum_p \delta_{edp} x_{dps} \leq y_{es}, \quad e = 1, 2, \ldots, E \quad s = 1, 2, \ldots, S \quad (12.2.5c)
\]

\[
\sum_q \theta_{eqs} z_{eq} \geq y_{es}, \quad e = 1, 2, \ldots, E \quad s = 1, 2, \ldots, S \quad (12.2.5d)
\]

\[
\sum_e \sum_q \gamma_{geq} z_{eq} \leq u_g, \quad g = 1, 2, \ldots, G. \quad (12.2.5e)
\]
Extension: Overlay/P2P Networks

- **Overlay Networks**
  - logical networks on top of physical networks
  - improved end user performance
  - new services:
    - Content distribution: Akamai
    - p2p file sharing: BitTorrent, EMule
    - Streaming/multicast: Skype/IPTV

- **Overlay Network Design**
  - efficiency: topologies, routing, scheduling, rate control
  - interaction with native IP networks

- **Reference**:
  "On the Interaction Between Overlay Routing and Traffic Engineering'",
  http://eeweb.poly.edu/faculty/yongliu/docs/info05.pdf