EL-630: Probability Theory
Midterm Exam, Spring 2002.

(Answer all problems)

1. (a) Let $p$ represent the probability of an experiment $A$. The experiment is repeated $n$ times. How small should the probability $p$ be so that the probability that $A$ does not occur in any of the $n$ trials is greater than the probability that $A$ occurs at least once during the $n$ trials.

(b) A biased coin has a known probability $p$ of coming up heads. Someone flips the coin seven times and tells you that heads appeared in more than half the tosses. What is the conditional probability that heads appeared in all seven tosses?

2. (a) Evaluate $P(X > t_2 | X > t_1)$, $t_2 > t_1$ where $X$ is an exponential random variable with parameter $\lambda$.

(b) Let

$$P(X = k) = c k^{k-1}, \quad k = 1, 2, \ldots, \infty.$$ 

Find $c$ so that $P(X = k)$ above represents a probability mass function.

3. (a) If $Y = \sqrt{X}$, and $X$ is an exponential r.v, show that $Y$ represents a Rayleigh r.v.

(b) Let

$$f_X(x) = \frac{|x|}{\pi^2}, \quad -\pi < x < \pi.$$ 

and define

$$Y = \cos X$$

Determine $f_Y(y)$. 