EL-630: Probability Theory

Final Exam, December 20, 1999.

(Assign all problems)

1. (a) Show that for any random variable $X$, 
\[ E(X^4) \geq [E(X^2)]^2. \]
(b) Let $X$ and $Y$ be independent Poisson r.v.s with common parameter $\lambda$. Find $P(X + Y = k | Y = j)$.

2. The joint p.d.f of $X$ and $Y$ is given by 
\[ f_{XY}(x, y) = \begin{cases} 
6x, & x > 0, \ y > 0, \ 0 < x + y \leq 1, \\
0, & \text{otherwise}.
\end{cases} \]
Define 
\[ Z = X - Y. \]
(a) Find the p.d.f of $Z$.
(b) Find the conditional p.d.f of $Y$ given $X$.
(c) Determine $Var(X + Y)$.

3. Find the probability density function of $Z = X/Y$ if $X$ and $Y$ have the joint p.d.f 
\[ f_{XY}(x, y) = \begin{cases} 
xye^{-(x^2+y^2)/2}, & x > 0, \ y > 0, \\
0, & \text{otherwise}.
\end{cases} \]
4. Let $X$ and $Y$ be independent identically distributed exponential random variables with common parameter $\lambda$. Define

$$W = \frac{X}{\min(X, 2Y)}.$$ 

Find the p.d.f of $W$.  

\[ (20) \]

5. Let

$$f_{XY}(x, y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < \infty, \\ 0, & \text{otherwise}. \end{cases}$$

Define

$$Z = X + Y, \quad W = \frac{Y}{X}.$$ 

Determine the joint p.d.f of $Z$ and $W$. Are $Z$ and $W$ independent random variables?  

\[ (25) \]