Consider the following two experiments:

(a) A single die is thrown 6 times. Find the probability $p_1$ of obtaining face # 1 at least once.

(b) A single die is thrown 12 times. Find the probability $p_2$ of obtaining face # 1 at least twice.

1) Let $x_1$ be the number of times face # 1 shows in 6 trials.

$x_1 \sim \text{Binomial (n = 6, } p = \frac{1}{6})$

$p_1 = P(\text{"face 1 appears at least once"})$

$= P(x_1 \geq 1) = 1 - P(x_1 = 0) = 1 - \left(\frac{5}{6}\right)^6$

$= 1 - \left(\frac{5}{6}\right)^6 = 0.665$

2) Let $x_2$ be the number of times face # 1 shows in 12 trials.

$p_2 = P(\text{"face 1 shows up at least twice"})$

$= P(x_2 \geq 2) = 1 - P((x_2 = 0) \cup (x_2 = 1))$

$= 1 - P(x_2 = 0) - P(x_2 = 1)$

But $x_2 \sim \text{Binomial (n = 12, } p = \frac{1}{6})$

$p_2 = 1 - \left(\frac{11}{6}\right)^{12} - \left(\frac{11}{6}\right)^{11}\left(\frac{5}{6}\right)^1$

$= 1 - \left(\frac{5}{6}\right)^{12} - 12 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{11} = 0.6187$