NON-INTERLEAVED REED-SOLOMON CODING OVER A BURSTY CHANNEL

Ted Berman and Dr. Jeffrey Freedman

Stanford Telecom
7501 Forbes Blvd.
Seabrook, MD 20706

ABSTRACT. This paper presents a new technique which analytically calculates the word, symbol, and bit error rates of a Reed-Solomon (RS) coding system in a bursty (e.g., pulsed Radio Frequency Interference (RFI)) environment with Poisson pulse arrivals using a Markov Chain model to represent burst errors. It is faster than the standard simulation method of analysis for low error rates, since its runtime is independent of the desired bit error rate (BER) whereas the runtime of a standard simulation is inversely proportional to the desired BER.

The model is very flexible: it can analyze channels both with and without ideal (infinite) RS interleaving; it can analyze RS or shortened RS codes of any length, rate, or number of bits per symbol; and, most importantly, it can analyze a system which has a bursty channel characterized by bursts with a fixed length, and Poisson distributed arrival times.

The paper presents results for the commonly used RS (255,223) code, which guarantees correction of any 16 symbol errors.

I. INTRODUCTION

The correlation of symbol errors in adjacent symbols in a bursty channel makes analytic analysis difficult. The following analysis accomplishes this task through the use of a Markov Chain (MC) modeling technique.

Reed-Solomon (RS) codes are the most powerful block codes that are commonly used for satellite communications. The Consultative Committee for Space Data Systems (CCSDS) Recommendation for "Advanced Orbiting Systems" of the 1990's [1] specifies the use of a RS (255,223) code.

Previous studies analyzed RS codes in a non-bursty environment, in which signal bit errors are evenly distributed over the codewords [2,3]. However, many channel environments are bursty, where errors tend to arrive in groups. Significant performance degradation can result from this error correlation, but this model can be difficult to analyze accurately.

This study analyzes a bursty channel that may be a result of pulsed Radio Frequency Interference (RFI). Pulsed RFI often causes satellite communication systems to experience burst errors, and RS encoding, used in this analysis, is one possible mitigation technique.

Performance analysis models are generally broken up into two categories: simulation models and analytic models. Simulations often have the advantage of simplicity, but they have the disadvantage of a runtime which is inversely proportional to the BER calculated. This paper describes an analytic model which makes use of a MC to model burst error events in the RS codewords. In this analytic model, the runtime is independent of the BER.

The system modeled here includes a data source/data decoder, a RS encoder/decoder, an optional RS Interleaver/De-Interleaver, and a bursty channel. See Figure 1.1.

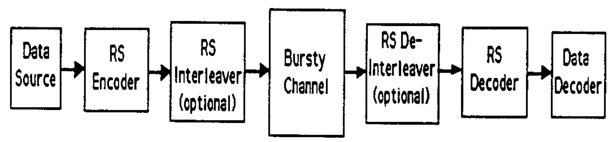

Fig. 1.1: A RS Coding System with a Bursty Channel

The analysis is summarized as follows: first a MC representing correlation between RS symbol errors is derived. From the MC, the probability that i symbol errors occur in a RS code word is calculated. From this, the word, symbol and bit error rates after RS decoding are found.

The paper is organized as follows: section II describes the channel model; section III describes the MC analysis; section IV describes the calculation of the error rates for RS codes; section V gives results for the RS (255,223) code; and section VI summarizes the capabilities of the model.

II. CHANNEL MODEL

The system is modeled as a binary symmetric channel (BSC). The crossover (bit error) probability of the BSC may take on two different values, corresponding to the two different states of the system.

Each bit can either be in the burst state (when the channel is hit by a burst) or the no burst state. The probability of bit error for the no burst state is P_{nb}, and the probability of bit error for the burst state is P_b. The analysis considers a bit to be in the burst state if any part of the bit is overlapped by any part of a burst. It makes the assumption that all burst hits are equally severe. It does not matter if a burst fully overlaps a bit, partially overlaps a bit, or if two or more bursts overlap a bit. The probability of error is the same.

Bursts of errors are assumed to arrive with a Poisson distribution. It is a basic property of a Poisson process that the probability of a burst arriving at any time is independent of the amount of time since the previous burst arrival, so two or more bursts may overlap in time. The Poisson distribution is defined as follows:

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$$P(\tau \leq t) = 1 - e^{-\lambda t}, t \geq 0 \quad (2.1)$$

where τ = random time between successive bursts (seconds)

λ = duty cycle / Δt

Δt = deterministic length of RFI pulses (seconds)
duty cycle = proportion of time system is in burst state
= proportion of time RFI is present

III. ANALYSIS

The objective of this section is to detail the analysis that is needed to compute $A_N(i)$, the probability that i symbol errors occur in a received RS codeword with N symbols. $A_N(i)$ is used to calculate the word, symbol, and bit error probabilities.

If symbol errors were randomly distributed, the analysis would be relatively simple, as $A_N(i)$ could be determined through a binomial distribution. For burst errors, an upper bound to $A_N(i)$ could be found by assuming that errors do not overlap. However, this may be not be a tight bound in some cases. The analysis here uses a MC to accurately calculate $A_N(i)$.

Section III is separated into four subsections: section A describes the development of the MC; section B analyzes the MC; section C calculates $A_N(i)$ for the ideal interleaving case; and section D calculates $A_N(i)$ for the no interleaving case.

A. The Markov Chain

The MC keeps track of the correlation of symbol errors. The correlation arises when pulsed RFI hits several RS symbols in a row. The state of the MC represents the number of RS symbols which remain to be hit (this includes the present symbol) by the most recently arriving RFI pulse.

The MC has L+1 states, where L is the maximum number of RS symbols which a single burst can hit. One state transition corresponds to the arrival of one RS symbol.

It is possible that a burst will only hit L-1 symbols, so two different types of bursts are defined. A Type A burst hits the maximum number of symbols (L) and a Type B burst hits one less than the maximum number of symbols (L-1).

The state of the MC corresponds to one more than the number of RS symbols which remain to be hit by the most recent burst arrival. For example, the state of the system is L if a Type A burst arrives during the present symbol. The state of the system is L-1 if a Type B burst arrives during the present symbol. The state is then reduced by one if the next symbol is not hit by another burst. It continues to be reduced after each symbol until it is either in the zero state or it is hit with another burst. An example, with L=3, is shown in Figure 3.1, and should clarify the relationship between burst arrivals and the state of the MC.

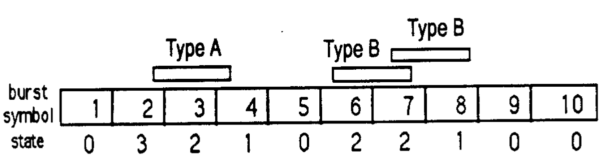

Fig. 3.1: The States of the Markov Chain

Notice that in the example, a Type A burst arrival results in a transition to state three, and a Type B burst arrival results in a transition to state two. Also, bursts can overlap one another, as happens in symbol seven.

The actual MC, including transition probabilities, is shown in Figure 3.2. The relevant parameters are L, α, β, and γ. These are defined in the following paragraphs.

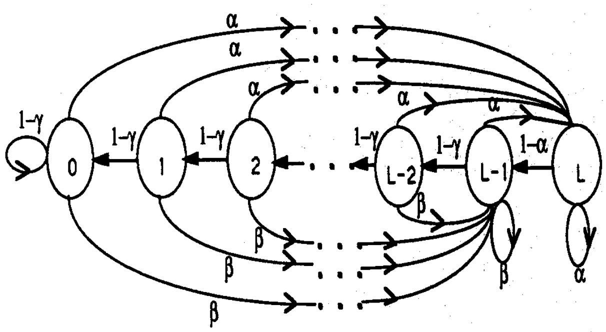

Fig. 3.2: The L State Markov Chain

The maximum number of RS symbols that a single burst can hit, L, is:

$$L = [\Delta t/S] + 1 \quad (3.1)$$

where [x] is the largest integer such that $[x] \leq x$
S = the length of a coded RS symbol (seconds)

The probability of having at least one burst arrival in any part of a coded RS data symbol, γ, (a data symbol is composed of several data bits) is:

$$\gamma = P(\tau \leq S) = 1 - e^{-\lambda S} \quad (3.2)$$

The probability that at least one Type A burst arrives is α and the probability that at least one Type B burst arrives is β. Note that $\gamma = \alpha + \beta$. Assuming that burst arrivals are uniformly distributed within a symbol, α and β are given by:

$$\alpha = 1 - e^{-\lambda S[1 + MOD(BL - 2,k)]/k} \quad (3.3)$$

$$\beta = (1 - e^{-\lambda S[k - 1 - MOD(BL - 2,k)]/k})e^{-\lambda S[1 + MOD(BL - 2,k)]/k} \quad (3.4)$$

$$= e^{-\lambda S[1 + MOD(BL - 2,k)]/k} - e^{-\lambda S}$$

where k = the number of bits per RS symbol
BL = the length of an RFI pulse in RS codeword bits

B. Analysis of the Markov Chain

$A_N(i)$ is calculated as a function of P_{us}, Q(i), and Q(j | i). P_{us} is the symbol error prior to decoding, Q(i), for i = 0,...,L, are the MC stationary probabilities and Q(j | i) is the probability that the previous state was j, given that the present state is i, for i,j =0,...,L.

$P_{us|i}$, for i = 0,...,L, is the probability of symbol error prior to decoding conditioned upon being in a certain state of the MC. P_{us} is the unconditioned probability of symbol error prior to decoding. These values are given by:

$$P_{us|0} = 1 - (1 - P_{nb})^k$$
$$P_{us|i} = 1 - (1 - P_b)^k, \; i = 1,...,L \tag{3.5}$$

$$P_{us} = \sum_{i=0}^{L} P_{us|i} Q(i) \tag{3.6}$$

The stationary (unconditional) probabilities of each state in the MC are as follows:

$$Q(L) = \alpha \tag{3.7}$$
$$Q(L-1) = \beta + \alpha(1 - \gamma)$$
$$Q(i) = Q(i + 1)(1 - \gamma), \; i = 1, ..., L-2$$
$$Q(0) = Q(1)(1 - \gamma)/ \gamma$$

The conditional probabilities are given below. The analysis calculates these values from the transition probabilities shown in Figure 3.2 and Equation (3.7).

$$Q(0 \mid 0) = Q(0) / [\, Q(0) + Q(1)\,] \tag{3.8}$$
$$Q(1 \mid 0) = Q(1) / [\, Q(0) + Q(1)\,]$$
$$Q(j \mid L) = Q(j), \; j = 0, ..., L$$
$$Q(L \mid L - 1) = (1 - \alpha)Q(L) / [\, \beta + \alpha(1 - \gamma)\,]$$
$$Q(j \mid L - 1) = \beta Q(j) / [\, \beta + \alpha(1 - \gamma)\,], \; j = 0, ..., L - 1$$
$$Q(i + 1 \mid i) = 1, \; i = 1, ..., L - 2$$
$$Q(j \mid i) = 0, \text{ otherwise}$$

C. Calculation of $A_N(i)$ for the Ideal Interleaving Case

$A_N(i)$ is the probability that i symbol errors occur in a RS codeword with N symbols. When ideal interleaving is used, the symbol errors are assumed to be independent and identically distributed. For a bursty channel, this will improve performance, as a single long burst or a small number of medium size bursts in a single codeword will not occur. Therefore $A_N(i)$ may be calculated using the standard Bernoulli distribution as follows:

$$A_N(i) = \binom{N}{i} P_{us}^i (1 - P_{us})^{N-i} \tag{3.9}$$

D. Calculation of $A_N(i)$ for the No Interleaving Case

The no interleaving case makes use of a recursion technique to calculate $A_N(i)$. We must calculate $A_t(i|j)$, the probability of i errors in the first t symbols of a codeword conditioned on the t'th symbol being in state j.

This is done recursively for t = 0,..., N. We have:

$$A_0(0|j) = 1, \; j = 1,...,L \tag{3.11}$$
$$A_0(i|j) = 0, \; i = 1,...,N; \; j = 1,...,L$$

$$A_t(i|j) = \sum_{h=1}^{L} \{\, P_{us|j} Q(h|j) A_{t-1}(i-1|h) + (1 - P_{us|j}) Q(h|j) A_{t-1}(i|h)\, \};$$

$$j = 1,...,L; \; i = 0,...,N; \; t = 1,...,N$$

The unconditional probability $A_N(i)$ is found through probabilistic summation as follows:

$$A_N(i) = \sum_{j=1}^{L} Q(j) A_N(i|j) \tag{3.12}$$

VI. PROBABILITY OF BIT ERROR AFTER RS DECODING

The probabilities of codeword error, symbol error, and bit error are functions of three values: $A_N(i)$, N, and l. The equations developed in this section are therefore valid for both the interleaved and the non-interleaved cases.

An (n,m) RS error correcting code can correct any l = (n-m)/2 symbol errors which occur in a codeword [4]. A symbol error occurs if one or more bits in a symbol are in error. Correction is not guaranteed if more than l symbol errors occur.

The probability of a RS word error, P_w, is equal to the probability that more than l symbol errors occur:

$$P_w = \sum_{i=l+1}^{N} A_N(i) \tag{4.1}$$

The probability of a RS symbol error, P_s, is equal to the weighted average of the proportion of symbol errors which occur:

$$P_s = \frac{1}{N} \sum_{i=l+1}^{N} i A_N(i) \tag{4.2}$$

The probability of a RS bit error, P_e, is equal to half of P_s, since on average half the bits in an erroneous symbol are in error.

$$P_e = P_s / 2 \tag{4.3}$$

V. RESULTS

This section presents results for a RS (255,223) code, which has eight bits per symbol and can correct any 16 of 255 symbols. Each run is a function of four values: P_b, P_{nb}, the duty cycle (the proportion of time that date is hit by RFI pulses), and the burst length (BL). The value of P_b is .5 for all runs, corresponding to the case of "infinite" noise. There is a one-to-one relationship between P_{nb} and E_b/N_0 for a BSC with Gaussian noise and an antipodal signal set. Results are given here in terms of E_b/N_0, since this is generally useful for communications applications. Results are given for a channel both with and without ideal RS symbol interleaving.

Figure 5.1 plots the BER vs. E_b/N_0, with an interference duty cycle of .005. Note that when there is no interleaving, a larger burst length corresponds to a greater BER, since bursts of symbol errors tend to occur in a single codeword.

When there is interleaving, a larger burst length corresponds to a smaller BER, as shown in figure 5.1. This effect occurs because the average number of bits which are in error remains constant for any burst length, so longer bursts result in a larger number of bit errors per symbol and a smaller number of symbols in error.

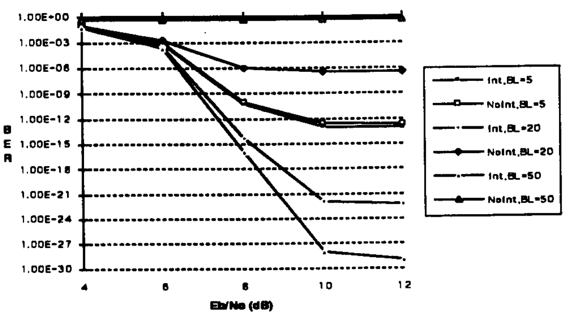

Fig. 5.1: RS BER vs. Eb/No; dc = .005, BL Measured in RS Coded Bits

Interleaving makes almost no difference when the burst length is short relative to the error correction capability of the code (BL=5), but results in a large improvement when the burst length is long (BL=20,50).

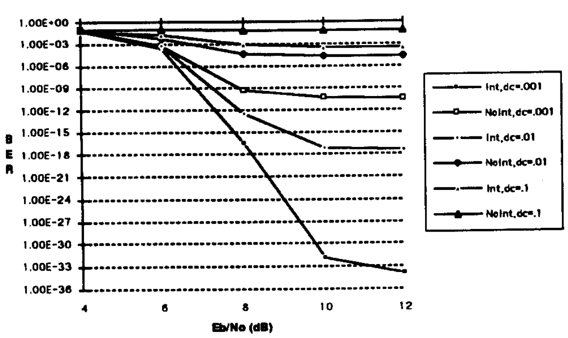

Fig. 5.2: RS BER vs. Eb/No; BL = 20 RS Coded Bits

Figure 5.2 plots the BER vs. the channel E_b/N_o, with a burst length of 20. Interleaving results in an improvement for all duty cycles since the burst length is relatively long. Also, the greater the duty cycle, the greater the BER, as expected.

Figure 5.3 plots the BER vs. BL, with a channel E_b/N_o of 8.0 dB. Interleaving always results in an improvement, and larger duty cycles have a higher BER, as in the other graphs. Longer bursts decrease the BER, as before. However, when there is no interleaving, decreasing the burst length reduces the BER (as before) until the BER reaches a minimum, and then the BER increases. This is because bursts are either L or L-1 symbols long and with shorter bursts there are more opportunities for L length bursts and more symbols will be effected.

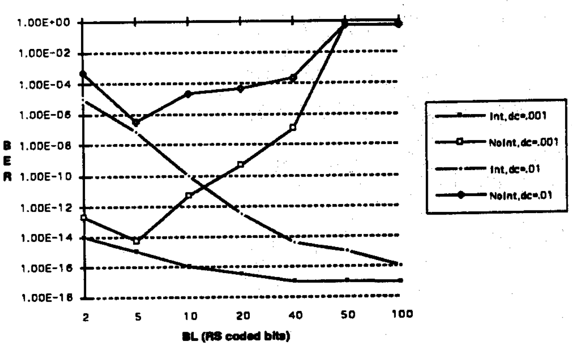

Fig. 5.3: RS BER vs. BL; Eb/No = 8.0 dB

VI. CONCLUSION

This paper presents an analytical algorithm which can calculate the BER of a RS coding system in a burst error channel using a Markov Chain model for burst events. This algorithm is very efficient, as the BER can be calculated in a few seconds on an HP 9000 series 735 workstation computer. It is more efficient than a simulation, as the run time is independent of the BER, and it is more flexible than previous analytic models, in that it can calculate the BER for the no interleaving case without assuming infinite noise during a burst and/or no noise at other times. The model confirms that RS interleaving often results in a significant improvement in the BER and that a non-interleaved RS decoder working in a bursty environment is most effective in correcting moderate length bursts.

ACKNOWLEDGMENT

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