

Beamformer Design with Feedback Rate Constraints : Criteria and Constructions

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I. INTRODUCTION

In slow block fading channels, channel information can be obtained at the transmitter using a low delay feedback loop. In such cases, even with a few bits of channel information, feedback based transmission schemes offer substantial performance gains over channel agnostic systems with multiple transmit antennas [1, 2]. In this paper, we study the design of beamformers when finite rate channel feedback is available in a multiple transmit antenna system.

In this work, we provide a geometrical framework for the analysis and design of beamformer codebooks with finite number of beamformer vectors. We present a design criterion for good beamformer codebooks and show the equivalence of the beamformer design problem to two other known problems. First, the beamformer design problem can be directly posed as a problem of packing 2 dimensional subspaces in a $2t$ dimensional Grassmannian manifold, t being the number of transmit antennas. And second, under certain conditions, the beamformer design problem is equivalent to the construction of unitary space time constellations. Further, in [3], we derived a lower bound on outage performance of beamforming with finite number of beamforming vectors. Our simulations demonstrate that the design criterion results in constructions which lead to outage performance very close to the lower bound derived in [3].

II. PROBLEM FORMULATION AND RESULTS

We consider a wireless system where the transmitter, equipped with t transmit antennas, communicates with a single antenna receiver. The channel fading is assumed to be quasi-static in time with the channel gains across the antenna elements being statistically independent. The $1 \times t$ channel vector is given by $\mathbf{h} = [h_1, h_2, \dots, h_t]$, where for each i , h_i is assumed to be circularly symmetric complex Gaussian random variable with zero mean and unit variance per complex dimension. We assume that perfect channel state information is available at the receiver. We also assume the existence of an error-free feedback channel of capacity B bits/frame. The channel information at the transmitter comprises of the channel statistics and B bits of quantized information about the channel realization available via the feedback channel. The transmitter chooses a beamforming vector from a codebook \mathcal{C} based on the feedback information. Hence, corresponding to B feedback bits, we have a beamformer codebook of size $N = 2^B$, given by $\{C_1, C_2, \dots, C_N\}$, where $C_i \in \mathbb{C}^t$ and $\|C_i\|_2 = 1$, for each i . For a given number of transmit antennas t and feedback bits B , we seek to construct beamformer codebooks which minimize the outage probability.

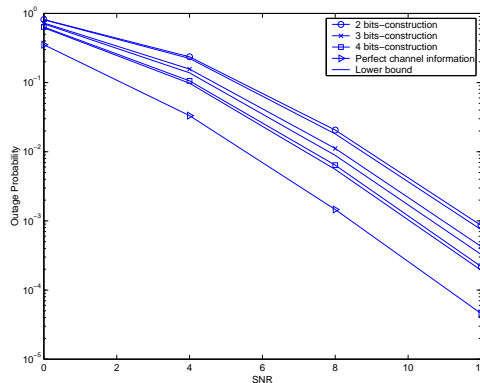


Figure 1: Performance of beamformers constructed for 4 transmit antennas transmitting at 2 bits/sec/Hz.

We characterize each codebook \mathcal{C} by the quantity $\gamma^*(\mathcal{C})$, referred to as *overlap radius*. The overlap radius can be computed as a function of the constituent vectors of \mathcal{C} as given in the following lemma.

Lemma 1 (Overlap radius [3]) For any beamformer codebook \mathcal{C} , we have

$$\gamma^*(\mathcal{C}) = \left(\frac{2\gamma_0}{1 + \max_{i,j:i \neq j} | \langle C_i, C_j \rangle |} \right) \quad (1)$$

In particular, it can be shown that larger overlap radius results in lower outage probability. Hence, the design criterion for good beamformers is to maximize the overlap radius. In the light of Lemma 1, the design criterion for a beamformer codebook \mathcal{C} comprising of N beamforming vectors for t transmit antennas is equivalently given by

$$\min_{\substack{\mathbf{c} \in \underbrace{\mathbb{C}^t \times \dots \times \mathbb{C}^t}_N}} \max_{\substack{i,j: \\ 1 \leq i \neq j \leq N}} | \langle C_i, C_j \rangle |. \quad (2)$$

The performance of the beamformers constructed for 4 transmit antennas is compared with the outage lower bound derived in [3]. Further results and discussion can be found in [3].

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